PROPOSAL OF GENERALIZED SLENDERNESS-BASED RESISTANCE CURVES FOR THE LOCAL AND INTERACTIVE BUCKLING OF RECTANGULAR HOLLOW SECTIONS

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The design against local and local+global buckling behaviour of slender square (SHS) and rectangular (RHS) hollow-sections currently contains a degree of conservatism that is seen as hindering a wider application of high-strength steel (HSS) or otherwise innovative hollow sections. The Eurocode and other design provisions define the local slenderness of sections on the basis of the width-to-thickness ratios c/t of individual plates. By using this approach, a significant number of SHS and RHS with fy ≥ 690 MPa – both cold formed and hot-finished – are considered to be slender and are consequently penalized. In addition to that, the classification approach itself presents discontinuities in the estimation of the buckling reduction factor, whereas a continuous design curve ranging from the plastic to the slender range would be more efficient and cost-effective both for steel producers and designers. In this paper, the cross-sectional strength and local buckling behavior of SHS and RHS sections of various slenderness will be analyzed through an extensive parametric FEM-model study calibrated against a large physical test campaign. The data from this study is then used for the development and calibration of new design rules defined on the basis of a "Generalised Slenderness-based Resistance Method" (GRSM). This method makes use of the results of (numerical) linear buckling analyses (LBA) for the overall cross-section or member to determine the slenderness and consequently applies an overall buckling reduction factor. The development of the new GSRM design curves is shown in a step by step approach and their increased accuracy is proven by examples. This study thereby illustrates part of the principal findings of the research project "HOLLOSSTAB", carried out under a grant agreement with the European Research Fund for Coal and Steel (RFCS).

Keywords: local buckling; overall buckling resistance; hollow sections; GSRM; high-strength-steel.

1 Introduction

This paper is concerned with the development of Generalised-Slenderness-based design rules for the cross-sectional (local) buckling behaviour of rectangular and square hollow sections (RHS and SHS). During the European project HOLLOSSTAB, an advanced, non-linear FEM-model, validated and verified through an extensive experimental test campaign in project deliverable D4.2 (Toffolon et al, 2019), was used to carry out an extensive parametric study that formed the basis

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of the calibrations of new design rules for both local and global buckling phenomena. The developed approaches (which in the project were developed for local, global and local+global interactive buckling) are termed the "Generalised Slenderness-based Resistance Method" (GSRM) in the following. This paper provides an overview of the derivation and background of the developed GSRM method. It starts with the description of the scope and methodology of the numerical parametric study. The detailed derivation, as well as the comparison to current design rules, for the local buckling case follows in the subsequent section. Finally, a summary of the design proposal is given. Recent examples of methods based on similar concepts to the GSRM are found e.g. in Schafer (2018), Gardner (2018) and Boissonnade (2019).

2 Scope and Methodology

Two types of cross-sections are the subject of the present study and form one part of the scope of the HOLLOSSTAB project. Figure 1. gives schematic representation. The sections are rectangular (SHS, RHS) hollow sections produced in accordance with the European fabrication standard EN 10219 (2006)0, which applies to cold-formed sections, with steel grades from (normal strength) S355 to (high strength) S700. This section of the paper describes the scope and methodology of this series of tests and numerical analyses.

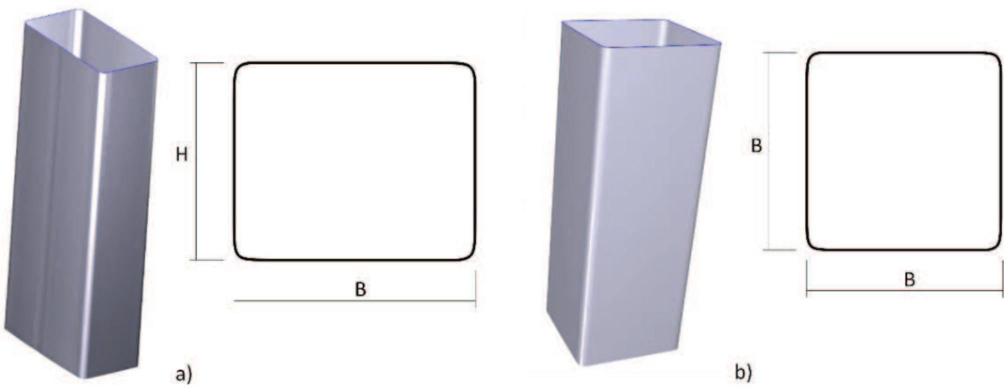


Figure 1. Studied types of cross-section: cold- and hot-finished RHS and SHS, mild to high-strength steel

2.2 *Numerical methodology and campaign*

The FEM model for the numerical campaign was developed and validated in Toffolon and Taras (2019), where the methodology for the numerical calculations is explained in detail. In summary of this methodology, two different types of analyses were typically carried out during the numerical test campaign for each "numerical specimen": a Linear Buckling Analysis (LBA) for the determination of the imperfection shape, and a Geometrically and Materially Non-linear Imperfection Analysis (GMNIA) for the determination of the ultimate amplification factor, i.e. the "resistance" in terms of a load amplifier for the applied loads.

Two different material models and calibrated imperfections were used in the parametric study for hot-rolled and cold-formed steel sections, see also a second paper by the authors at this conference and Toffolon et al. (2019). Figure 2 a) shows a schematic representation of the material model for hot-finished steel and cold-formed steel is shown. In Figure 2 b) and c) the shape of a local buckling mode (taken from an LBA) is illustrated, as well as the chosen imperfection amplitude for the parametric study of e_0=b/400, where *b* is the longest cross-section side of the RHS and SHS.

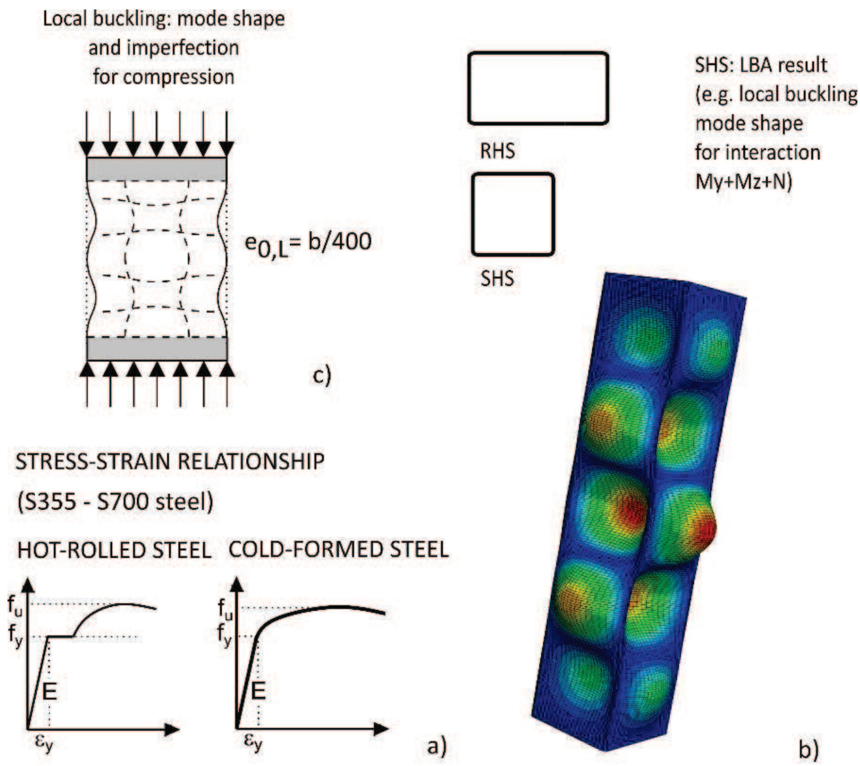


Figure 2 Material model chosen for the EN10219 and EN10210 standards; b) and c) LBA shape for the local imperfections

Table 1. Parameters of the parametric study on local buckling.

Thickness (mm)	L/L_{cr} (-)	Steel grade	Local Imperfection amplitude	ϕ_y (°)	ϕ_z (°)	h/b (-)	Manufacturing standard
2.0	0.1	S355	b/400	0	0	1	EN10219
2.5	0.15	S460		15	15	1.5	EN10210
3.0	0.2	S550		30	30	2	
3.5		S700		45	45		
4.0				60	60		
5.0				75	75		
6.3				90	90		
8.0							
10.0							
12.0							

The numerical test campaign conducted for the study of local buckling consisted of around 30000 numerical tests (LBA+GMNIA), the parameters of which are shown in Table 1 in form of a numerical test matrix. All shown parameters were combined with all other given parameters.

Thereby, in the table, the following variables are used:

- $\frac{L}{L_{cr}} = min(\frac{I_y}{A};\frac{I_z}{A})\pi\sqrt{\frac{E}{fy}}$; $\phi_y = arctan\left(\frac{m_y}{n}\right)$; $\phi_z = arctan\left(\frac{m_z}{n}\right)$, and
- $m_y = \frac{M_{y,Ed}}{M_{y,pl}}$; $m_z = \frac{M_{z,Ed}}{M_{z,pl}}$; $n = \frac{N_{Ed}}{N_{pl}}$.

The aim of the extensive numerical parametric study on the local buckling behaviour of slender and non-slender cross-sections was the realistic determination of the cross-section capacity under different load combinations and for various degrees of local slenderness. For this reason, a large number of thicknesses and load combinations were considered.

2.3 *Presentation format*

In the GSR method developed in the HOLLOSSTAB project, a generalized definition of the cross-section resistance is used, based on a generalized definition of the slenderness $\bar{\lambda}$ and an overall knock-down factor χ. Figure 3 gives an overview of the GSRM design steps and corresponding parameters as applied to the local case. The definition of the "resistances" R as load amplification factors for all GSRM parameters is shown in Figure 3 b).

In order to find the most effective definition of slenderness and buckling reduction factor, a number of attempts were made (Taras et al., 2019). For the case of local buckling, this paper makes use of the following definition:

$$\bar{\lambda}_L = \sqrt{\frac{R_{cr}}{R_{el}}} \qquad \chi_L = \frac{R_{b,L}}{R_{el}} \tag{1}$$

where R_{el} is the elastic (first-yield) cross section resistance, R_{cr} is the critical load amplification factor and $R_{b,L}$ is the buckling resistance (again as load amplification factor). The final design curve of the HOLLOSSTAB project is formulated in this format and is schematically shown in Figure 3 a) together with the steps of the calculation of $R_{b,L}$.

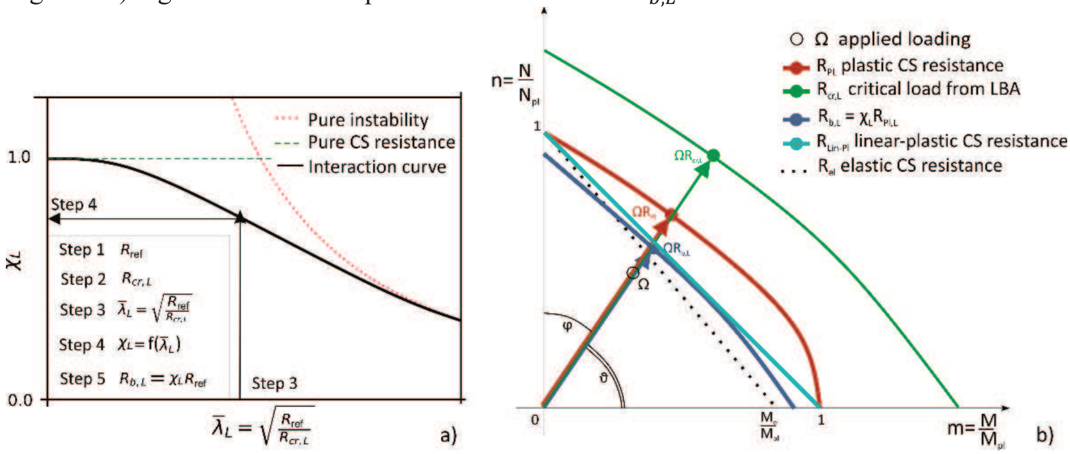


Figure 3 a) steps for the calculation of the buckling reduction factor and a calibrated design curve; b) various load amplification factors for the local case.

3 Derivation and Calibration of Design Formulae

The derivation and calibration of new, GSRM-type design formulae was carried out for two distinct ranges. In 3.1 the GSRM design curve for the elastic range will be derived from the theory of plate buckling and the calibration of modified Winter formulae, familiar from plate buckling cases as defined e.g. in Eurocode 3 (EN 1993-1-5). For the "plastic" range of slenderness a different formulation is provided as a simplified and calibrated linear function between $\bar{\lambda}_L$ and χ_L.

3.1 *Winter-type rules in the elastic range*

Early studies on plate buckling investigated the simplified model of a thin plate (high width to thickness ratio) supported on both sides and subjected to a constant in-plane compression load. The reduction factor ρ and the plate slenderness $\bar{\lambda}_p$ can thus be defined as follows:

$$\rho = \frac{b_{eff}}{b} \quad ; \quad \bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{crit}}} \tag{2}$$

Using the terms $\bar{\lambda}_p$ and ρ, the plate buckling knock-down factor according to Winter reads:

$$\rho = \frac{1}{\bar{\lambda}_p}(1 - \frac{0.22}{\bar{\lambda}_p}) \tag{3}$$

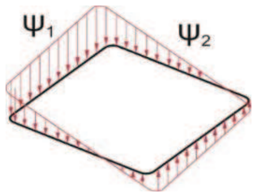
where the coefficient 0.22 was finally chosen after different calibrations and proposals (see Winter 1946). The Eurocode method for plate buckling applies the findings of Winter and assesses separately ρ for each cross-section plate introducing the parameter ψ, as the ratio between the stress along each plate. The reduction factor and the plate slenderness are defined as follows:

$$\rho = \frac{1}{\bar{\lambda}_p}\left(1 - 0.055\frac{3+\psi}{\bar{\lambda}_p}\right) \quad ; \bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{crit}}} = \frac{\bar{b}/t}{28.4\sqrt{\frac{235}{f_y}}\sqrt{k_\sigma}} \tag{4}$$

The chosen approach for the GSR Method developed in HOLLOSSTAB makes use of Winter's formula, generalising it to describe the behaviour of the entire cross-section as observed in the project's physical and numerical tests. The slenderness and the reduction factors refer to the cross-section properties (geometry and steel grade) and to the applied load. Thus, in order to use the Winter formulation and fit it to the results of the parametric study, ρ and $\bar{\lambda}_p$ are rewritten as χ_L and $\bar{\lambda}_L$ in the GSRM and using the coefficient A for calibration:

$$\chi_L = \frac{1}{\bar{\lambda}_L}(1 - \frac{A}{\bar{\lambda}_L}) \tag{5}$$

Thus, in HOLLOSSTAB, the parameter A was calibrated to the results of the extensive numerical parametric study on local buckling. In this calibration, it was seen to be conducive to good results to define A as a function of ψ_1 and ψ_2, in partial correspondence with the Eurocode approach. ψ_1 and ψ_2 are the stress ratios in the two plates adjacent to the corner with the highest compressive stress in the section, see Figure 4. They are determined in a simplified manner, discounting the presence of a rounding at the edges of RHS and SHS sections. This is justified by the small difference in the actual stress state and the increased use of ease of the formulations.



$$\Psi_1 = MAX\left(\frac{\frac{N}{A}+\frac{M}{Wy}-\frac{M}{Wz}}{\frac{N}{A}+\frac{M}{Wy}+\frac{M}{Wz}}\,;\,\frac{\frac{N}{A}-\frac{M}{Wy}+\frac{M}{Wz}}{\frac{N}{A}+\frac{M}{Wy}+\frac{M}{Wz}}\right)$$

$$\Psi_2 = MIN\left(\frac{\frac{N}{A}+\frac{M}{Wy}-\frac{M}{Wz}}{\frac{N}{A}+\frac{M}{Wy}+\frac{M}{Wz}}\,;\,\frac{\frac{N}{A}-\frac{M}{Wy}+\frac{M}{Wz}}{\frac{N}{A}+\frac{M}{Wy}+\frac{M}{Wz}}\right)$$

Figure 4 Definition and graphic representation of ψ_1 and ψ_2 as used in the GSRM formulation for the cross-sectional capacity

Initially, a formulation for the parameter A was sought that describes the cases with compression and mono-axial (or "in-plane") bending, about either axis. This corresponds to all cases where the stress ratio ψ_1 is equal to 1,0 (pure compression in one of the plates). For cold-formed (EN 10219) and hot-finished sections (EN 10210), the following linear functions were determined through calibration and the final choice of practical, easy-to-use functions and coefficients.

$$A = 0.225 + 0.025\psi_2 \text{ (cold-formed sections)} \tag{6}$$

$$A = 0.20 + 0.02\psi_2 \text{ (hot-finished sections)} \tag{7}$$

The validation of these calibrated functions fir an exemplary combined N+M_y load case for hot-finished sections is shown in Figure 5 a). In Figure 5 b), the design curve for cold-section is shown: the pure compression case ($\psi_1 = +1$) and the pure bending case ($\psi_1 = -1$) are displayed with different colors, and area between the lines corresponds to the N+M_y combinations.

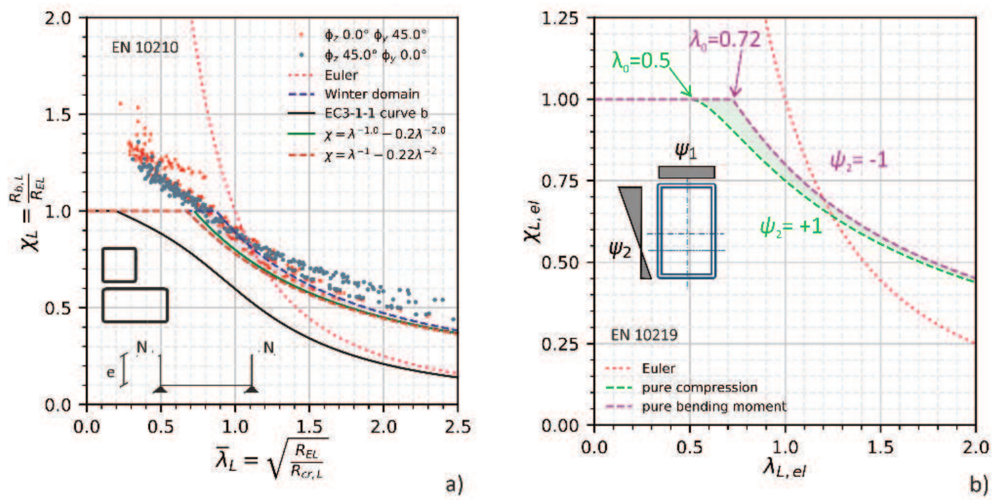


Figure 5 a) exemplary calibration (mono-axial case) for hot-finished sections; b) overview of the GSRM design curve (mono-axial case) for cold-sections.

Once the calibration for the mono-axial cases was achieved, the formulation was expanded to account for different stress ratios in the "most compressed" plate as well, i.e. for cases with bi-axial bending, with the results given in Equ. (8) and (9). The additional multiplier as a linear function covers the biaxial load case and is formulated as a generalization of the one found in EN 1993-1-5 for linear stress fields in an individual plate:

$$A = (0.225 + 0.025\psi_2)\frac{(1+\psi_1)}{2} \text{ for cold-formed sections} \tag{8}$$

$$A = (0.2 + 0.02\psi_2)\frac{(1+\psi_1)}{2} \text{ for hot-finished sections} \tag{9}$$

The point where the slender range ends is denoted as the "elastic limit slenderness" λ_0 and is calculated as follows: $\lambda_0 = 0.5 + \sqrt{0.25 - A}$

3.2 *Bilinear function in the stocky range*

A bilinear relation was chosen the represent the resistance in the stocky range, where the cross-sectional capacity exceeds R_{el}. Two "anchor" points are needed for this purpose:

- $\chi_L = 1$ at $\bar{\lambda}_0$
- α_{pl} at $\bar{\lambda}_L = \bar{\lambda}_{pl} = 0.3$

The proposed formulation for the prediction of the cross-section capacity for the stocky range is as follows, with the values for $\bar{\lambda}_{pl}$ and the maximum value α_{pl} taken to represent the data with acceptable safety and accuracy. Thus, in summary, in the stocky range the GSRM design proposal reads as follows:

$$\text{For } \bar{\lambda}_L \leq \bar{\lambda}_0 : \chi_L = 1 + (\alpha_{pl} - 1)\left(\frac{\bar{\lambda}_0 - \bar{\lambda}_L}{\bar{\lambda}_0 - \bar{\lambda}_{pl}}\right) \leq \alpha_{pl} \tag{10}$$

$$\text{where } \bar{\lambda}_{pl} = 0.3; \quad \alpha_{pl} = \frac{R_{pl}}{R_{el}} \leq 1.5\ ;\ \lambda_0 = 0.5 + \sqrt{0.25 - A} \tag{11}$$

4 Comparison of FEM Results vs. GSRM and EC3 Rules

For the validation of the design rule in the GRSM, the results of the design method were compared in HOLLOSSTAB with the GMNIA results and with the current EC3 design method. Only a short overview of these validation efforts can be shown in this paper, see Figure 6. The results of the GMNIA calculations are in most of the cases higher than the GSRM results, but fall in a range fairly close to the design values, providing generally low values of scatter and much better predictions than Eurocode 3. The scatter is represented in the plots by the horizontal lines for each class of cross-section (Classes 1-4, cl.1+2, cl.3 and cl.4 in the figures) by indicating the mean value (upper line) and the m-2s value, whereby s is the calculated standard deviation for the considered cross-sectional class. Thus, as can be seen in Figure 6, the proposed formulation leads to a relatively stable degree of scatter and therefore to a fairly homogenous (and low) level of average conservatism throughout slenderness classes, for all load cases and cross-sectional shapes.

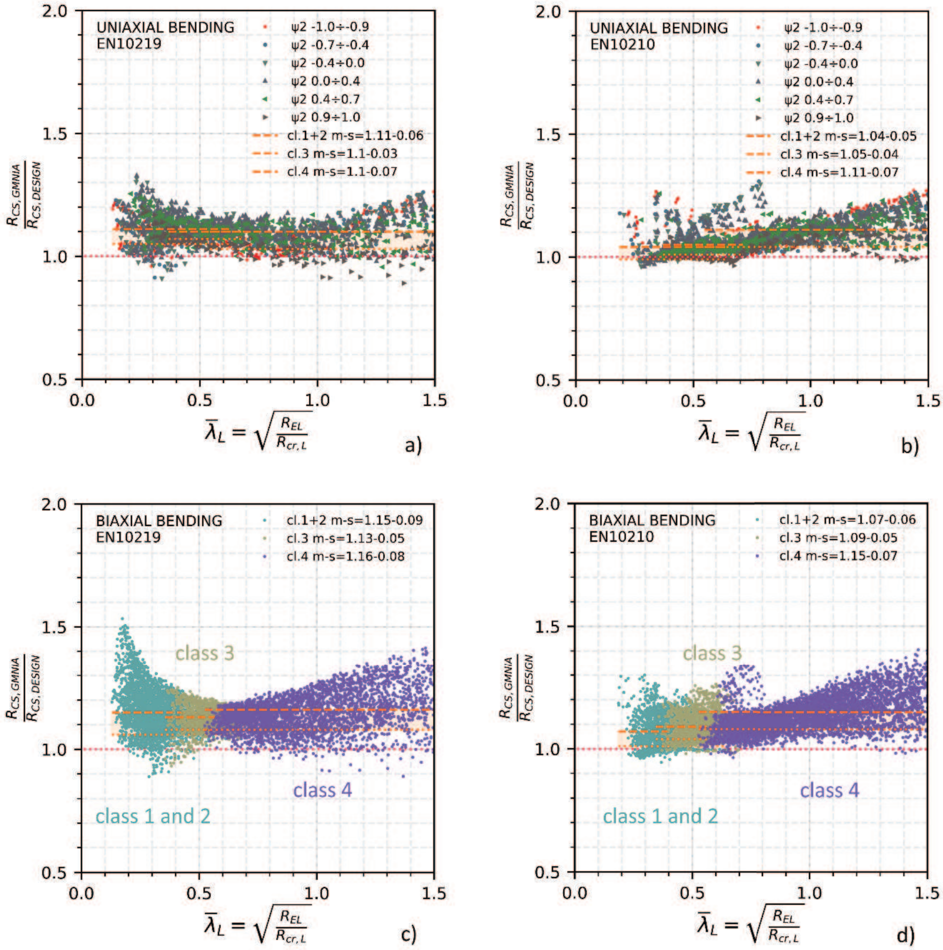


Figure 6 Graphical validation of uniaxial bending + compression cases for a) cold-formed and b) hot-finished sections, where marker colours correspond to different load cases; c) and d): cases with compression and biaxial bending, categorized by EC3 classes.

5 Conclusions and Outlook

This paper presented a new design approach for the determination of the cross-sectional strength and resistance against local buckling of rectangular and square hollow sections made of mild and high-strength steel grade. The method was developed during the European (RFCS) project HOLLOSSTAB and is termed the "Generalised Slenderness-based Resistance Method". Due to lack of space, only the rules for the resistance at the cross-sectional level were shown here. However, in the project new rules for global and interactive (local+global) buckling were developed as well, see Taras et al. (2019a). The overall results of the newly-developed GSRM approaches for hollow sections lead to significant increases of economy and mechanical accuracy.

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