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Hysteresis phenomena in the reflection of shock waves

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1 Introduction

When a supersonic flow, $M_0>1$, encounters a straight compressive wedge a straight and attached to the leading edge of the reflecting wedge shock wave is formed, provided the reflecting wedge angle, θ_w, is smaller than the maximum flow deflection angle appropriate to flow-Mach number, M_0, i.e., $\theta_w<\delta_{max}(M_0)$.

If the reflecting wedge is positioned over a straight surface (which can be considered as a line of symmetry) the oblique shock wave will be reflected from the surface resulting in either a regular reflection, RR, or a Mach reflection, MR. Schematic illustrations of the wave configurations of an RR and an MR are shown in Fig. 1. While passing through the incident shock wave, i, the oncoming flow is deflected by an angle $\theta_1=\theta_w$, to become parallel to the reflecting wedge surface. The supersonic deflected flow behind i approaches obliquely the bottom surface with an incident angle equal to θ_w. The supersonic flow can negotiate this obstacle only with the aids of either an RR or an MR as shown in Fig. 1.

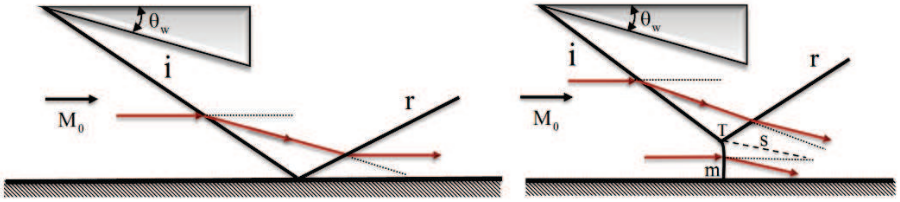

Fig. 1 Illustrations of the wave configurations of an RR (a) and an MR (b) in the reflection of straight oblique shock waves from straight surfaces in steady flows

Two out of a variety of conditions, which were proposed by various investigators, for the RR↔MR transition, in the past 125 years, are extreme. They are the detachment condition beyond which an RR wave configuration is theoretically impossible and the von Neumann condition beyond which an MR wave configuration is theoretically impossible. Von Neumann [1] was the first to introduce these two conditions as possible RR↔MR transition criteria.

Hornung & Robinson [2] showed that the RR↔MR transition criterion in steady flows depends upon whether M_0 is smaller or larger than M_{0C}, which is the value appropriate to the point at which the transition lines arising from the above mentioned von Neumann and detachment criteria intersect. Molder [3] calculated the exact value of M_{0C} to be 2.20 for a perfect diatomic gas and 2.47 for a perfect monatomic gas. Based on their experimental results Hornung & Robinson [2] concluded that the both the RR→MR and the MR→RR transitions occurs at the von Neumann criterion for

$M_0 \geq M_{0C}$, and at the sonic condition, which is very close to the detachment criterion, for $M_0 \leq M_{0C}$.

By defining the angles of incidence of the incident shock wave that are appropriate to the von Neumann and the detachment conditions as β^N and β^D, respectively, one obtains that only RR is theoretically possible in the range $\beta<\beta^N$, and only MR is theoretically possible in the range $\beta > \beta^D$. In the intermediate range $\beta^N \leq \beta \leq \beta^D$ both RR and MR are theoretically possible. For this reason, the intermediate domain, bounded by β^N and β^D is known as the *dual-solution-domain*.

As a consequence the (M_0,θ_w)-plane can be divided into three domains (see Fig. 2):

- A domain inside which only RR are theoretically possible;
- A domain inside which only MR are theoretically possible;
- A domain inside which both RR and MR are theoretically possible.

The existence of a domain inside which only RR is theoretically impossible, a domain inside which only MR is theoretically impossible, and a domain inside which both RR and MR are theoretically possible (see Fig. 2) led Hornung et al. [4] to hypothesize that a hysteresis could exist in the RR↔MR transition process.

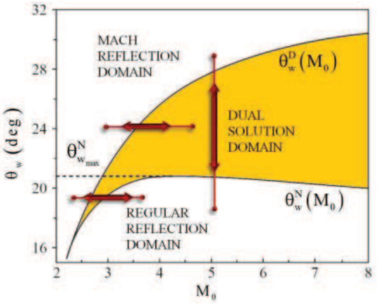

Fig. 2 Domains of possible shock wave reflection wave configurations in the (M_0,θ_w)-plane

1.1 Hysteresis in Steady Supersonic Flow Over a Symmetric Geometry

An inspection Fig. 2 suggests that two general hysteresis processes are theoretically possible:

- *A wedge-angle (θ_w)-variation-induced hysteresis process*,

in which the flow Mach number is kept constant and the wedge angle is changed.

- *A flow-Mach-number (M_0)-variation-induced hysteresis process*

in which the wedge angle is kept constant and the flow-Mach number is changed.

It is noted that since $\beta=\beta(M_0,\theta_w)$, the above mentioned two hysteresis processes are, in fact, *angle-of-incidence (β)-variation-induced hysteresis processes*.

Henderson & Lozzi [5,6] and Hornung & Robinson [2] failed in their experimental attempts to record the wedge-angle-variation-induced hysteresis process and concluded that the RR wave configuration is unstable inside the dual-solution domain, and that as a consequence both the MR→RR and RR→MR transitions occur at the von Neumann condition.

Teshukov [7] used a linear stability technique, and proved that the RR wave configuration is stable inside the dual-solution domain. Li & Ben-Dor [8] applied the principle of minimum entropy production and proved that the RR wave configuration is stable in most of the dual-solution-domain.

Chpoun et al. [9] were the first to experimentally record both stable RR wave configurations inside the dual-solution-domain, and a wedge-angle-variation-induced hysteresis in the RR↔MR transition.

Vuillon et al. [10] were the first to numerically obtain stable RR and MR wave configurations for the same flow-Mach numbers and reflecting wedge angles but different aspect ratios inside the dual-solution-domain. Using a Navier-Stokes solver, Chpoun et al. [11] were the first to numerically simulate and thereby verify the existence of a wedge-angle-variation-induced hysteresis in the RR↔MR transition. Unfortunately, since their study was published in a French scientific journal, it has not caught the attention of the relevant scientific community.

The above mentioned experimental and numerical findings that the RR is stable inside the dual-solution-domain and the experimental finding that a hysteresis in the RR↔MR transition indeed exists, re-initiated the interest of the scientific community in the reflection process in steady flows, in general, and the hysteresis process in the RR↔MR transition, in particular. The revived interest led to the publication of tens of papers that eventually shattered the state-of-knowledge that existed until the early 1990's and consequently led to a new state-of-knowledge.

In all the above mentioned numerical simulations of the hysteresis process the obtained transition angles did not agree well enough with the appropriate theoretical von Neumann and detachment angles. The numerical MR→RR transition angle was about 1° larger than the theoretical von Neumann angle. This was probably due to the fact that the very small Mach stem, in the vicinity of the von Neumann transition angle, was not resolved well enough in the computations. Grid refinement studies confirmed that the numerically obtained MR→RR transition angle approached the theoretical value as the grid was refined. The RR→MR transition angle did not depend on the grid resolution for fine enough grids but strongly depended on the numerical dissipation inherent in any shock-capturing solver. Large numerical dissipation or low order reconstruction could result in significant differences between the numerical and the theoretical values of the transition angles. For example, the RR→MR transition angle, for M_0=4.96, in the computations of Chpoun & Ben-Dor [12] who used an INCA code, was more than 5° larger (33° instead of 27.7°). The use of a high-order shock-capturing scheme gave a transition wedge angle 27.95°, which was much closer to the theoretical value.

Why had the hysteresis phenomenon been recorded in the course of some experimental investigations and not in others soon became a research question. Although the answer to this question has not been fully resolved, two possible major reasons were suggested and put forward:

- **Reason 1:** The extent of the hysteresis depends on the type the wind tunnel inside which the experiment was conducted.

  Fomin et al. [13] and Ivanov et al. [14] showed experimentally, that while in a closed test section wind tunnel the hysteresis was hardly detected, a clear hysteresis was obtained in an open test section wind tunnel. Not surprisingly Henderson & Lozzi [5,6], Hornung et al. [4] and Hornung & Robinson [2] who did not detect the hysteresis, used closed section wind tunnels, while Chpoun et al. [9] and Fomin et al. [13] who did detect the hysteresis, used open jet type wind tunnels.
- **Reason 2:** Three-dimensional edge effects affect the experiment and promote the hysteresis.

Skews et al. [15], Skews [16,17], Ivanov et al. [18] and Kudryavtsev et al. [20], claimed and showed that the experimental investigations, in which hysteresis in the RR↔MR transition were well recorded, were all contaminated by 3D edge effects and hence could not be considered as purely two-dimensional. Skews [19] showed that 3D edge effects are evident in actual wave configurations associated with the reflection of plane shock waves over plane wedges.

It should be noted here that using the same reflecting wedge (i.e., same aspect ratios) a hysteresis was observed in an open section wind tunnel by Chpoun et al. [9] and was not observed in a closed section wind tunnel by Ivanov et al. [14], in spite of the fact that almost identical 3D effects were present in both cases. These results clearly indicate that 3D effects by themselves are not enough to promote the hysteresis and that the type of the wind tunnel (open or closed) has a significant, not yet understood, role in the occurrence of hysteresis in the RR↔MR transition in steady flows. Kudryavtsev et al. [20] demonstrated numerically and experimentally that increasing the aspect ratio could reduce the influence of the 3D edge effects. They concluded that an actual MR cannot be considered as free of 3D edge effects as long as the height of its Mach stem is smaller than the Mach stem height that is appropriate to a calculated 2D Mach reflection. It should be noted here that this condition is a necessary but not a sufficient one.

Ivanov et al. [21] and Onofri & Natusi [22] illustrated numerically that keeping the wedge angle, θ_w, constant and changing the flow-Mach number, M_0, can also lead to a hysteresis process in the RR↔MR transition. Figure 2 indicates that there are two possible hysteresis processes for this case:

- If $\theta_w > \theta_{w,max}^N$ the Mach number can be changed along the path *BB'B* from a value inside the dual-solution domain where both RR and MR wave configurations are theoretically possible, to a value outside the dual-solution domain for which only an MR wave configuration is theoretically possible and then back to the initial value. If one starts inside the dual-solution domain with an RR then after transition to an MR the wave configuration never returns to be an RR because the MR→RR transition is not compulsory on the return path. It should be noted that this loop does not represent a full hysteresis loop, though both RR and MR wave configurations can be observed for the same values of θ_w and M_0.
- If $\theta_w < \theta_{w,max}^N$ the Mach number can be changed from a value for which only an RR wave configuration is theoretically possible to a value for which only an MR wave configuration is theoretically possible and then back to the initial value crossing the $\theta_w^N(M)$ and $\theta_w^D(M)$ curves (path *CC'C* in Fig. 2). In this case, a full hysteresis loop is obtained.

1.2 Hysteresis in Steady Supersonic Flow Over an Asymmetric Geometry

Li et al. [23] conducted a detailed analysis of the 2D reflection of asymmetric shock waves in steady flows. In similar to the interaction of symmetric shocks in steady flows, the interaction of asymmetric shocks leads to two types of overall wave configurations, namely; an overall regular reflection, oRR, and an overall Mach reflection, oMR. An oRR wave configuration consists of two incident shocks, two reflected shocks and one slipstream. These five discontinuities meet at a single point (R). The slipstream results from the fact that the streamlines of the oncoming flow pass through two unequal shock wave sequences. In addition to the incident and reflected shock waves a Mach stem appears in an oMR wave configuration. The Mach stem bridges two triple points from which two slipstreams emanate.

Li et al. [23] showed that three different oMR wave configurations are theoretically possible. They are:

- An oMR wave configuration that consists of two direct-Mach reflections, DiMR.
- An oMR wave configuration that consists of one DiMR and one stationary-Mach reflection, StMR; and
- An oMR wave configuration that consists of one DiMR and one inverse-Mach reflection, InMR.

Details regarding the DiMR-, the StMR- and the InMR wave configurations can be found in Ben-Dor [26].

In the course of their study, Li et al. [23] identified, two extreme transition criteria, which were analogous to the above mentioned detachment and von Neumann criteria. In similar to the case of the reflection of symmetric shock waves, the two extreme transition criteria also resulted in a dual-solution-domain.

The $(\theta_{w1},\theta_{w2})$-plane for a given flow Mach number, M_0, can be divided into three parts:

- A domain inside which only oRR wave configurations are theoretically possible;
- A domain inside which only oMR wave configurations are theoretically possible;
- An intermediate domain inside which both oRR and oMR wave configurations are theoretically possible.

As a result, in similar to the case of symmetric shocks, two general hysteresis processes are possible:

- *A wedge-angle-variation-induced hysteresis process*

  in which the flow-Mach number and the wedge angle of one of the two wedges are kept constant and the wedge angle of the other wedge is changed.
- *A flow-Mach-number-variation-induced hysteresis process*

  in which the two wedge angles are kept constant and the flow-Mach number is changed.

Chpoun & Lengrand [25] verified experimentally the existence of the above mentioned wave configuration and the existence of the wedge-angle-variation-induced hysteresis process in the oRR↔oMR transition. Ivanov et al. [26] verified them numerically.

It is important to note that the experimental and geometrical set-ups of the reflection experiments over asymmetric wedges were similar to those over symmetric wedges. Hence, the 3D effects in both cases should have been probably similar. However, the fact that very good agreements between the analytical predictions and the experimental results were obtained regarding both the transition and the wave angles might suggest that the influence of the 3D effects was not too significant.

Similarly, to the flow-Mach-number-variation-induced hysteresis process, in the reflection of symmetric shocks, which was numerically illustrated and verified both by Ivanov et al. [21] and Onofri & Nasuti [22], it is reasonable to assume that a similar flow-Mach-number-variation-induced hysteresis process also exists in the reflection of asymmetric shock waves. Owing to the fact that conducting experiments in a wind tunnel in which the flow-Mach number is continuously changed is complicated, the existence of a flow-Mach-number-variation-induced hysteresis process in the reflection of asymmetric shock waves still awaits a numerical proof.

1.3 Hysteresis in Steady Supersonic Flow Over an Axisymmetric Geometry

In order to better understand the extent of 3D effects on the hysteresis process Chpoun et al. [27] and Ben-Dor et al. [28] designed an axisymmetric geometrical set-up, which by definition was free of 3D effect. A schematic illustration of the experimental set-up fulfilling this requirement is shown in Fig. 3. A 70-mm in diameter and 28-mm wide conical ring was placed in the center of a 127-mm supersonic jet, which emanated from the wind tunnel. The head angle of the conical ring was $\theta=8.5°$. A curvilinear cone was placed downstream of the conical ring. The base diameter and the length (height) of the curvilinear cone were 30.4 mm and 40 mm, respectively. The conical ring generated an incident converging straight conical shock wave, i_1. This incident converging straight conical shock wave interacted with the incident diverging curvilinear conical shock wave, i_2, which was generated by the curvilinear cone. Depending on the angle of interaction between these two incident shock waves three different types of overall wave configurations were recorded in the course of the experimental investigation conducted by Ben-Dor et al. [28]. Two types were similar to an oRR (one was viscous-dependent) and one to an oMR.

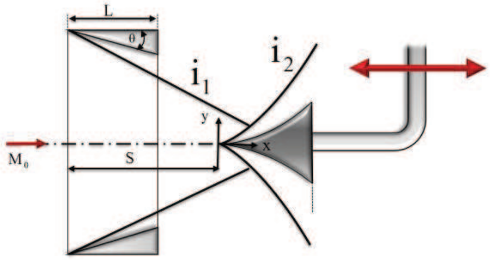

Fig. 3 Illustration of the geometrical set-up for investigating the reflection of conical shock waves in an axisymmetric flow

An inspection of the geometrical set-up shown in Fig. 3 indicates that the angle of interaction between the converging and diverging incident conical shocks, i_1 and i_2, depends on either the axial distance between the conical ring and the curvilinear cone or the oncoming flow-Mach number. This gives rise to the following two possible hysteresis processes in the oRR↔oMR transition:

- *A geometrical-variation-induced hysteresis process*
  In this process the axial distance between the conical ring and the curvilinear cone is changed for a given oncoming flow-Mach number, and
- *A flow-Mach-number-variation-induced hysteresis process*
  In this hysteresis the oncoming flow-Mach number is changed for a fixed axial distance between the conical ring and the cone.

It should be noted again that the changing angle of interaction between the two incident shock waves is the mechanism inducing the hysteresis in both processes.

Ben-Dor et al. [28] investigated experimentally and numerically (inviscid) the geometrical-variation-induced hysteresis process. They found that in addition to a major hysteresis process in the oRR↔oMR transition, there were minor hysteresis processes associated with oMR↔oMR transitions processes in which the Mach stem heights of the two oMRs were different.

Ben-Dor et al. [29] numerically investigated the flow-Mach-number-variation-induced hysteresis process for three cases that differed in the location of the curvilinear cone w.r.t. the conical ring. They found that there are situations in which two hysteresis loops overlapped. As a result, three different wave configurations were theoretically possible for the same flow-Mach number. It was shown that the different wave configurations for identical values of M_0 were associated with significantly different pressure distributions along the curvilinear cone surface. Ben-Dor et al.'s [29] study also revealed that in all the cases where the Mach stem of the oMR was long enough pressure peaks that were 40-50 times larger than the ambient pressure were reached.

It is important to note here that in spite of the fact that the early reasons for the interest in studying the hysteresis process in the RR↔MR transition were purely academic, it turned out that the existence of the hysteresis process might have an important impact on flight performance at supersonic and hypersonic speeds. Consequently, there is a clear aeronautical and aerospace engineering interest in better understanding this phenomenon. Some of the geometries that were investigated in recent years resembled geometries of supersonic/hypersonic intakes. The findings regarding the existence of hysteresis processes, in general, and overlapping hysteresis processes, in particular, can be relevant to the flight performances of vehicles flying at supersonic and hypersonic speeds. The possible dependence of the flow pattern that is established inside an intake, in general, and the accompanied pressure distribution, in particular, on the preceding variations in the speed of flight of a supersonic/hypersonic aircraft should be accounted for in designing intakes and flight conditions for supersonic and hypersonic vehicles. Especially due to the fact that different flow fields would result in different flow conditions that can significantly affect the combustion process and the entire performance of the vehicle.

References

[1] J. von Neumann, *Oblique reflection of shocks*, Explos. Res. Rept. 12, Navy Dept Bureau of Ordinance, Washington, DC, U.S.A, (1943).

[2] H.G. Hornung, M.I. Robinson, *Transition from regular to Mach reflection of shock waves. Part 2. The steady flow criterion*, J. Fluid Mech., **123** 155-164 (1982).

[3] S. Molder, *Particular conditions for the termination of regular reflection of shock waves*, CASI Trans. **25** 44-49 (1979).

[4] H.G. Hornung, H. Jr. Oertel H, R.J. Sandeman, *Transition to Mach reflection of shock waves in steady and pseudo-steady flow with and without relaxation*, J. Fluid Mech., **90** 541-560 (1979).

[5] L.F. Henderson, A. Lozzi, *Experiments and transition of Mach reflection*, J. Fluid Mech., **68** 139-158 (1975).

[6] L.F. Henderson, A. Lozzi, *Further experiments and transition of Mach reflection. Experiments and Transition of Mach reflection*, J. Fluid Mech., **68** 139-158 (1979).

[7] V.M. Teshukov, *On stability of regular reflection of shock waves*. Prilk. Mekhanika i Techn. Fizika, (translated to English in Appl. Mech., Tech. Phys.), **2** 26-33 (1989) (in Russian).

[8] H. Li, G. Ben-Dor, *Application of the principle of minimum entropy production to shock wave reflections. I. Steady flows*, J. Appl. Phys., **80(4)** 2027-2037 (1996).

[9] A. Chpoun, D. Passerel, H. Li, G. Ben-Dor, *Reconsideration of oblique shock wave reflection in steady flows. Part 1. Experimental Investigation*, J. Fluid Mech., **301** 19-35 (1995).

[10] J. Vuillon, D. Zeitoun, G. Ben-Dor, *Reconsideration of oblique shock wave reflections in steady flows. Part 2. Numerical investigation*, J. Fluid Mech., **301** 37-50 (1995).

[11] A. Chpoun, D. Passerel, J.-C. Lengrand, H. Li, G. Ben-Dor, *Mise en evidence experimentale et numerique d'un phenomene d'hysteresis lors de la transition reflexion de Mach-reflexion reguliere*, C. R. Acad. Sci. Paris, **319(II)** 1447-1453 (1994).

[12] A. Chpoun, G. Ben-Dor, *Numerical confirmation of the hysteresis phenomenon in the regular to the Mach reflection transition in steady flows*, Shock Waves, **5(4)** 199-204 (1995).

[13] V.M. Fomin, H.G. Hornung, M.S. Ivanov, A.M. Kharitonov, G.P. Klemenkov, A.N. Kudryavtsev, A.A. Pavlov, *The study of transition between regular and Mach Reflection of shock waves in different wind tunnels*, Proc. 12th Int. Mach Reflection Symp., Ed. B. Skews, Pilanesberg, South Africa, 137-151 (1996).

[14] M.S. Ivanov, A.M. Kharitonov, G.P. Klemenkov, A.N. Kudryavtsev, S.B. Nikiforov, V.M. Fomin, *Influence of test model aspect ratio in experiments on the* $\mathrm{RR} \leftrightarrow \mathrm{MR}$ *transition*, Proc. 13th Int. Mach Reflection Symp. Ed. G. Ben-Dor, Beer-Sheva, Israel, 3 (1998).

[15] B.W. Skews, S. Vukovic, M. Draxl, *Three-dimensional effects in steady flow wave reflection transition*, Proc. 12th Int. Mach Reflection Symp. Ed. B. Skews, Pilanesberg, South Africa, 152-162 (1996).

[16] B.W. Skews, *Aspect ratio effects in wind tunnel studies of shock wave reflection transition*, Shock Wave, **7** 373 (1997).

[17] B.W. Skews, *Oblique shadowgraph study of shock wave reflection between two wedges in supersonic flow*, Proc. 13th Int. Mach Reflection Symp. Ed. G. Ben-Dor, Beer-Sheva, Israel, 3 (1998).

[18] M.S. Ivanov, S.F. Gimelshein, A.N. Kudryavtsev, G.N. Markelov, *Transition from regular to Mach reflection in two- and three-dimensional flows*, Proc. 21st Int. Symp. Shock Waves, Eds. A.F.P. Houwing, A. Paull, R.R. Boyce, R.R. Xanehy, M. Hannemann, J.J. Kurtz, T.J. MxIntyre, S.J. McMahon, D.J. Mee, R.J. Sandeman, H. Tanno, Panther Publ., Fyshwick, Australia, **II** 813-818 (1998).

[19] B.W. Skews, *Three dimensional effects in wind tunnel studies of shock wave reflection*, J. Fluid Mech., **407** 85-104 (2000).

[20] A.N. Kudryavtsev, D.V. Khotyanovsky, G.N. Markelov, M.S. Ivanov, *Numerical simulation of reflection of shock waves generated by finite-width wedge*, Proc. 22nd Int. Symp. Shock Waves, Eds. G.J. Ball, R. Hillier, G.T. Roberts, University of Southampton, **2** 1185-1190 (1999).

[21] M.S. Ivanov, G. Ben-Dor, T. Elperin, A.N. Kudryavtsev, D.V. Khotyanovsky, *Mach-number-variation-induced hysteresis in steady flow shock wave reflections*, AIAA J., **39(5)** 972-974 (2001).

[22] M. Onofri, F. Nasuti, *Theoretical considerations on shock reflections and their implications on the evaluations of air intake performance*, Proc. 22nd Int. Symp. Shock Waves, Eds. G.J. Ball, R. Hillier, G.T. Roberts, University of Southampton, **2** 1285-1290 (1999).

[23] H. Li, A. Chpoun, G. Ben-Dor, *Analytical and experimental investigations of the reflection of asymmetric shock waves in steady flows*, J. Fluid Mech., **390** 25-43 (1999).

[24] G. Ben-Dor, *Shock Wave Reflection Phenomena*, (Springer-Verlag, New York, 2007).

25] A. Chpoun, J.-V. Lengrand, *Confermation experimentale d'un phenomene d'hysteresis lors de l'interaction de deux chocs obliques de familles differentes*, C. R. Acad. Sci. Paris, **304** 1 (1997).

[26] M.S. Ivanov, G. Ben-Dor, T. Elperin, A.N. Kudryavtsev, D.V. Khotyanovsky, *The reflection of asymmetric shock waves in steady flows: A numerical investigation*, J. Fluid Mech., **469**, 71-87 (2002).

[27] A. Chpoun, F. Chauveux, L. Zombas, G. Ben-Dor, *Interaction d'onde de choc coniques de familles opposees en ecoulement hypersonique stationnaire, Mecanique des Fluides/Fluid Mechanics*, C. R. Acad. Sci. Paris, **327(1)** 85 (1999).

[28] G. Ben-Dor, E.I. Vasiliev, T. Elperin, A. Chpoun, *Hysteresis phenomena in the interaction process of conical shock waves: Experimental and numerical investigations*, J. Fluid Mech., **448** 147-174 (2001).

[29] G. Ben-Dor, T. Elperin, E.I. Vasilev, *A numerical investigation of the flow-Mach-number-induced hysteresis phenomenon in the interaction of conical shock waves*, J. Fluid Mech., **496**, 335-354 (2003).