Prognostics and Maintenance Optimization in Bridge Management

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This paper has been written in collaboration with the Norwegian Public Roads Administration (NPRA). In Norway, bridges are a vital part of the transportation infrastructure. With more than 18,000 road bridges across the country, an efficient bridge management system is of critical importance to avoid high costs from over expending, to ensure safety of the public and availability of the transportation system. In the bridge management system applied by NPRA, the inspections are mainly carried out periodically based on pre-defined rules and the decision about when to perform the maintenance is based on the findings of these inspections. The objective of this paper is to propose a modelling framework that makes it possible to challenge these pre-defined rules by doing degradation prognostic and maintenance optimization. We propose to use a Piecewise Deterministic Markov Process to encompass different modelling assumptions as non-negligible maintenance delays and time dependent inspections. State probabilities and performance indicators are assessed through Monte Carlo simulations and a numerical scheme. The experimental values provided at the end show that optimal maintenance and optimization strategies should be investigated and further developed.

Keywords: Bridge management, stochastic modelling, piecewise deterministic Markov process, prognostics, numerical assessment, Monte Carlo simulation, road bridges.

1. Introduction

The use of automobiles experienced a rapid growth during the 20th century and with this growth came the development of a massive transportation infrastructures. In Comission (2008), the Council of the European Union includes the transport sector in the list of Critical Infrastructures, considering that modern societies depend on the availability of this service and that its disruption or unavailability poses risks with serious consequences to the health, safety, economic or social well-being of people and vital societal functions. A systematic approach to maintenance and rehabilitation strategies for the transportation system was not identified until the late 1960s. The Highway Safety Act of 1968 was a development that resulted from the collapse of the Silver bridge across the Ohio River, USA in 1967, and the concerns related to the bridge management problem. This Act required state road officials to inspect and rate the condition of the bridges as mentioned by Scherer and Glagola (1994).

Bridge management can be understood as the optimal planning of inspections and maintenance activities of road bridges, with the goal of preserving the asset value of the infrastructure by optimizing the costs over its lifetime, while ensuring the safety of users and offering a sufficient quality of service, as Woodward et al. (2000). More than 50 years after the collapse of the Silver bridge, despite the advances in technology, rehabilitation techniques and safety assessments, bridge collapses continue to occur. Moreover, the construction of new bridges has been slowing down in most countries, which now face a stock of aging bridges, requiring an effective and efficient bridge management.

1.1. *Bridge management in Norway*

In Norway, bridges are a vital part of the transportation infrastructure. With more than 18,000 road bridges across the country, an efficient bridge management system is of critical importance to avoid high costs from over expending and to ensure safety of the public and availability of the transportation system.

As pointed by Kallen (2007), there are many factors that make bridge management a challenging task, such as: the varying weight and intensity of the traffic, the evolution of the building codes over the years, the weather influence on the structures, large number of structures spread over a large area, and others. All these factors create uncertainty, which makes the bridge management a problem of decision making under uncertainty.

In the bridge management system applied

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by the Norwegian Public Roads Administration (NPRA), the agency responsible for planning, building, operating, and maintaining national and country bridges in Norway, the inspections are mainly carried out periodically based on predefined rules and the decision about when to perform the maintenance is based on the findings of these inspections. The handbooks for management and inspections of bridges, Statens Vegvesen (2014a,b), establish types of inspections for the bridges and the period in which they must be performed, e.g. a main inspection of a bridge, with an overview of all the elements of the bridge, must (in general) be performed every five years. Statens Vegvesen (2014b) also establishes how the inspections must be logged in a database, how the findings must be reported and provides guidelines on when to perform the repairs for found damages.

When an inspection is performed on a bridge, the severity of the found damages is assessed in a scale of one to four, as:

- 1 - Small damage
- 2 - Medium damage
- 3 - Large damage
- 4 - Critical damage

Based on the severity of the damage, a maintenance action is scheduled:

- Severity: 1 - No maintenance action is required
- Severity: 2 - A maintenance action must take place between four and ten years
- Severity: 3 - A maintenance action must take place between one and three years
- Severity: 4 - A maintenance action must take place in less than six months

This bridge management system can be characterized as a condition-based maintenance program, in which the maintenance decisions are based on recommendations from the information gathered through condition monitoring. However, following this program is a challenging task for the NPRA. With such a large stock of bridges throughout the country, it is difficult to keep up to date the inspection program due to budget and resources constraints.

A problem raised for some years by the NPRA is to question if this bridge management system can be optimized by moving from diagnostics to prognostics.

1.1.1. *Diagnostics to Prognostics*

The current trend in many fields and with critical infrastructures is to move the decision making in condition-based maintenance from diagnostics to prognostics.

Diagnostics involve the techniques and practice of determining whether a fault is present, identifying its nature and estimating its severity. Prognostics on the other hand, is the practice of forecasting the likely development of such fault.

Through fault diagnosis, it is possible to implement maintenance decisions by following pre-established rules and recommendations saying when to perform what. This process tends to be dependent on the technical and mechanical education of the maintenance staff and their hands on expertise, and as pointed out by Rausand and Høyland (2004), although the expertise is key in maintenance management and performance, it should not be the only basis for making the decisions.

Prognostics allow to take the analysis one step further in order to question such pre-established rules, to reduce overestimated margins and to optimize decision rules. With the use of mathematical models, it may be possible to simulate different maintenance strategies and to assess the associated effects, the maintenance costs and the operational performance in the long run. Therefore, these simulations can be very helpful for deciding the most appropriate maintenance strategy to implement.

In this sense, the maintenance decision-making in the bridge management of the NPRA may be improved by using information available in a national data base (BRUTUS), the NPRAs tool for management and supervision of bridge-related work tasks, and a model capable of describing the deterioration of the bridge and the effect of decision criteria, such as: inspection interval, condition thresholds for performing preventive repairs, and type of repair (complete renewal or partial repairs).

The objective of this paper is to demonstrate the implementation of a Piecewise-Deterministic Markov Process (PDMP) as a framework to model the deterioration process of a structure and maintenance strategies applicable by the NPRA, in order to assess the effects of such strategies and assist the decision-making process. The remainder of this paper is organized as follows: section 2 states the assumptions, the problem statement and model formulation. Section 3 describes the implementation and quantification of the model in terms of next-event simulation and Monte Carlo simulation. Section 4 presents discussions around the framework and results.

2. Modelling Framework and Assumptions

In the field of civil engineering and bridge management, it is widely common to assess the severity and condition of the structures in a discrete scale similar to the one used by the NPRA. To quantify for the uncertainties involved in the deterioration process of a structure, described in a discrete scale, finite-state Markov processes have been applied often for modelling the deterioration of bridges, as Kallen (2007), Cesare et al. (1992), and Morcous (2006). More recently, semi-Markov processes have been studied in order to account for the aging of the structures as Mašović et al. (2015), Thomas and Sobanjo (2016) and Zambon et al. (2019).

2.1. *Assumptions*

For modelling the deterioration process of a structure and inspections and maintenance strategy consistent with the bridge management of the NPRA, the following assumptions are made:

(i) The observed condition of the unit is represented by a discrete variable ranging from small or no damage to critical damage
(ii) The deterioration process of the unit can be modelled with a homogeneous Markov chain with constant transition rates
(iii) The unit is periodically inspected and not continuously monitored
(iv) Inspections are perfect and reveal the true state of the unit
(v) When an inspection reveals a damage with severity medium or higher, a maintenance action is scheduled
(vi) There is a significant delay before a maintenance is performed
(vii) The duration of the delay is deterministic
(viii) Maintenance interventions occur at the scheduled date instantaneously, i.e. the duration of the intervention is null
(ix) After a maintenance action, the unit is as good as new

A Markov process is not suitable to model the inspection and maintenance strategy of the NPRA due to assumptions iii and vii. Here, we propose a PDMP, as a framework to model the deterioration of the structure and the effect of inspection and maintenance strategies.

2.2. *Modelling framework*

A PDMP is an extension of a Markov chain that incorporates continuous states with evolution that follow discrete state-dependent deterministic differential equations. The resulting stochastic process is a Markov process with a mixture of random jumps and deterministic motion. They were introduced by Davis (1984), as a general class of non-diffusion stochastic models that provides a framework for studying optimization problems.

A PDMP is a hybrid process $\{I_t, X_t\}_{t>0}$ with values in a discrete-continuous space $E \times R$, as described by Lair et al. (2011, 2012). The first component I_t is discrete, with values in a finite state space E and corresponds to the unit states. The second component X_t takes values in a Borel subset $R \subset \mathbb{R}^k$ and it stands for the environmental conditions, which in our case will refer to the time until next inspection and next maintenance action.

2.2.1. *Discrete component I_t*

The discrete component I_t of the PDMP in our case, is used to model the deterioration process of the structure and to indicate a type of maintenance that has been scheduled.

First, a variable indicating the condition of the structure can be denoted $i_A(t)$.

$i_A(t) = \{1, 2, 3, 4\}$, where:

- $i_A = 1$: Small or no damage
- $i_A = 2$: Medium damage
- $i_A = 3$: Large damage
- $i_A = 4$: Critical damage

Only when the unit is inspected, the degree of deterioration of the unit is detected, and a maintenance is scheduled accordingly. The type of scheduled maintenance can be denoted $i_B(t)$.

$i_B(t) = \{1, 2, 3, 4\}$, where:

- $i_B = 1$: No maintenance is scheduled
- $i_B = 2$: Slow maintenance is scheduled, (i.e. a maintenance intervention takes place between four and ten years)
- $i_B = 3$: Medium maintenance is scheduled, (i.e. a maintenance intervention takes place between one and three years)
- $i_B = 4$: Fast maintenance is scheduled, (i.e. a maintenance intervention before six months)

The discrete component of the PDMP is then $I_t = i$, with $i = (i_A, i_B)$, given that all the combinations are not possible and should be taken only in the finite state space E of the PDMP, E = {(1,1), (2,1), (2,2), (3,1), (3,2), (3,3), (4,1), (4,2), (4,3),(4,4)}. To simplify, we denote hereafter $i = (i_A, i_B)$, without reminding that the possible couples of values (i_A, i_B) are limited to E.

2.2.2. *Continuous component X_t*

The continuous component here is not related to any physical phenomena, but it is used as an artefact to model a process that requires a combination of stochastic random jumps and continuous variables to count time. The environmental condition in this case, stands for the date of the next inspection, the date of the next maintenance action and time.

Let $X_t = x$, with $x = (x_A, x_B, t)$, where:

- x_A : date of next inspection
- x_B : date of next maintenance action
- t : time

2.2.3. *PDMP*

The complete process to consider $\{I_t, X_t\}$ is made of $\{(i_A, i_B), (x_A, x_B, t)\}$. The process may experience jumps at random or at deterministic times.

Jumps at random times are used in our case to simulate the deterioration of the unit. The unit makes a transition to a more degraded state. This degradation is not detected immediately, so the scheduled type of maintenance does not change. The discrete component jumps from $(i_A, i_B) = (j, k)$ to $(i_A, i_B) = (m, k)$, while the continuous component does not change. The deterioration process of the unit with random jumps is shown in figure 1.

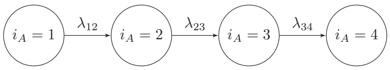

Fig. 1. Deterioration process.

Jumps at deterministic times are used to model the inspection and maintenance actions.

When an inspection is performed, the date to the next inspection (x_A) is updated, a maintenance action is scheduled (x_B) and the type of scheduled maintenance (i_B) is updated according the condition of deterioration of the unit.

When a maintenance action is performed, the discrete component (i_A, i_B) jumps to $(1, 1)$ (as good as new), the date to the next inspection (x_A) does not change, and the date of the next maintenance action (x_B) is set to infinite (no maintenance scheduled).

Between two consecutive jumps, only the continuous variable t evolves, with speed of one.

3. Quantification

Solving the PDMP analytically is generally impossible due to complex system behaviour. For reliability assessments, Monte Carlo simulation and numerical scheme based on finite-volume methods are two commonly used approaches to solve PDMP. In our case, both approaches are used for validating the results and compare the advantages or disadvantages from each.

3.1. *Monte Carlo simulation*

The simulation procedure of the PDMP is shown in figure 2. It includes five main steps to simulate a realization of the PDMP until the horizon time t_{hor}.

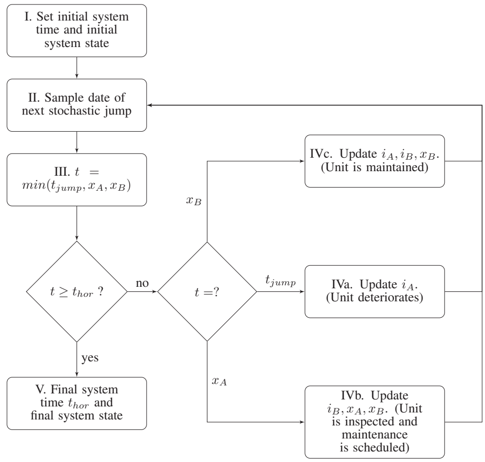

Fig. 2. Simulation procedure.

(i) Set initial system time and initial system state
In our case, initial time is set to zero, the unit is set to be in new condition with no maintenance action scheduled and the date of the first inspection is set to the period. (i.e. $t = 0$, $i_A = 1$, $i_B = 1$, $x_A = T$ and $x_B = \infty$), where T is the inspection period.

(ii) Sample date of next stochastic jump, if enabled
The date of the next stochastic jump t_{jump} is sampled from the corresponding probability density function and the corresponding parameter(s). In our case, exponential distribution is considered with rates as shown in figure 1.

(iii) Identify next event
The date of the next stochastic jump t_{jump} is compared with date of next inspection x_A, the date of next maintenance action x_B and the horizon time t_{hor}.
The system time is updated as: $t = \min(t_{jump}, x_A, x_B, t_{hor})$. If the simulation time has reached the horizon time, $t = t_{hor}$, the simulation continues to step v, otherwise it continues to step iv.

(iv) Update system state
The system state is updated according to the jump that takes place at time t, deterioration, inspection or maintenance.

(a) Deterioration: ($t = t_{jump}$)
Only i_A is updated in this jump

(b) Inspection: ($t = x_A$)
Variables i_B, x_A, x_B are updated. The after jump values ($^+$) are:
$i_B^+ = i_A$;
$x_A^+ = t + T$;
$x_B^+ = t + M_{i_B}$; where M_{i_B} is the delay for maintenance action of the type i_B.

(c) Maintenance: ($t = x_B$)
Variables i_A, i_B, x_B are updated. The after jump values ($^+$) are:
$i_A^+ = i_B^+ = 1$;
$x_B^+ = \infty$.

(v) Set final system time and final system state
The final system time is t_{hor} and the final system state is the state resulting from the last jump to take place no later than t_{hor}.

This simulation procedure is replicated N

times, to approximate quantities of interest, such as deterioration state probabilities.

3.2. *Numerical scheme*

The probability of the state of the system of a PDMP can be completely described by the Chapman-Kolmogorov equations, as demonstrated by Cocozza-Thivent et al. (2006). A numerical scheme based in finite-volume methods to approximate these probability measures is proposed by Cocozza-Thivent et al. (2006), with proof of the convergence to the unique solution.

The principle of the scheme is the discretization of the continuous component X_t into cells. The time evolution of the probability masses in each cell of the environmental space is followed, and at each step, a balance between the in-coming and out-going probability masses is written, allowing us to solve a linear system, as Lair et al. (2012).

Let $\mathcal{M}$ denote the mesh of the discretization of the environmental state space R and δ_t denote the environmental state space step (we use the same step for x_A, x_B and t in our case, since x_A, x_B and t have units of time). A cell w of $\mathcal{M}$ has cubic shape $w = [n_1\delta_t; (n_1{+}1)\delta_t) \times [n_2\delta_t; (n_2{+}1)\delta_t) \times [n_3\delta_t; (n_3 + 1)\delta_t)$, with $(n_1, n_2, n_3) \in N^3$.

The evolution of the process, between t and $t + \delta_t$ can be written as:

$$p_{t+\delta_t}\{i, x\} = \sum_{\substack{u \in E \\ w \in \mathcal{M}}} p_t\{u, w\} G_{\{u,w\}}^{\{i,x\}} \qquad (1)$$

Where $G_{\{u,w\}}^{\{i,x\}}$ is the probability that the system moves from state $\{u, w\}$ to state $\{i, x\}$ in the time interval $[t; t + \delta_t)$. The conditional probabilities for this model are included in the appendix.

The probability for the unit to be in the state of deterioration j, $Pr(i_A = j)$, at time t is:

$$Pr(i_A = j)_t = \sum_{k,r,s} p_t((j, k), (r, s, t)) \qquad (2)$$

4. Results and Discussions

Both quantification approaches are used to approximate the deterioration states probabilities shown in figure 3. The parameters used are shown in table 1. The deterioration rates have been estimated from previous works carried by the NPRA based on the information available on their database for inspections and maintenance actions, BRUTUS.

4.1. *Monte Carlo simulation vs numerical scheme*

To compare the results of the quantification from both approaches, the residuals or difference between the state probabilities is shown in figure 4.

Table 1. Model parameters.

Deterioration rates (h^{-1})	Maintenance delays (y)	Inspection interval (y)
$\lambda_{12} = 1.5\text{e}{-5}$	$M1 = \infty$	$T = 5$
$\lambda_{23} = 6\text{e}{-6}$	$M2 = 8$	
$\lambda_{34} = 1.4\text{e}{-6}$	$M3 = 3$	
	$M4 = 0.5$	

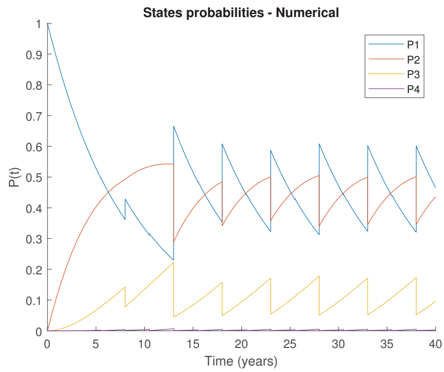

Fig. 3. Deterioration states probabilities

It can be observed that the difference in results is small, with an order of magnitude of 10^{-3}. In addition, the difference is reduced by performing a higher number of replications of the Monte Carlo simulation, showing same convergence.

The Monte Carlo simulation method is widely used in practice, conceptually easy to apply and without particular restrictions on the dimension of the PDMP. On the other hand, the numerical scheme has high accuracy with short computation times, as pointed by Lin et al. (2018). In our case, the Monte Carlo simulation with 100,000 replications took approximately one hour to obtain time-dependent probabilities, while with the numerical scheme the results are obtained in one second.

4.2. *Strategy assessment*

The PDMP allows to test different inspection and maintenance strategies and assess their effect on the structure condition. In a first attempt, we can challenge the inspection period, evaluating the effect on the condition of the structure. Figure 5 shows how the critical condition of the unit ($i_A = 4$), varies with time for different inspection periods. This allows to support the decision process related to inspections by evaluating the associated risk on the structure.

The PDMP framework supports the modelling of a strategy in which the inspection is not performed periodically, but that can instead be dependent on the condition of the structure. The

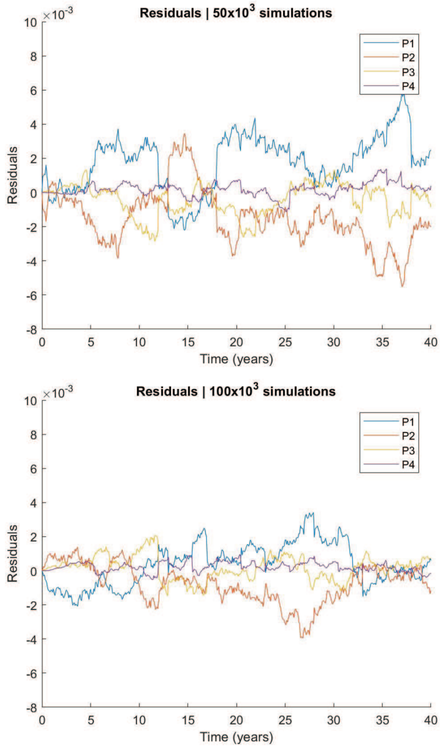

Fig. 4. Residuals between quantification approaches

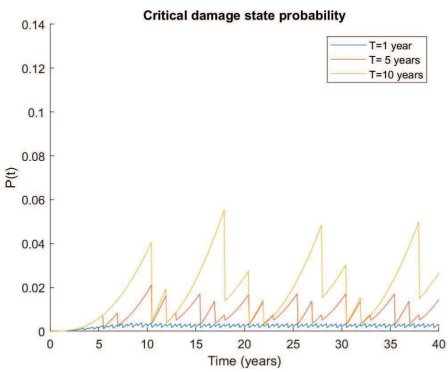

Fig. 5. Critical damage probability for different inspection intervals

model proposed here can be modified to allow for this strategy, in a similar way to how different maintenance delays have been set dependent on the condition of the unit.

Moreover, to assist the decision process in bridge management, the cost of a strategy can be evaluated in addition to the effect on the condition of a structure. In this way the strategy can be optimized, by finding an inspection/maintenance strategy that minimizes the mean cost over a time period, with acceptable risk for the structure. The cost function can be set as:

$$C(t) = N_{insp}(t) \cdot C_{insp} + N_{mr}(t) \cdot C_{mr} + N_{lr}(t) \cdot C_{lr}(t) + N_{cr}(t) \cdot C_{cr}(t) \quad (3)$$

Where:

- C_{insp}: Cost of inspection
- C_{mr}: Cost of medium repair (unit with medium damage)
- C_{lr}: Cost of large repair (unit with large damage)
- C_{cr}: Cost of critical repair (unit with critical damage)
- $N_{insp}(t)$: Mean number of inspections until t
- $N_{mr}(t)$: Mean number of medium repairs until t
- $N_{lr}(t)$: Mean number of large repairs until t
- $N_{cr}(t)$: Mean number of critical repairs until t

The number of inspections and repairs can be counted from Monte Carlo simulations or expressed in terms of the marginal distributions of the PDMP and approximated with the numerical scheme. For example, the mean number of medium repairs until t, can be approximated as the probability that the system jumps from state: $\{u, w\}$ to state $\{i, x\}$ with $u = (2, 2)$ and $i = (1, 1)$ before time t, when δt is small so that the probability of two or more medium repairs in $(t, t + \delta t]$ is negligible, as:

$$N_{mr}(t) \approx \sum_{z=0}^{t} p_t\{(2,2), w\} G_{\{(2,2),w\}}^{\{(1,1),x\}} \quad (4)$$

5. Conclusions and Further Works

Diagnostics allow the application of condition-based maintenance by following pre-established rules and guidelines that state when to perform inspection and maintenance activities. Prognostics empower the decision makers by enabling them to evaluate the effect and cost of a given strategy, therefore allowing to allocate resources in a more efficient manner and optimize the bridge management.

In this paper, we propose a PDMP as a statistical data driven approach to model the deterioration of a structure as a stochastic process, relying on available past observed data, and to make prognosis for a unit that is not monitored continuously but periodically and with significant delay for a maintenance action to be performed.

Two approaches for solving the PDMP are presented. In general, the Monte Carlo simulation approach is conceptually easier to apply while the numerical scheme can provide better accuracy in the results with faster computation times. In the PDMP presented here, the evolution of the continuous component is reduced to a trivial equation. This makes it relatively simple to apply the numerical scheme, presenting a convenient alternative for optimization problems which require testing different strategies, thus repeating the quantification procedure several times.

With support from the NPRA, the work presented here can be developed further. More advanced estimation of parameters for the PDMP can be explored, with sensitivity analysis. Other strategies can be evaluated, such as a condition-based inspection policy rather than inspections performed at equal time intervals, and other maintenance alternatives than as-good-as-new replacements. A PDMP is a framework suitable to model such strategies. In addition, the proposed cost function needs to be addressed together with the definition of constraints on the risk, to optimize the bridge management.

A PDMP presents a framework for hybrid models prognostics, a combination between data-driven and physics-based models, that could be explored for bridge management. It is also of interest to study the application of the PDMP for maintenance models for multi-units systems, accounting for their dependencies, and evaluating the advantages and disadvantages of the numerical scheme and Monte Carlo simulation in these applications.

Acknowledgement

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Appendix. Conditional probabilities for the numerical scheme

Consider the environmental state space cells $w = w_1 \times w_2 \times w_3$ and $x = x_1 \times x_2 \times x_3$. Where w_j, x_j are intervals, e.g. $w_j = [n_j\delta_t, (n_j+1)\delta_t)$ with $n_j \in N$, $j = \{1,2,3\}$. For simplicity, we denote: $w_j = [\underline{w_j}, \overline{w_j})$, where $\overline{w_j} = \underline{w_j} + \delta_t$

Due to the deterioration of the unit, modelled with random jumps, the probability masses move from w to x, which are neighboring cells of the mesh $\mathcal{M}$, i.e. $x_1 = w_1$, $x_2 = w_2$ and $x_3 = w_3 + [\delta_t, \delta_t)$, since only the environmental variable t evolves with speed of one between two consecutive jumps.

The non-null transition probabilities due to a random jump, can be written as:

- $G_{\{(1,1),(w)\}}^{\{(1,1),(x)\}} = 1 - (\lambda_{12}\delta_t)$
- $G_{\{(1,1),(w)\}}^{\{(2,1),(x)\}} = \lambda_{12}\delta_t$
- $G_{\{(2,1),(w)\}}^{\{(2,1),(x)\}} = 1 - (\lambda_{23}\delta_t)$
- $G_{\{(2,1),(w)\}}^{\{(3,1),(x)\}} = \lambda_{23}\delta_t$
- $G_{\{(2,2),(w)\}}^{\{(2,2),(x)\}} = 1 - (\lambda_{23}\delta_t)$
- $G_{\{(2,2),(w)\}}^{\{(3,2),(x)\}} = \lambda_{23}\delta_t$
- $G_{\{(3,1),(w)\}}^{\{(3,1),(x)\}} = 1 - (\lambda_{34}\delta_t)$
- $G_{\{(3,1),(w)\}}^{\{(4,1),(x)\}} = \lambda_{34}\delta_t$
- $G_{\{(3,2),(w)\}}^{\{(3,2),(x)\}} = 1 - (\lambda_{34}\delta_t)$
- $G_{\{(3,2),(w)\}}^{\{(4,2),(x)\}} = \lambda_{34}\delta_t$
- $G_{\{(3,3),(w)\}}^{\{(3,3),(x)\}} = 1 - (\lambda_{34}\delta_t)$
- $G_{\{(3,3),(w)\}}^{\{(4,3),(x)\}} = \lambda_{34}\delta_t$
- $G_{\{(4,1),(w)\}}^{\{(4,1),(x)\}} = 1$
- $G_{\{(4,2),(w)\}}^{\{(4,2),(x)\}} = 1$
- $G_{\{(4,3),(w)\}}^{\{(4,3),(x)\}} = 1$
- $G_{\{(4,4),(w)\}}^{\{(4,4),(x)\}} = 1$

In our case, jumps between non-neighboring cells of the environmental space occur only at inspection and maintenance dates.

At inspection dates ($w_1 = w_3$), the probability masses may move from cell w to cell x, when a maintenance is scheduled or re-scheduled, or may move from cell w to cell y when no maintenance action needs to be scheduled or re-scheduled, with: $x_1 = w_1 + [T, T)$, $\underline{x_2} = min(\underline{w_2}, \underline{x_3} + M_{iB})$, $x_3 = w_3$, $y_1 = w_1 + [T, T)$, $y_2 = w_2$ and $y_3 = w_3$. The non-null transition probabilities of this type are:

- $G_{\{(1,1),(w)\}}^{\{(1,1),(y)\}} = 1$
- $G_{\{(2,1),(w)\}}^{\{(2,2),(x)\}} = 1$
- $G_{\{(2,2),(w)\}}^{\{(2,2),(y)\}} = 1$
- $G_{\{(3,1),(w)\}}^{\{(3,3),(x)\}} = 1$
- $G_{\{(3,2),(w)\}}^{\{(3,3),(x)\}} = 1$
- $G_{\{(3,3),(w)\}}^{\{(3,3),(y)\}} = 1$
- $G_{\{(4,1),(w)\}}^{\{(4,4),(x)\}} = 1$

- $G_{\{(4,2),(w)\}}^{\{(4,4),(x)\}} = 1$
- $G_{\{(4,3),(w)\}}^{\{(4,4),(x)\}} = 1$
- $G_{\{(4,4),(w)\}}^{\{(4,4),(y)\}} = 1$

At maintenance dates, ($w_2 = w_3$), the probability masses move from cell w to cell x, with $x_1 = w_1$, $x_2 = \infty$ and $x_3 = w_3$. The non-null transition probabilities of this type are:

- $G_{\{(2,2),(w)\}}^{\{(1,1),(x)\}} = 1$
- $G_{\{(3,2),(w)\}}^{\{(1,1),(x)\}} = 1$
- $G_{\{(3,3),(w)\}}^{\{(1,1),(x)\}} = 1$
- $G_{\{(4,2),(w)\}}^{\{(1,1),(x)\}} = 1$
- $G_{\{(4,3),(w)\}}^{\{(1,1),(x)\}} = 1$
- $G_{\{(4,4),(w)\}}^{\{(1,1),(x)\}} = 1$

References

Cesare, M. A., C. Santamarina, C. Turkstra, and E. H. Vanmarcke (1992). Modeling bridge deterioration with markov chains. *Journal of Transportation Engineering 118*(6), 820–833.

Cocozza-Thivent, C., R. Eymard, and S. Mercier (2006). A finite-volume scheme for dynamic reliability models. *IMA journal of numerical analysis 26*(3), 446–471.

Cocozza-Thivent, C., R. Eymard, S. Mercier, M. Roussignol, et al. (2006). Characterization of the marginal distributions of markov processes used in dynamic reliability. *International Journal of Stochastic Analysis 2006*, 1–18.

Comission, E. (2008). Council directive 2008/114/ec of 8 december 2008 on the identification and designation of european critical infrastructures and the assessment of the need to improve their protection (text with eea relevance). *Official Journal of the European Union*.

Davis, M. H. (1984). Piecewise-deterministic markov processes: A general class of non-diffusion stochastic models. *Journal of the Royal Statistical Society. Series B (Methodological)*, 353–388.

Kallen, M. J. (2007). *Markov processes for maintenance optimization of civil infrastructure in the Netherlands*. Ph. D. thesis, Delft University of Technology, Delft, Netherlands.

Lair, W., S. Mercier, M. Roussignol, and R. Ziani (2011). Piecewise deterministic markov processes and maintenance modeling: application to maintenance of a train air-conditioning system. *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability 225*(2), 199–209.

Lair, W., R. Ziani, S. Mercier, and M. Roussignol (2012). Maintenance effect modelling and optimization of a two-components system. *Advances in Safety, Reliability and Risk Manag*.

Lin, Y.-H., Y.-F. Li, and E. Zio (2018). A comparison between monte carlo simulation and finite-volume scheme for reliability assessment of multi-state physics systems. *Reliability Engineering & System Safety 174*, 1–11.

Mašović, S., S. Stošić, and R. Hajdin (2015). Application of semi-markov decision process in bridge management. In *IABSE Symposium Report*, Volume 105, pp. 1–8. International Association for Bridge and Structural Engineering.

Morcous, G. (2006). Performance prediction of bridge deck systems using markov chains. *Journal of performance of Constructed Facilities 20*(2), 146–155.

Rausand, M. and A. Høyland (2004). *System Reliability Theory. Models, Statistical Methods, and Applications.* John Wiley & Sons.

Scherer, W. T. and D. M. Glagola (1994). Markovian models for bridge maintenance management. *Journal of Transportation Engineering 120*(1), 37–51.

Statens Vegvesen (2014a). *Bruforvaltning. Handbook R411*. Norway: Statens Vegvesen.

Statens Vegvesen (2014b). *Inspeksjonshndbok for bruer. Handbook V441*. Norway: Statens Vegvesen.

Thomas, O. and J. Sobanjo (2016). Semi-markov models for the deterioration of bridge elements. *Journal of Infrastructure Systems 22*(3), 04016010.

Woodward, R., P. Vassie, and M. Godart (2000). Bridge management in europe (brime): overview of project and review of bridge management systems. In *Bridge Management 4-Inspection, Maintenance, Assessment and Repair*.

Zambon, I., A. Vidović, A. Strauss, and J. Matos (2019). Condition prediction of existing concrete bridges as a combination of visual inspection and analytical models of deterioration. *Applied Sciences 9*(1), 148.