LATERAL TORSIONAL BUCKLING BEHAVIOUR OF RECTANGULAR HOLLOW SECTIONS (RHS) WITH CIRCULAR WEB OPENINGS

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Circular web penetrations in beams are commonly used for services in order to reduce floor to floor height in construction. This will create multiple local variations in section stiffness properties, most usually minor axis bending stiffness (I_y) and torsional stiffness (J). This paper presents a finite element study that firstly examines the reduction in bending or torsional stiffness caused by a variety of web hole sizes and spacing, and the subsequent impact on lateral torsional buckling behaviour. Preliminary design recommendations have been developed based on the results of the FE models and parametric study. The design methods are presented as simple reduction equations for both second moment of area and torsional stiffness.

Keywords: Lateral buckling; web penetrations; finite element analysis; structural steel hollow sections (SSHS).

1 Introduction

The aim of this paper is to investigate a simple mechanism to account for the impact of web penetrations on the various geometric stiffness properties of hollow sections, and the consequential reduction in lateral torsional buckling strength. Web penetrations in beams are commonly used for services in order to reduce floor to floor height in construction. Figure 1 shows components of the Slimfloor system, where traditional I shapes are used for internal beams, but edge beams are sometimes specified as hollow sections.

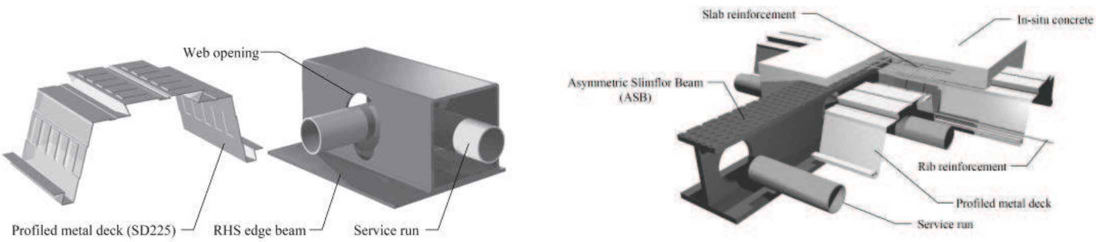

Figure 1. Examples of beams with web penetrations – Slimfloor System (Ridley-Ellis 2000) – using either (a) Hollow Sections or (b) I sections

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A hole will create multiple local variations in section stiffness properties. Naturally where there is no hole, the section maintains its original geometric properties, but at cross-sections that intersect with a hole, there will be reductions in stiffness. If there are a series of penetrations along the length of a beam, there will be periodical variations of these properties.

For I sections:

- The major axis bending stiffness, I_x, has a small reduction as the hole is in the web centred on the major neutral axis
- The minor axis bending stiffness, I_y, has (almost) no change, as the hole sits entirely on the minor neutral axis.
- The warping torsion stiffness, I_w, has (almost) no change, as the hole sits entirely on the web (warping torsion is essentially a function of the flanges only)
- The uniform shear torsion stiffness, J, could be notably reduced depending on the hole size.

However for hollow sections, the relative impact on section properties is quite different, as the webs do not lie on the minor neutral axis

- The major axis bending stiffness, I_x, has a small reduction as the hole(s) is in the web centred on the major neutral axis
- The minor axis bending stiffness, I_y, has a potentially notable change, as the hole sits at maximum distance from the minor neutral axis.
- The impact on torsional behavior is potentially complex. Closed cell hollow section have no warping behavior, but exhibit extremely high uniform shear torsion stiff J (approximately 1000 times the value for a similarly dimensioned I section). The presence of (multiple) holes, particularly if close together, may cause a hollow section to demonstrate more open-section- like torsional behavior, which is potentially much less stiff The warping torsion stiffness, I_w, has (almost) no change, as the hole sits entirely on the web (warping torsion is essentially a function of the flanges only)

It is therefore usually accepted that web penetrations in I-sections do not have major impact on bending behavior, but there is often considerable drop in shear strength. This is often ameliorated via the use of stiffeners, or placing the holes in areas of relatively low shear. There are widely used design guides for such cases (for example SCI (2011)), which contain comprehensive reference lists of the experimental, theoretical and finite element bases for the design recommendations. However, there is little previous research on the impact of web penetrations of bending behavior of hollow sections.

Ridley-Ellis (2000 & 2001) performed bending and torsion tests on RHS with a single hole (on both webs), and extended the study via FEA to consider multiple holes. His research primarily focussed on section capacity and localized buckling effects around the hole. He included recommendation about critical hole size and spacing in order to achieve Class 1 and 2 bending behavior.

Feng et al (2017) performed experiments on aluminum SHS and RHS under bending particularly focusing on flange and web plate slenderness ratios. Multiple perforations did not necessarily change the buckling failure modes observed. The different material (aluminium) exhibited reasonably similar behavior to the carbon steel results of Ridley-Ellis.

The loss of minor axis and torsional stiffness is likely to impact the lateral torsional buckling strength of such beams. Hence, the aim of this paper is to investigate a simple mechanism to account for the impact of web penetrations on the various geometric stiffness properties of hollow sections, and the consequential reduction in lateral torsional buckling strength.

2 Project Aim, Methodology and Scope

This paper summarises an undergraduate thesis topic performed at the University of Sydney, undertaken by authors 1 and 2 (Badyari 2018, and Black 2018), under the supervision of the 3rd author, and significant finite element input from the 4th author. An undergraduate thesis represents 25 % of the enrolment load of students across both semesters of their final year. They also undertake significant coursework while performing their research thesis, and by necessity the scope of such a project must be kept quite narrow, hence only linear elastic buckling (global, not local) was considered.

Figure 2 defines the geometry and nomenclature for the geometry of the hollow sections and the holes (size and spacing). The potential variables include:

- Diameter of the hole (relative to RHS depth), D/d
- Number of holes, N
- Hole spacing density, N/L,
- Aspect ratio of the RHS, d/b
- Thickness of the RHS, t
- Length of the RHS, L.

The presence of multiple holes down the length will create a variation of the different stiffness properties at each cross section. One key aim is to attempt to define and calculate a single effective stiffness (either flexural or torsional) for the member

Rather than (initially) perform any type of finite element buckling analysis, the first step was to model simple elastic deflections for minor axis bending, or simple twisting rotations for torsional stiffness. Deformations could be compared against simple well known closed form solutions to elastic deformation problems, in order to obtain estimates of $I_{y,\ \mathrm{effective}}$ and $J_{\mathrm{effective}}$. under various hole geometries. A relationship between the relative drop in stiffness compared to a section with no holes could then be obtained. These effective stiffness values can be used in well known elastic buckling equations, such as $M_{cr,\,effective} = \frac{\pi}{L}\sqrt{EI_{y,\,effective} \cdot GJ_{effective}}$.

Linear elastic finite element buckling analysis was finally performed on models with different hole geometries to obtain a finite element prediction of the buckling moment. The finite element buckling results were compared against the finite element buckling predictions for sections with no holes. A process of back calculation using well established buckling formula was then used to determine the drop in effective stiffness as a function of different hole patterns.

The two different versions of the change in stiffness could then be compared.

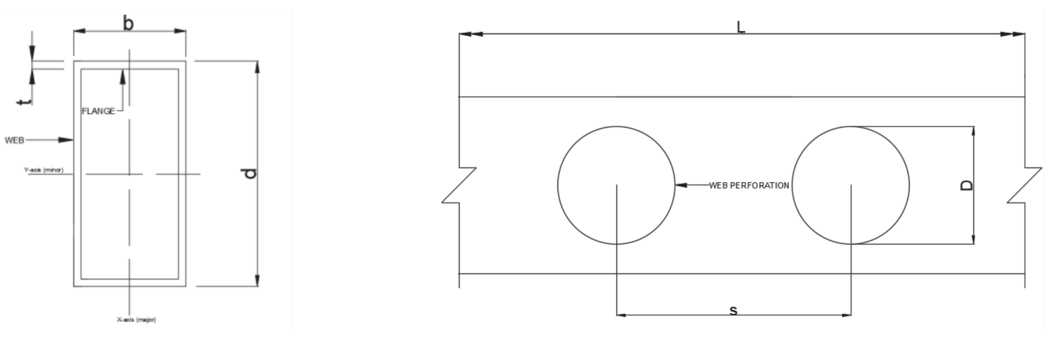

Figure 2. Definition of geometric variables

3 Finite Element Modelling

ABAQUS was used to perform the finite element analysis using shell elements. ABAQUS has a "circular cut" tool, which allows for automatic inclusion of holes and remeshing. Details of the entire finite element process and the mesh refinement process (in particular) are available in Badyari (2018) and Black (2018), but Figure 3 shows a portion of a typical mesh of 12 element down a web, morphing into a irregular mesh around the hole which had roughly twice the mesh density in order to account for higher stress and strain gradients around the holes.

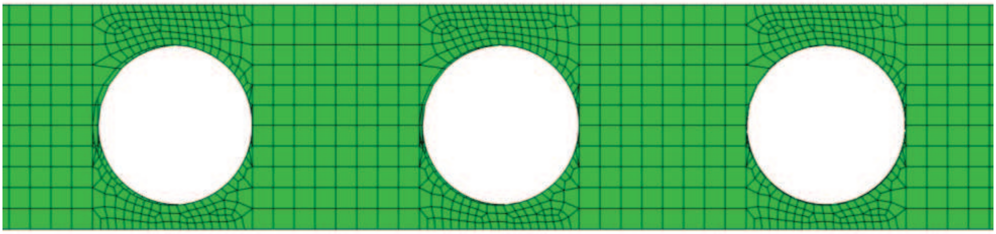

Figure 3. Typical meshing around holes

4 Elastic deformation modelling

Simple elastic bending theory for a simply supported beam with uniform end moments, and hence a uniform bending moment, as shown in Figure 4, gives the mid-span deflection as $v = ML^2/8EI$. Similarly, the end rotation of a column with one fixed end a uniform torque applied at the free end is $\phi = TL/GJ$.

Figure 4. Simply supported beam with end moments

Various beams with different hole patterns were subject to either simple bending or torsional load. The deflection or end twist rotation was extracted from ABAQUS, and the effective flexural or torsional stiffness values obtained by re-arranging the simple elastic formulae above. The results are best represented by expressing the (reduced) effective stiffness as a ratio (less than one) compared to the stiffness of a model with no holes. Figures 5, 6 and 7 show some typical results of the impact of hole space density (for different hole sizes) against the reduction in stiffness.

Naturally effective stiffness dropped with bigger hole size, and a higher hole density (ie closer spacing). Figure 7 (plus other results shown in the full theses of Black and Badyari) indicate that reduction rate was fairly insensitive to changing aspect ratio or RHS thickness. The only significant parameters were hole size and spacing.

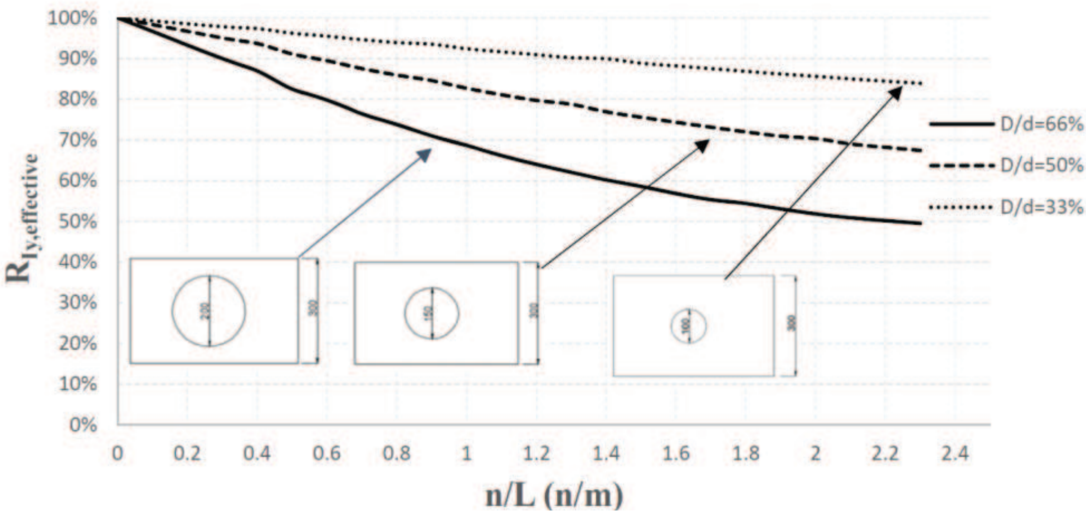

Figure 5. Effective bending stiffness reduction with increasing hole density

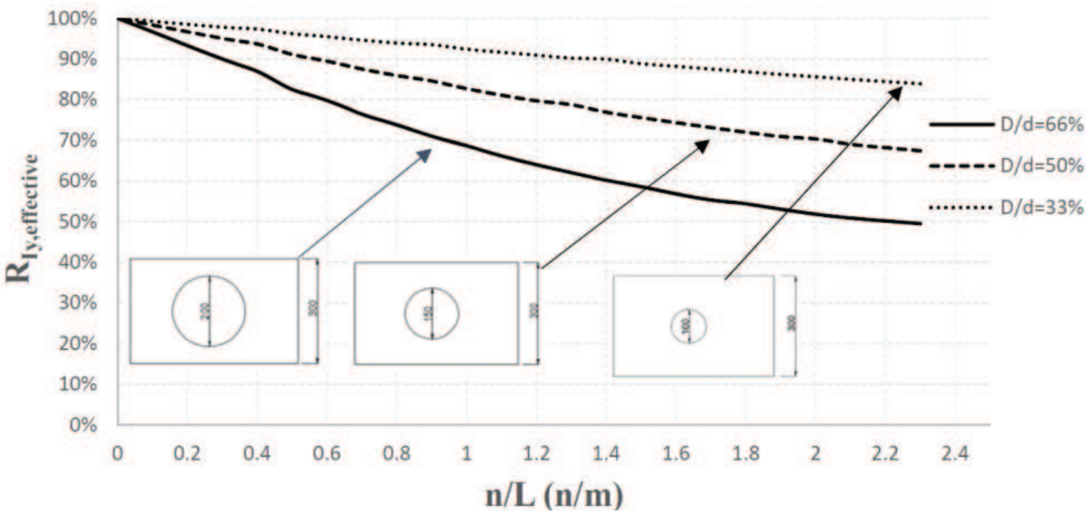

Figure 6. Effective torsional stiffness reduction with increasing hole density

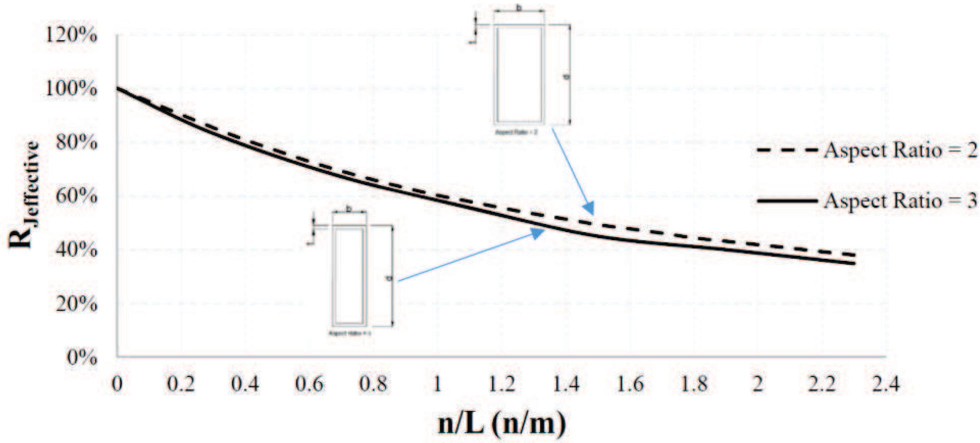

Figure 7. Effective torsional stiffness reduction with increasing hole density – impact of aspect ratio

5 Comparison to Finite Element Linear Buckling Analyses

Linear buckling analyses were performed on specimens with no holes, and specimens with various hole sizes and spaces. These finite element results were in the form of a critical elastic buckling moment, and a hence a reduction factor (in terms of moment) could be created. Similarly, using the deflection calculated effective stiffness values from Section 4 above, and using the standard elastic lateral torsional buckling equation, a second critical moment reduction relationship could be obtained. These two differently derived reduction relationships are compared in Figure 8 below. The similarity in the two approaches (as evidenced by the ratio being approximately 100% regardless of hole size or spacing), suggests the two approaches are equally valid.

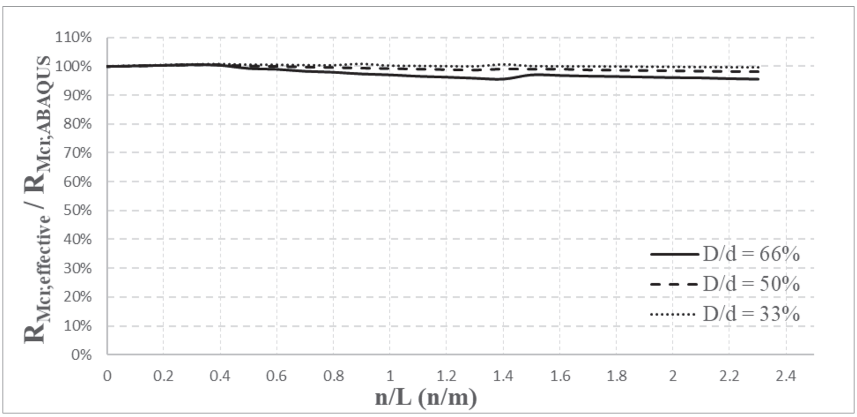

Figure 8: Comparison of methods to establish critical buckling moment reduction factors

6 Parameterization of geometric effects obtained from simple stiffness simulations

It was previously identified that only hole size and spacing had a meaningful effect on flexural and torsional stiffness reduction, and hence these parameters are isolated in the 3D plots of Figure 9.

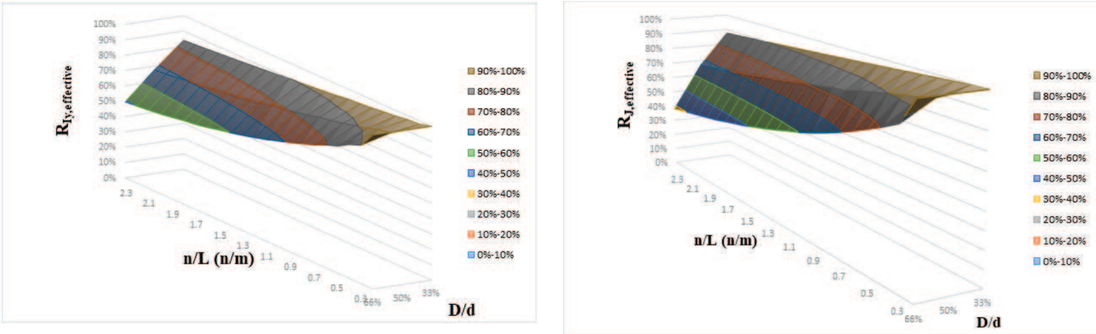

Figure 9. Stiffness reduction factors as functions of hole spacing and size

Polynomial regressions utilise the least squares method to predict the relationship between the independent and dependent variables. This method attempts to minimise the sum of squared residuals in the data. The polynomial regression was conducted in Microsoft Excel. Upon initial investigation into Figure 4-5 and 4-7, it was found that using the trendline application, that the graphs were best represented using a quadratic model. This information combined with the curvilinear relationship, was the reason a quadratic model was proposed when applying the least squares method to predict the relationship. The polynomial regression found:

$$R_{c,Iy} = 1.05 - 0.43\tfrac{D}{d} \times \tfrac{n}{L} - 0.03\left[10\left(\tfrac{D}{d}\right)^2 - \left(\tfrac{n}{L}\right)^2\right] \quad R_{c,Iy} \le 1$$

$$R_{c,J} = 1.10 - 0.50\tfrac{D}{d} \times \tfrac{n}{L} - \left[0.66\left(\tfrac{D}{d}\right)^3 - 0.04\left(\tfrac{n}{L}\right)^2\right] \quad R_{c,J} \le 1$$

7 Maximum spacing density

The results of Ridley-Ellis (2000) identified “zones of influence” for which holes close together would impact each other. These zones were considered to be approximately three times the hole diameter. As part of this project, the simple elastic stress distributions (shear stress due to torsion) were investigated to establish any similarity with those recommendations.

In order to determine what the maximum spacing density should be, a criterion needed to be determined. The logic behind the criteria was based on the reduction in cross section due to the introduction of the holes. The ‘large’ holes reduced the web by 66%. This meant that 66% less area that could be utilised to distribute the applied load. Therefore, it was decided that the maximum allowable change in stress across the region between holes was to be related to the reduction in cross section. The initial stress was taken as where the stress was uniform across the depth of the section.

Hole density was thus increased until there was a maximum of three times the stress experienced across the depth of the section between adjacent holes. This is depicted in Figure 10. In this case, the hole size is 66% of the web, so under uniform stress conditions, this would create a stress increase of 3. The uniform stress in the situation shown was the light blue region adjacent to the end of the beam. Looking at the region between the holes, the maximum stress is roughly 3 times larger than the stress at the ends. In this case, the geometry reasonably aligned with the recommendation of Ridley-Ellis (2000).

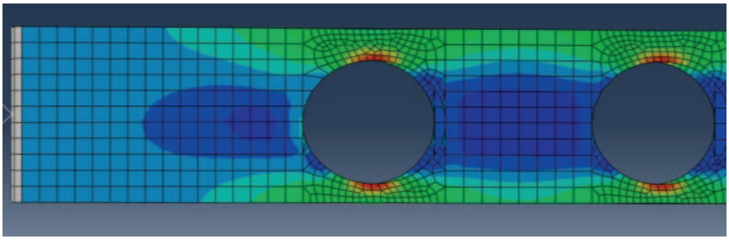

Figure 10. Determination of the maximum allowable spacing density

8 Conclusions

This paper describes a small scale research project using finite element analysis investigating the elastic lateral torsional buckling behavior of rectangular hollow sections with large web penetrations. Parameters such as RHS aspect ratio and thickness were found to have limited relative impact on the buckling moments. Relative hole size, and hole spacing were found to have the most profound effect.

The presence of multiple web holes along the length of a beam create continually changing cross sections and hence change stiffnesses.

Bucking analyses can often be difficult, so the project sought to investigate a simpler procedure. Simple elastic stress and stiffness models first investigated creating a simple effective flexural or torsional stiffness that could be singularly applied to the entire beam. Polynomial regression equations were proposed to predict these reduced stiffnesses.

These reduced stiffnesses were used in the traditional well known elastic buckling formula, and compared to the elastic buckling predictions of a more complete finite element analysis, and very similar trends were observed. This would suggest this simplified approach has merit.

It is recommended that extending further finite element analyses to incorporate material and geometric non-linearity be undertaken before adopting this process.

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