

Influence of Obstacle Section Shape on Electro-thermo-convection

Mingdai Yang¹, Xiaolong Leng¹, Waqas Ul Arifeen¹, P.Rosaiah¹ and Tae Jo Ko^{1,#}

¹ School of Mechanical Engineering, Yeungnam University, 280, Daehak-ro, Gyeongsan-si, Gyeongsangbuk-do, 712-749, Republic of Korea
Corresponding Author / Email: tjko@yu.ac.kr, TEL: +82-053-810-3836

KEYWORDS: Electro-thermo-convection, Multi-physical coupling, Lattice Boltzmann model

In this work, the Lattice Boltzmann method is used to simulate the complex transport phenomenon of coupled heat transfer under electrothermal convection. First, four LBM equations of electric potential, charge density, flow field and temperature field in unified form are established. Secondly, the correctness of the model is confirmed by comparing the numerical results of the model's electrostatics with the analytical solution results. Finally, the influence of the cross-sectional shape of the obstacle on the electrothermal convection of the low-density dielectric liquid is analyzed. The simulation results show that for the fluid field, temperature field and charge field, the shape of the obstacle section has little effect. There is a clear difference in the electric field strength. The range and magnitude of the electric field intensity distribution of the circular section are obviously higher than those of the irregular circular section.

NOMENCLATURE

u = fluid velocity	p = corrected pressure
g = gravitational acceleration	
ν = liquid kinematic viscosity	
E = electric field strength	
β = liquid Thermal Expansion Coefficient	
q = charge density	ϵ = dielectric constant
θ = temperature	K = ion mobility
ϕ = electric potential	
D = charge diffusion coefficient	
Ra = Thermal Rayleigh number	
T = Electric Rayleigh number	
C = charge injection strength	
M = dimensionless ion mobility	
α = dimensionless charge diffusion coefficient	

1. Introduction

Electro-thermo-convection (ETC) refers to the process of coupled heat transfer in electrical convection. Electric convection describes the motion of charged particles and their interaction with electric fields and surrounding flow fields. In this context, the charging of the particles results from the injection of electric charges in the dielectric

liquid. Then, under the action of an electric field in the dielectric liquid, the charged particles in the electric field are subjected to the Coulomb force. At the macroscopic scale, this force drives fluid flow.

When the temperature field is coupled in the process of electric convection, there is a temperature difference in the fluid, and the temperature gradient causes the density of the fluid to change to generate a buoyant force. This paper takes a low-density fluid as the research object. Due to the low density, the effect of buoyancy is small and can be ignored. This paper discusses the influence of the cross-sectional shape of the obstacle on the electrothermal convection in this case.

The electrothermal convection problem involves complex multiphysics coupling process and the solution of strongly nonlinear equations. The traditional numerical solution of fluid mechanics is difficult to describe the essence of such cross-scale problems, and at the same time, the amount of calculation is too large and the calculation efficiency is low.

Lattice Boltzmann Method (LBM), as a new numerical solution algorithm, uses the mesoscopic dynamic model to simulate complex transport phenomena. It has the advantages of simple programming and calculation, easy to deal with complex boundaries and parallel operations. Suitable for complex shapes and parallel computing and many other advantages.

This paper mainly refers to the work of literature [3], [5] on electrothermal convection, and establishes four unified LB equations to solve the electric potential, charge density, flow field and

temperature field. The influence of the cross-sectional shape of the obstacle on the electrothermal convection of the low-density dielectric liquid is discussed.

2. Physical Model and Governing Equations

As shown in the figure, plate electrodes with high potential Φ_1 , high temperature θ_1 and low potential Φ_0 and low temperature θ_0 are placed on the left and right walls of the cavity respectively. The upper and lower walls are insulated and insulated, and free charges are injected uniformly from the left electrode plate. The square cavity can be described by the continuity equation, electric field definition equation, charge conservation equation and energy equation [1, 2]:

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) = -\nabla p + \nabla \cdot \{v[\nabla \mathbf{u} + (\nabla \mathbf{u})^T]\} + q\mathbf{E} + \mathbf{g}[1 - \beta(\theta - \theta_{\text{ref}})] \quad (1)$$

$$\nabla^2 \phi = -\frac{q}{\epsilon} \quad (2)$$

$$\mathbf{E} = -\nabla \phi \quad (3)$$

$$\frac{\partial q}{\partial t} + \nabla \cdot [q(\mathbf{KE} + \mathbf{u})] = \nabla \cdot (D\nabla q) \quad (4)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (5)$$

According to [3] the system can be essentially controlled by the following dimensionless parameters:

$$Ra = \frac{g\beta(\theta_h - \theta_c)\delta^3}{\nu\chi}, T = \frac{\epsilon(\phi_1 - \phi_0)}{\mu K}$$

$$C = \frac{q_1\delta^2}{\epsilon(\phi_1 - \phi_0)}, M = \frac{1}{K}\left(\frac{\epsilon}{\rho}\right)^{\frac{1}{2}}$$

$$\alpha = \frac{D}{K(\phi_1 - \phi_0)} \quad (6)$$

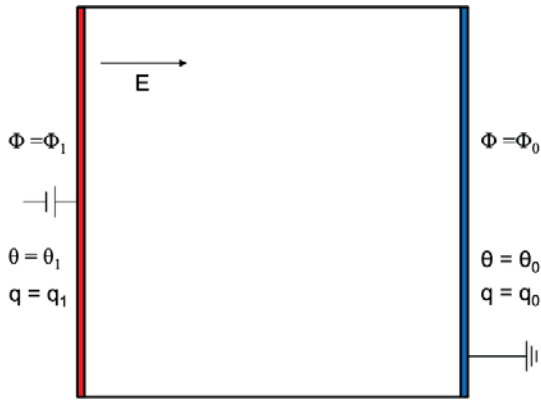


Fig. 1 Schematic diagram of the physical model.

Ra and T are thermal Rayleigh number and electric Rayleigh number, respectively, which are used to express the ratio of buoyancy force and Coulomb force to viscous force; C is the charge injection strength. The more; M represents the dimensionless ion mobility, the value of which is determined by the physical parameters of the ion and fluid. In this paper, Ra=0 is set.

3. Numerical Methods

To achieve the simulation, four unified LB equations are established to solve the electric potential^[4], the charge density^[5], the flow field^[6], and the temperature field^[7,8].

3.1 Lattice Boltzmann Model of Charge Density

The charge density adopts the D2Q9 model, and its equilibrium distribution function, relaxation time τ_q , the relationship of charge diffusion coefficient and the solution of charge density are shown in formulas (7), (8), (9), (10) respectively:

$$h_i(\mathbf{x} + \mathbf{c}_i\Delta t, t + \Delta t) - h_i(\mathbf{x}, t) = -\frac{1}{\tau_q} [h_i(\mathbf{x}, t) - h_i^{(\text{eq})}(\mathbf{x}, t)] \quad (7)$$

$$h_i^{(\text{eq})}(\mathbf{x}, t) = q\omega_i \left\{ 1 + \frac{\mathbf{c}_i \cdot (\mathbf{KE} + \mathbf{u})}{c_s^2} + \frac{[\mathbf{c}_i \cdot (\mathbf{KE} + \mathbf{u})]^2 - c_s^2 |\mathbf{KE} + \mathbf{u}|^2}{2c_s^4} \right\} \quad (8)$$

$$D = c_s^2(\tau_q - 0.5)\Delta t \quad (9)$$

$$q = \sum_i h_i \quad (10)$$

3.2 Lattice Boltzmann Model of Charge Density

The potential equation adopts the D2Q5 model, and its equilibrium distribution function, the relationship between the relaxation time τ_q and the charge diffusion coefficient, and the solution of the potential are shown in formulas (11), (12), (13), (14) respectively:

$$g_i(\mathbf{x} + \mathbf{c}_i\Delta t, t' + \Delta t) - g_i(\mathbf{x}, t') = -\frac{1}{\tau_\phi} [g_i(\mathbf{x}, t') - g_i^{(\text{eq})}(\mathbf{x}, t')] + \Delta t \hat{\omega}_i \xi R \quad (11)$$

$$g_i^{(\text{eq})}(\mathbf{x}, t) = \begin{cases} (\tilde{\omega}_0 - 1)\phi, & i = 0 \\ \tilde{\omega}_i \phi, & i = 1, 2, 3, 4 \end{cases} \quad (12)$$

$$\tau_g = 0.5 + \zeta / (\tilde{c}_s^2 \Delta t) \quad (13)$$

$$\mathbf{E} = \frac{1}{\tau_g \tilde{c}_s^2} \sum_i c_i g_i \quad (14)$$

4. Results and discussion

To verify the accuracy and rationality of the model. In this paper, the numerical simulation of electrostatics is carried out at first. When C=10, after the system is stable, the numerical results of the charge density and the distribution of the electric field along the horizontal line of the center of the square cavity are consistent with the analytical solutions in [5]. On this basis, we analyze the influence of the cross-sectional shape of the obstacle on the electrothermal convection of the low-density dielectric liquid.

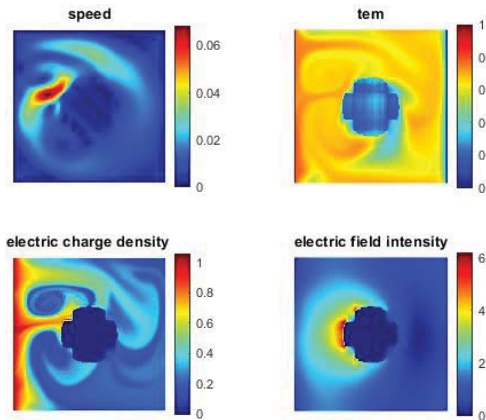


Fig. 2 cross section of obstacle is circular.

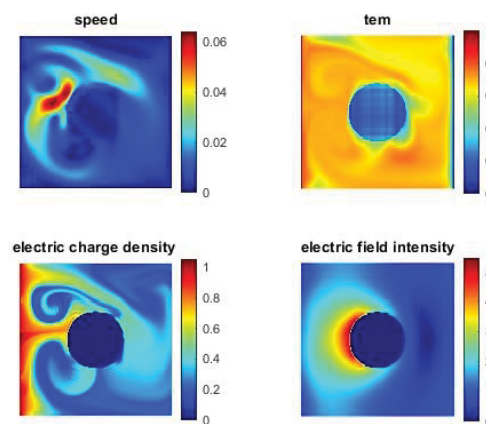


Fig. 3 cross section of obstacle is irregular circular.

For the fluid field, temperature field and charge field, the cross-sectional shape of the obstacle has little effect. There is a clear difference in the electric field strength. The range and magnitude of the electric field intensity distribution of the circular section are obviously higher than those of the irregular circular section.

ACKNOWLEDGEMENT

This study was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) and funded by the Ministry of Science, ICT, and Future Planning [Grant Number NRF- 2020R1A4A1019227].

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