

Inverse Kinematics Selection Algorithm for 6-DOF Manipulator Based on Neural Network to Pass Through the Singular Point

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The transformation of a robot's end-effector trajectory into each joint angle command is an inverse kinematics problem, which is the most fundamental problem in robot control systems under an offline teaching. This study developed a neural network-based algorithm to avoid the rapid rotation of some joints when a six-degree-of-freedom manipulator passes through the vicinity of a singular point, which is one of the most challenging problems in achieving offline teaching technology in industrial robotics. We defined the vicinity of a singular point as the extent when the speed of the servomotor is more than the maximum rotation speed and constructed the inverse kinematics model using an artificial neural network (ANN). Conventional methods can only solve for continuous functions, and ANN intentionally defines a discontinuous function by limiting the training data to suppress rapid rotations. The results show that the proposed method enables a manipulator to pass through the vicinity of the singular points without decreasing trajectory and velocity accuracy.

NOMENCLATURE

\mathbf{p} = position and posture vectors of the robot's end-effector
 $\boldsymbol{\theta}$ = vector related with each joint angle of the robot
 \mathbf{J} = Jacobi matrix of partial derivatives of the robot's position and posture at each joint angle
 \mathbf{R} = rotation matrix between each coordinate system configured for each link

1. Introduction

General industrial robots are taught using the teaching playback method, in which a dedicated controller memorizes tasks while the robot is operated, and the same task is repeated. This can only be performed by skilled operators and is time-consuming. To solve this problem, offline teaching, in which the trajectory of a robot's end-effector is created in the cyber world, has been introduced. In this study, a neural network-based algorithm was developed to avoid the rapid rotation of some joints when a six-degree-of-freedom (6-DOF) manipulator passes through the vicinity of a singular point, which is one of the most challenging problems in achieving offline teaching technology.

To design the trajectory of a robot's end-effector, the trajectory $\mathbf{p}(t)$ needs to be transformed into joint angles $\boldsymbol{\theta}(t)$ at that time. This is an inverse kinematics problem. Analytically solving the inverse kinematics of a 6-DOF manipulator is not possible. Therefore, it is solved numerically using the Jacobi matrices \mathbf{J} as shown in Eq. (1).

$$\delta \mathbf{p} = \mathbf{J}(\boldsymbol{\theta}) \delta \boldsymbol{\theta} \quad (1)$$

The condition for solving the inverse kinematics is that the Jacobi matrix is regular, that is, $\det \mathbf{J}(\boldsymbol{\theta}) \neq 0$. A singular point is defined as $\boldsymbol{\theta}$, where $\det \mathbf{J}(\boldsymbol{\theta}) = 0$. Equation (1) can be transformed into Eq. (2), such that at a singular point, the displacement of the joint angle is large for a small displacement of the end effector ⁽¹⁾.

$$\delta \boldsymbol{\theta} = \mathbf{J}^{-1}(\boldsymbol{\theta}) \delta \mathbf{p} \quad (2)$$

At a singular point, the robot is unstable owing to the rapid rotation of some joints, and the trajectory accuracy is reduced. Therefore, various algorithms have been proposed to enable a manipulator to pass through singular points, such as the singularity low-sensitive motion resolution (LM) ^{(2), (3)} and singularity consistent (SC) ^{(4), (5)}. These algorithms were particularly developed for cases in which the manipulators pass through a singular point, and thus do not consider problems that occur when passing through the vicinity of the singular

point. When a manipulator passes through the vicinity of the singular point, some joints must rotate by approximately 180° in a short period ⁽⁶⁾. In this study, we focused on this issue to achieve a novel motion control using an artificial neural network.

2. Experimental Method

A large 6-DOF industrial robot, SRA166-01 manufactured by NACHI-FUJIKOSHI CORP., was used in this study, as shown in Fig. 1.

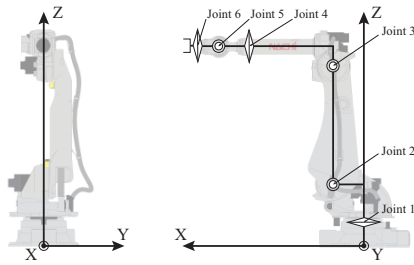


Fig. 1 Model of a 6-DOF manipulator

The 6-DOF manipulator had three singular postures. This study focused on the wrist singular posture in which the angle of Joint 5 was 0° . The robot was instructed to move its end-effector in a linear trajectory from A to B, as shown in Fig. 2. The trajectory was set to pass 10 mm below a singular point; hence, it has to pass through the vicinity of the singular point.

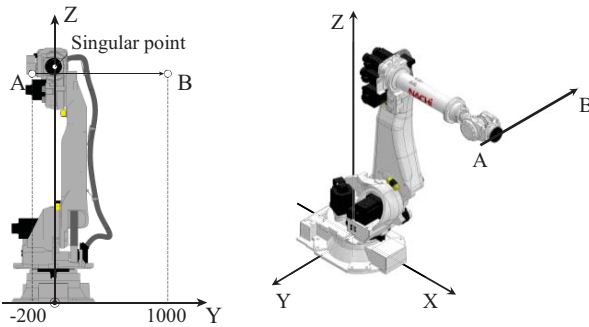


Fig. 2 Commanded trajectory of the robot's end-effector

3. Experimental Results

When the robot passed through the vicinity of the singular point, Joints 4 and 6 rotated rapidly. Figure 3 shows the rotational speeds of Joints 4 and 6.

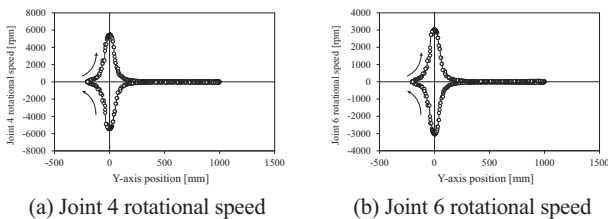


Fig. 3 Relation between the Y-axis position and the joint rotation speed of Joints 4 and 6

Figure 3 shows that the rotational speeds of Joints 4 and 6 are maximum at $Y = 0$ mm in the vicinity of the singular point. Owing to the maximum rotation speed of the servomotor, the commanded angle was not achieved, and the trajectory accuracy decreased. Figure 4 shows the commanded and measured Y-axis coordinates of the end effector. The figure indicates that there was a difference between the commanded and measured values at $Y = 0$, which was in the vicinity of the singular point.

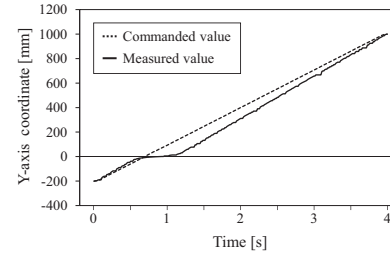


Fig. 4 Commanded and measured values of Y-coordinate

4. Discussion

4.1 Definition of the Vicinity of Singular Point

To define the extent of the vicinity of the singular point, the Z-coordinate of the linear trajectory in Fig. 2 was changed, and a comparison of the commanded maximum speed in the simulation and the measured maximum speed is shown in Fig. 5, where $Z = 2030$ mm is the location of the singular point.

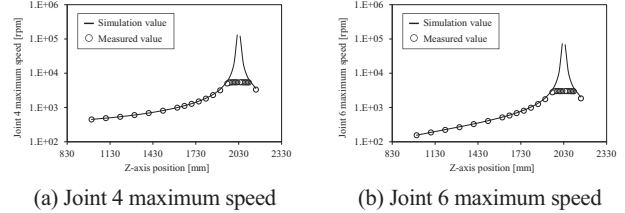


Fig. 5 Relation between the maximum rotational speeds of Joints 4 and 6 and the Z-axis position

Figure 5 shows that the simulated speed was faster than the measured speed near the singular point. This is because the servomotor could not rotate above its maximum rotational speed. In this area, the trajectory accuracy decreased. Therefore, this area, where the command rotational speed exceeded the maximum rotational speed of the servomotor, was defined as the vicinity of the singular point.

The commanded rotational speed of the joint was calculated using Eq. (3).

$$\dot{\theta} = J^{-1}\dot{p} \quad (3)$$

Equation (3) indicates that the rotational speed of the joint depends on the speed of the robot's end-effector. The extent of the vicinity of the singular point is shown in Fig. 6 when the speed of the robot end-effector was set to $\dot{Y} = \{10, 100, 500, 1200\}$ mm/s, where $\dot{Y} = 300$ mm/s is Fig. 5 (a).

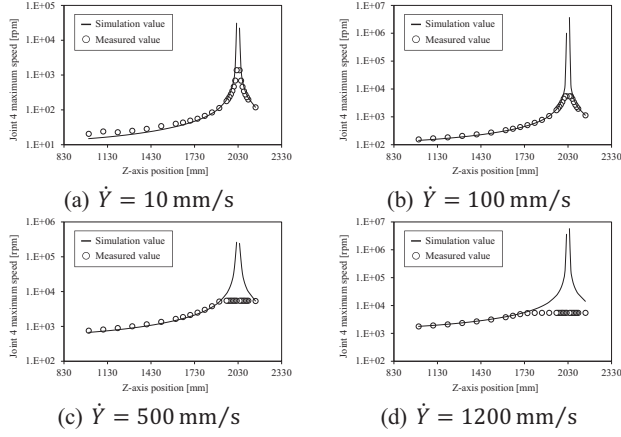


Fig. 6 Relation between the maximum rotational speed of Joint 4 and the Z-axis position for each speed of the robot end-effector

Figure 6 shows that the extent of the vicinity of the singular point expanded as the speed of the end-effector increased. Therefore, it was necessary to define the extent, considering the speed of the end-effector. Figure 7 shows the extent of the vicinity of the singular point for each end-effector speed.

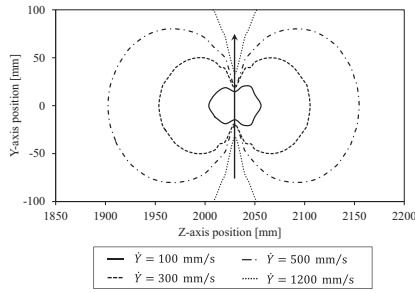


Fig. 7 Extent of the vicinity of the singular point for each end-effector speed

4.2 Solving Inverse Kinematics with Neural Network

The LM and SC methods were proposed to avoid the rapid rotation of some joints. Using these algorithms, it is possible for a manipulator to pass through singular points. However, another problem occurs when passing the vicinity of a singular point. This property of the inverse kinematics solution causes some joints to rotate approximately 180° in a short period. Figure 8 shows the results of simulating the command joint 4 angle using GI (based on Moore's generalized inverse matrix ⁽⁷⁾), LM, and SC methods.

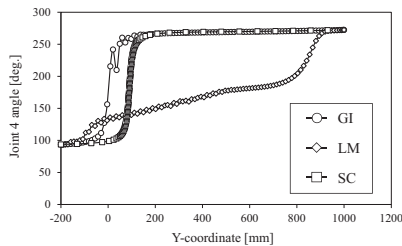


Fig. 8 Simulating Joint 4 angle for GI, LM, and SC methods

The figure shows that the rapid rotation at Y = 0, which was confirmed in GI, was avoided in LM by changing the trajectory

through it and in SC by changing the rotational speed through it. However, all algorithms still changed the joint angles from 90° to 270°. This is due to the analytical solution of the inverse kinematics of Joint 4 angle. The rotation matrix from Joints 4 to 6 is given by Eq. (4), and the Joint 4 angle is given by Eq. (5).

$${}^3R_6 = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \quad (4)$$

$$= \begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_4C_5S_6 - S_4C_6 & -C_4S_5 \\ S_5C_6 & -S_5S_6 & C_5 \\ -S_4C_5C_6 - C_4S_6 & S_4C_5S_6 - C_4S_6 & S_4S_5 \end{bmatrix} \quad (5)$$

$$\theta_4 = \text{atan2}(\pm R_{33}, \mp R_{13})$$

Figure 9 (a) shows the inverse kinematics solution for Joint 4 angle.

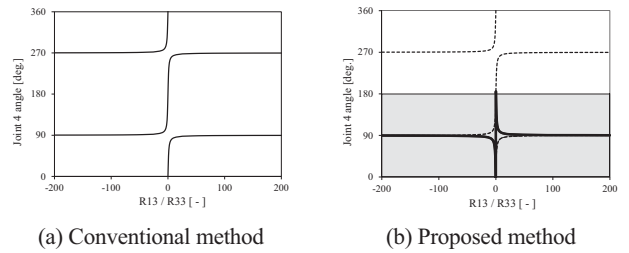


Fig. 9 Inverse kinematics solution for Joint 4 angle and its selection

Figure 9 (a) shows that the number of inverse kinematics solutions for the joint 4 angle is infinite because of the property of the inverse tangent function; the conventional method resulted in a 180° change because the solutions were continuously changing. However, if discontinuous solutions are selected, as shown in Fig. 9 (b), the rapid rotation of the joints can be suppressed. We focused on an artificial neural network (ANN) to select an appropriate inverse kinematic solution. That is, by deliberately limiting the training data of the ANN to the range shown in Fig. 9 (b), it was possible to limit the definition range of the Joint 4 angle. To construct the inverse kinematics model using ANN, a training dataset was prepared. The 3R_6 with geometric errors in the link parameters of Joints 4, 5, and 6 was more complicated than that in Eq. (4); thus, solving it analytically was difficult. Calculating Joints 4, 5, and 6 angles resulted in solving nine nonlinear equations simultaneously. The ANN was constructed with 3R_6 as the input and $\theta_4, \theta_5, \theta_6$ as the outputs. The procedure for constructing the ANN is as follows:

- (1) Set $0^\circ \leq \theta_4 \leq 180^\circ, -100^\circ \leq \theta_5 \leq 100^\circ, 0^\circ \leq \theta_6 \leq 180^\circ$ as 0.1° wide 3D grid point groups and extract 5,000 points randomly.
- (2) Substitute the extracted $\theta_4, \theta_5, \theta_6$ into Eq. (4) to calculate 3R_6 .
- (3) Create a dataset where the input data is $[R_{11}, R_{21}, R_{31}, R_{12}, R_{22}, R_{32}, R_{13}, R_{23}, R_{33}]^T$ and the output data is $[\theta_4, \theta_5, \theta_6]^T$.

The regression plot for the test data as a performance measure of the ANN is shown in Fig. 10. The figures show that the ANN can learn the inverse kinematics model with good accuracy.

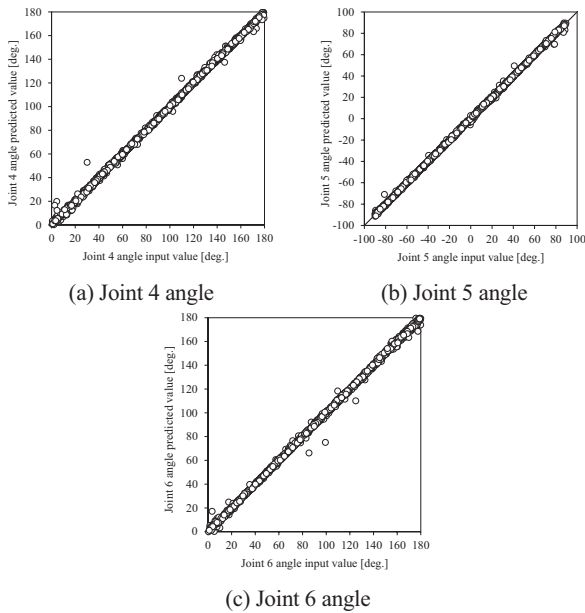


Fig. 10 Performance of the constructed ANN

Figure 11 shows the simulation results for Joint 4 angle with the new algorithm that limits the training data of the neural network.

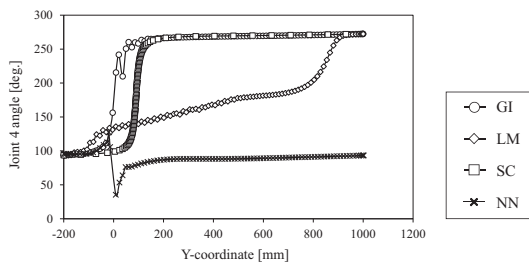


Fig. 11 Simulation results for Joint 4 angle using the neural network algorithm

As shown in Fig. 11, the neural network algorithm achieved a stable motion with a lesser amount of rotation (less than 90°) of Joint 4 angle compared with those obtained using the other methods. Therefore, the results show that the proposed method enables a manipulator to pass through the vicinity of a singular point without decreasing trajectory and speed accuracy.

5. Conclusions

To stabilize the motion of a 6-DOF manipulator in the vicinity of a singular point, we developed a new control algorithm using a neural network to address the issues of numerical methods and obtained the following results:

- (1) The vicinity of the singular point was defined as the extent where the commanded joint rotational speed exceeded the maximum rotational speed of the servomotor based on the speed of the end-effector.
- (2) Inverse kinematics models for Joints 4, 5, and 6 were constructed with high accuracy using neural networks.

- (3) The rapid rotation of some joints of a manipulator passing through the vicinity of a singular point can be suppressed by deliberately limiting the training data of the neural network, thus stabilizing the robot's motion.

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