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Bas-relief modeling combining multiple range data based on Poisson equation

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Reliefs are widely used as decorations for various things as medals, buildings and porcelains. If we can generate a bus-relief that represents multiple objects overlaid, the range of motifs that can be expressed on reliefs will be expanded. In this work, we propose a novel bas-relief modeling algorithm from two range data each of which represents the base and foreground pattern of a relief. The algorithm solves Poisson equation to conform the gradient of the relief height function to the gradient of the foreground pattern. Empirically it was shown that the algorithm can generate bas-relief models with foreground patterns not affected by the base features.

1. Introduction

A relief is a 2.5D sculpture in which a foreground pattern is added to a base as a flat or curved surface. Reliefs are widely used for decorating various things as medals, coins, buildings and porcelains. There are two types of reliefs: high-relief with undercuts that are elements detached from the relief base and bas-relief without undercuts. Fig. 1 shows the difference between a bas-relief and a high-relief.

In recent years, with the pervading additive manufacturing technology, relief production using 3D printers has been attracting attention. 3D printers have the ability to automate relief fabrication of arbitrary shapes. Generally, this process takes as input 3D models of reliefs which were created by mature designers.

Previous studies that generate 3D models of relief can be divided into four main categories based on the types of input and output. The two types of input are images and 3D models, and the two types of output are bas-relief and high-relief. Weyrich et al. [1] proposed a method to generate bas-reliefs from 3D models, and Arpa et al. [2] proposed to generate high-reliefs from 3D models. Ji et al. [3] develop an algorithm to reduce the relief thickness while preserving detailed shape features, using the height map of existing bas-reliefs as input. Most of the previous studies have proposed algorithms that take a single image or 3D model as input and generate a bas-relief or high-relief. If it is possible to generate a relief that combines multiple 3D models, it will expand the range of motifs that can be expressed on reliefs. For these reasons, in this work, we propose a novel bas-relief modeling algorithm from two range data each of which represents the base and the foreground pattern of a relief.

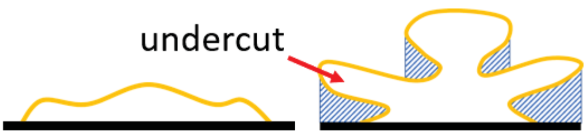

Fig. 1 Cross sections of bas-relief (left) and high-relief (right).

2. Algorithm

2.1 Overview

The input for this work is two range data, one for the base and the other for the foreground pattern of the bas-relief. Range data is an image whose pixel value means the distance from a view point or a certain plane. The output is a 2.5D surface mesh $\{(x, y, h(x, y))\}$ of a bas-relief. The flow of the proposed algorithm is as follows.

1. Initialize the height $h(x, y)$ of the relief vertices with the pixel values of the base range data.
2. Calculate the gradient $\boldsymbol{v}(x, y)$ of the pixel values of the range data of the foreground pattern.
3. In the foreground pattern area Ω, modify $h(x, y)$ to match its gradient $\nabla h(x, y)$ to $\boldsymbol{v}(x, y)$.
4. Repeat the update in Step 3 until the amount of change of $h(x, y)$ becomes small enough.

Fig. 2 shows the input and the result of each step of the algorithm.

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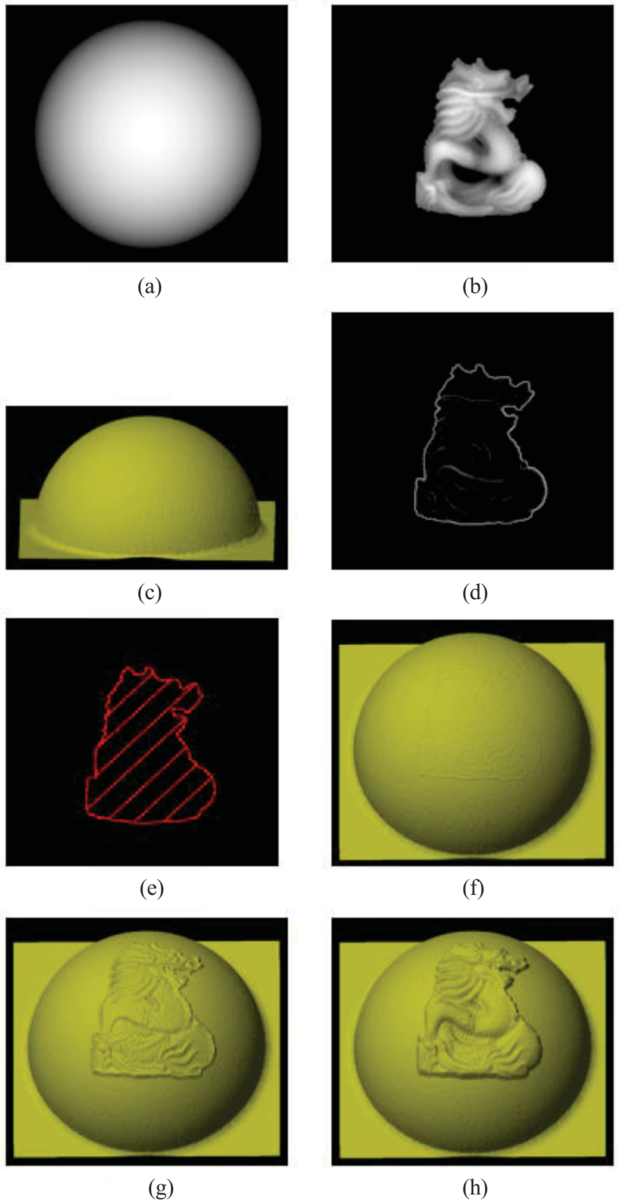

Fig. 2 (a) Input range data of base, (b) input range data of foreground pattern (image courtesy of [1]), (c) initialized surface mesh (Step1), (d) gradient $\boldsymbol{v}(\boldsymbol{x},\boldsymbol{y})$ of foreground pattern (Step2), (e) area $\boldsymbol{\Omega}$, (f)-(h) after 5, 100, and 30,000 iterations of Step 3.

Fig. 2(a) and (b) show the input range data for the base and foreground patterns of a hemisphere and a dragon respectively. Fig. 2(c) shows the hemisphere of the base formed in Step 1 of the algorithm. Fig. 2(d) is the gradient of the dragon which is the foreground pattern obtained in Step 2. Fig. 2(e) indicates the region Ω for Step 3. Fig. 2(f), (g) and (h) show how the vertices in Ω are repeatedly moved in Step 3, in which the foreground pattern is gradually reproduced on the base.

2.2 Method of determining the position of relief vertices by Poisson equation

In order to reproduce the foreground pattern on the base, the proposed algorithm matches the gradient of the relief height $\nabla h(x,y)$ to the gradient of the pixel value of the foreground pattern $\boldsymbol{v}(x,y)$ in Step 3. Fig. 3 shows the concept of matching $\nabla h(x,y)$ and $\boldsymbol{v}(x,y)$. When $\nabla h(x,y)$ is equal to $\boldsymbol{v}(x,y)$ in Ω, equation (1) holds.

$$\nabla h(x,y) = \boldsymbol{v}(x,y) \text{ over } \Omega \#(1)$$

Ideally, equation (1) should be satisfied for all (x,y) in Ω. The optimal solution of $h(x,y)$ is obtained by solving the following minimization problem (2) [4].

$$\iint_{\Omega} \|\nabla h(x,y) - \boldsymbol{v}(x,y)\|^2 \, dxdy \to \min \#(2)$$
$$\text{s.t. } h(x,y)|_{\partial\Omega} = I_b(x,y)|_{\partial\Omega}$$

Here $\partial\Omega$ is the boundary of Ω, and $I_b(x,y)$ is the pixel value of the base range data. The minimization problem (2) means minimizing the difference between $\nabla h(x,y)$ and $\boldsymbol{v}(x,y)$ for all the points in the region Ω.

We Solve this minimization problem as Poisson Image Editing [4]. The divergence of the both sides of equation (1) gives the Poisson equation:

$$\Delta h(x,y) = \operatorname{div} \boldsymbol{v}(x,y) \text{ over } \Omega. \#(3)$$

To satisfy this equation, the height h_p of the pixel of interest p in the region Ω is iteratively updated by the update equation (4).

$$h_p \leftarrow \frac{1}{4}\left(\sum_{q \in N_p} h_q + \sum_{q \in N_p} v_{pq}\right) \#(4)$$
$$v_{pq} = (g_p - g_q)/s$$

In the equation (4), N_p is the set of pixels in the 4 neighborhoods of p, q is an element of N_p, g_p is the pixel value of foreground pattern at pixel p, v_{pq} is the component of $v(x,y)$ corresponding to the direction of the vector pq, and s is the edge length of a pixel. In the experiments, v_{pq} is multiplied by $\lambda\,(\lambda < 1)$ to mitigate abrupt height changes.

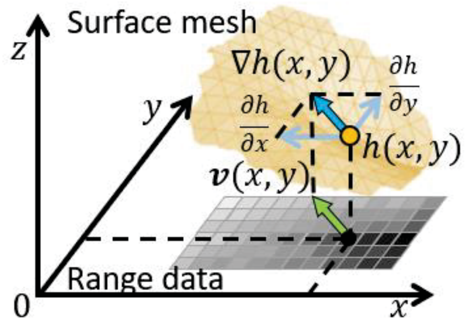

Fig. 3 Concept of matching the gradient of the height function $\nabla \boldsymbol{h}(\boldsymbol{x},\boldsymbol{y})$ to the gradient of the foreground pattern $\boldsymbol{v}(\boldsymbol{x},\boldsymbol{y})$.

3. Results

Our implementation is based on C++. The computer environment is configured as follows: Windows 10 Home 64-bit, 3.9 GHz AMD Ryzen 7 3800XT CPU, and 32GB main memory.

Fig. 4 shows the results of an experiment using range data of a hemisphere and dragon as input. The resolution is 675 × 615 and the number of pixels in Ω is 87,623. Fig. 4(c) and (d) show that the dragon shape is well reproduced on the hemisphere.

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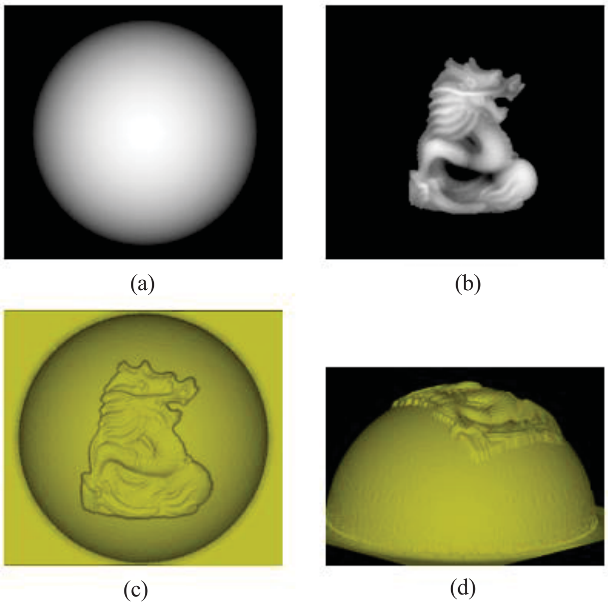

Fig. 4 Result for hemisphere and dragon, (a),(b) input range data, (c),(d) result relief.

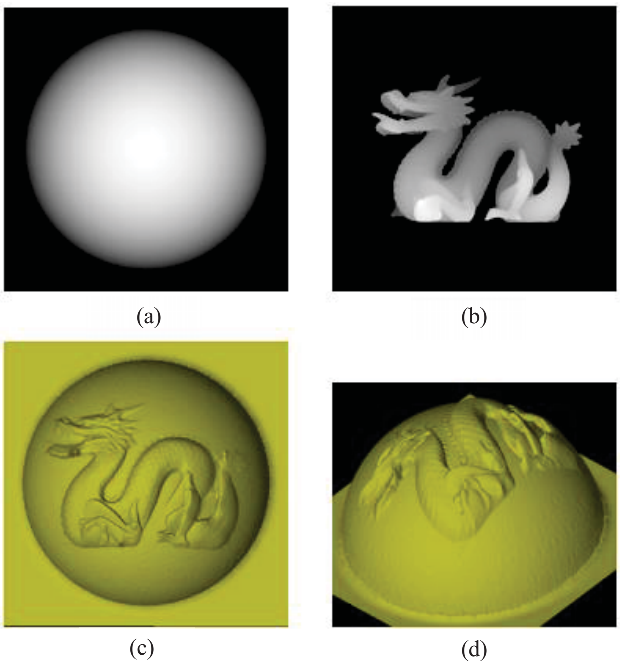

Fig. 5 Result of hemisphere and Stanford dragon, (a),(b) input range data, (c),(d) result relief.

Fig. 5 shows a result for range data generated from a surface mesh of the Stanford dragon. The resolution is 700×700 and the number of pixels in Ω is 116,588. In this experiment, we applied median filter to $\boldsymbol{v}(x,y)$ to suppress the values of $\boldsymbol{v}(x,y)$ calculated in Step 2.

Fig. 6 shows how the foreground pattern is preserved by our algorithm. The foreground range data was generated from a surface mesh of the Stanford bunny. Fig. 6(c) and (d) show that the foreground pattern is not affected by the base tier.

Fig. 7 shows the experimental results of house and statue. The base and foreground Images courtesy of [5] and [6]. The resolution is 700×700 and the number of pixels in Ω is 19,834. In this experiment, we applied median filter to unsharp masking filter to $\boldsymbol{v}(x,y)$ to emphasize the foreground pattern.

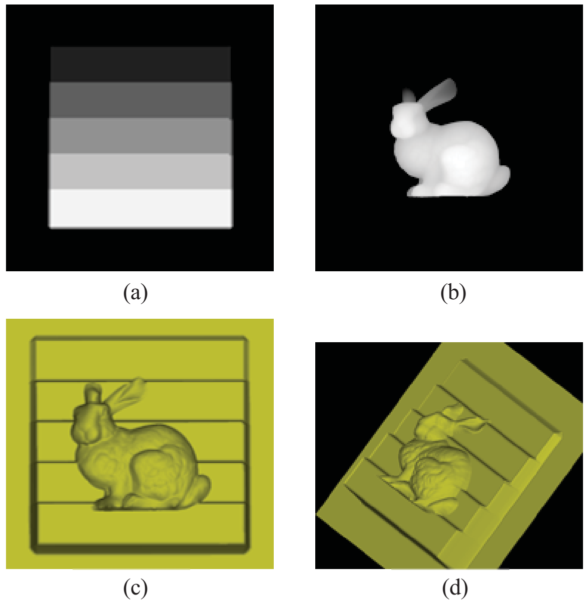

Fig. 6 Result of staircase and Stanford bunny, (a),(b) input range data, (c),(d) result relief.

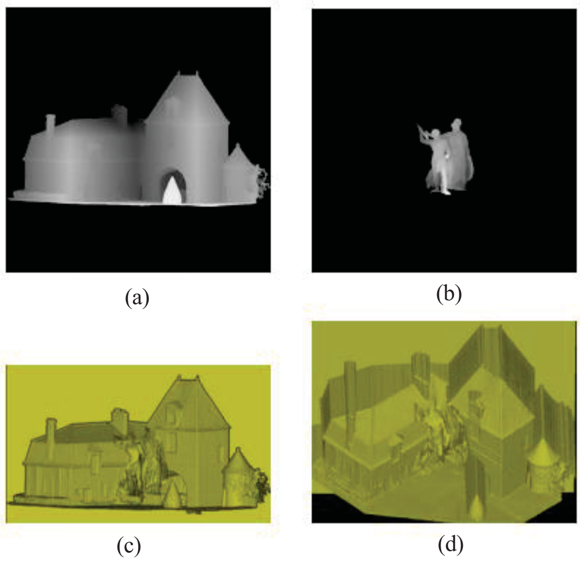

Fig. 7 Result of house and statue, (a),(b) input range data, (c),(d) result relief.

4. Conclusions

In this paper, we proposed an algorithm to generate a bas-relief surface mesh from two range data, one for the base and the other for the foreground pattern. Solving the Poisson equation which matches the gradient of the relief height to the gradient of the range data of the foreground pattern, enables to generate a bas-relief with the

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foreground superimposed on the base without losing the features of the foreground pattern.

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