LOCAL BUCKLING BEHAVIOUR OF RECTANGULAR HOLLOW SECTION UNDER COMBINED BENDING AND SHEAR

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Classification of beam cross sections for local buckling under bending loads according to whether the capacity reaches the yield of plastic moment is usually based on experiments or computer simulation of normal stresses only to produce slenderness (*b/t*) limits. However for short beams, or near the end of fixed ended beams there can be significant shear loads. This paper presents a finite element study designed to link the local buckling behaviour of simple plates, to those of rectangular hollow sections. The main complexity is the non-linear shear stress distribution in an RHS web under shear. The paper presents some interaction curves to assist in the design on beams under bending and shear.

Keywords: local buckling; bending; shear; finite element analysis; structural steel hollow sections (SSHS).

1 Introduction

1.1 *Local buckling of beams in bending*

Typically, the response of a steel beam under bending can be characterized by a moment – curvature graph, such as the one shown in Figure 1. The initial response is linear, until, the extreme fibres reach the yield stress at the yield moment (M_y), where $M_y = f_y Z$ and Z is the elastic section modulus. At larger curvatures and strains, yielding spreads inwards toward the neutral axis. For the elastic - plastic - strain hardening material, the section yields almost completely and at high values of curvature the plastic moment (M_p), where $M_p = f_y S$ and S is the plastic section modulus. However, at any stage during this bending process, the beam may fail by either local instability or lateral torsional instability (though lateral torsional instability is rare for RHS due to the extremely high torsional stiffness J of closed cell shapes).

It is very well established that the elastic local buckling stress (f_{cr}) of plate of width b and thickness t is given by $f_{cr} = k\pi^2 E / 12\left(1-\nu^2\right)\left(b/t\right)^2$ where k is the plate buckling coefficient. The value of k depends on the nature of the stress distribution across the plate and the support conditions of the plate. For an RHS web in flexure, k = 23.9 as the normal stress varies from maximum tension to maximum compression.

By equating the local buckling stress to the yield stress, one calculates a critical *b/t* value that differentiates when yielding occurs before buckling (and vice versa). This creates the concept of a slenderness limit between non-compact/slender or Class 3/4 behaviour.

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Appropriate modifications must be made to account for imperfections and early yield from residual stress. It is likewise possible to obtain slenderness limits between the other classes of behavior.

Consolidating the results and recommendations of a number of CIDECT research projects, Wilkinson (2003) provides a summary of much of the previous research in this area of RHS slenderness limits. However, more often than not, experiments are based on 4 point bending arrangements, and finite element analyses often focus on pure bending creating only normal stress. Hence there is less knowledge of the impact of shear.

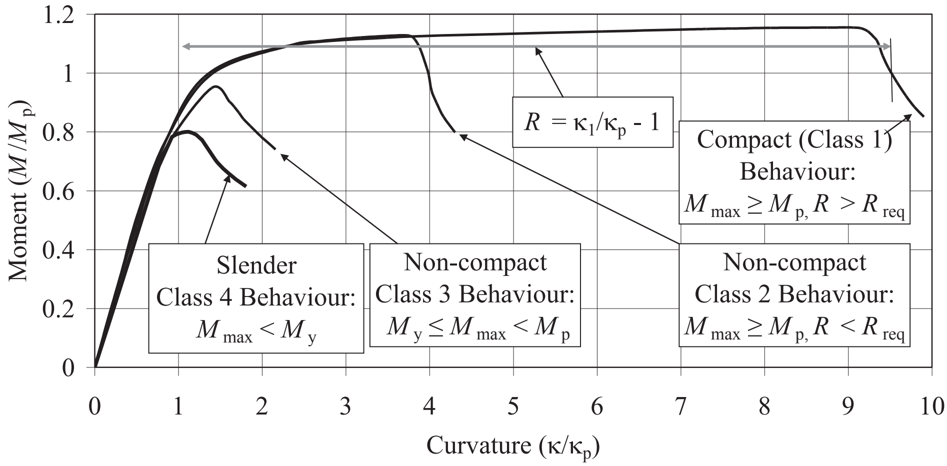

Figure 1: Moment-curvature behaviour of different types of steel sections

1.2 *The impact of shear*

A bending moment gradient, as opposed to uniform bending moment, is required to have non-zero shear force in a beam. In a real frame, the ends of beams near rigid moment connections are likely to experience combined shear and bending. The relative magnitude of the shear compared to the magnitude is often a function of the beam span.

Investigating the relationship between moment and shear is challenging. Experimentally, four point bending tests usually fail in the central maximum (and constant) moment zone with no shear, while three point bending tests are usually on short beams and have relatively high shear force. More recently, test setups that use two actuators (Motallebi et al 2019) can more easily replicate a range of shear to moment ratios. It is sometimes problematic in finite element simulations to introduce a shear force that creates the same non-linear shear stress distribution experienced in an RHS beam.

The authors were unable to find any meaningful studies on hollow sections the primarily focused on the impact of shear stress on bending behavior, though studies on I-section webs and isolated plates were found. A key issue is that web shear stress in an I-section is usually almost uniform, which is not the case for RHS.

The effect of shear stress on tapered steel plate was conducted (Bedynek, Real, & Mirambell, 2013) with both experimental and numerical research. Although the research was conducted on a different section, the method of their research can be utilized for this study, especially the numerical simulation. Shear stresses were applied on the cross-section in different directions, leading to positive and negative influence on the section. A study on tapered bridge girders were performed (Abu-Hamd & El Dib, 2016). The method of analysis used in the tapered bridge girder research is very similar to the analysis method in this study. Pure bending and shear stress was initially applied to the specimen respectively, and section geometry was altered to obtain different slenderness ratio of the section. Buckling stress was recorded for each slenderness ratio and loading to plot graphs on buckling stress versus web slenderness. Then load combinations were applied to sections with different slenderness ratios. Load combinations were altered by change the proportions of bending and shear applied on the section. Directions of bending and shear were adjusted to investigate the influence of positive and negative resultant moment on the buckling stress.

The presence of shear is likely to impact the local buckling behavior of an RHS in bending. Hence, the aim of this paper is to use finite element analysis investigate whether a simple plate model can reasonably match the behavior mechanism to account for the impact of web penetrations on the various geometric stiffness properties of hollow sections, and the consequential reduction in lateral torsional buckling strength.

2 Project Aim, Methodology and Scope

The presence of shear is likely to impact the local buckling behavior of an RHS in bending. Hence, the aim of this paper is to use finite element analysis investigate whether a simple plate shear-bending model can reasonably match the local buckling behaviour of a complete RHS under bending and shear.

This paper summarises an undergraduate thesis topic performed at the University of Sydney, undertaken by authors 1 and 2 (Tang 2018, and Yau 2018), under the supervision of the 3rd author, and significant finite element input from the 4th author. An undergraduate thesis represents 25 % of the enrolment load of students across both semesters of their final year. They also undertake significant coursework while performing their research thesis, and by necessity the scope of such a project must be kept quite narrow, hence only linear elastic buckling was considered. Figure 2 defines the geometry considered for both plates and RHS.

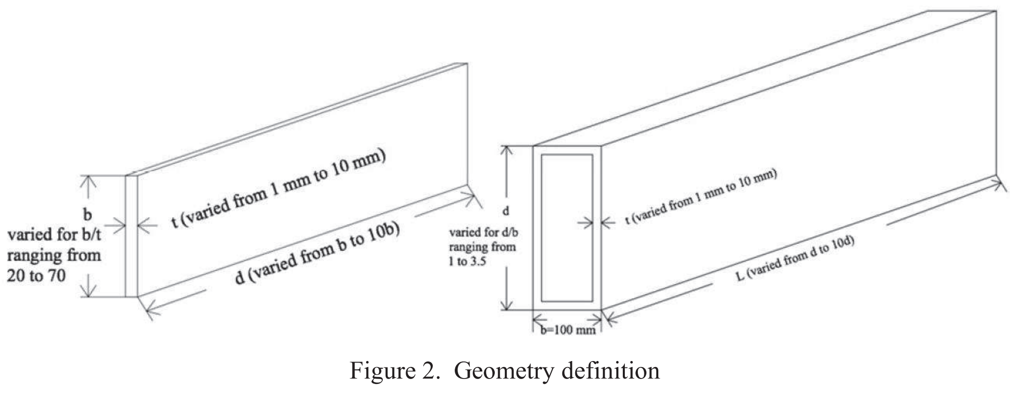

Figure 2. Geometry definition

3 Isolated plate – finite element elastic buckling

3.1 *Modelling*

Full details of the finite element analysis process are given in King (2018) and Tang (2018). As a quick summary, Figure 3 illustrates the isolated plate that represents the web of the equivalent RHS. Shell elements were used. Plate slenderness (b/t) ratios were varied, but were kept within the typical range of values in currently manufactured RHS (typically 20 ~ 100). Both thickness of the plate and the aspect ratio were selected as 1 initially to model a square, slender plate with high b/t ratio. Then the thickness varied between 1 and 10 to investigate the effect of thickness on local buckling behaviour for identical plates. Then a new model with aspect ratio of 2 was built and analysis with different thicknesses were performed repetitively. Loading was applied as a moment at each end, and as stresses around the 4 edges as shown in Figure 4. Elastic buckling analysis was carried out, eigenvalues for each model were recorded, and corresponding bucking stresses were calculated.

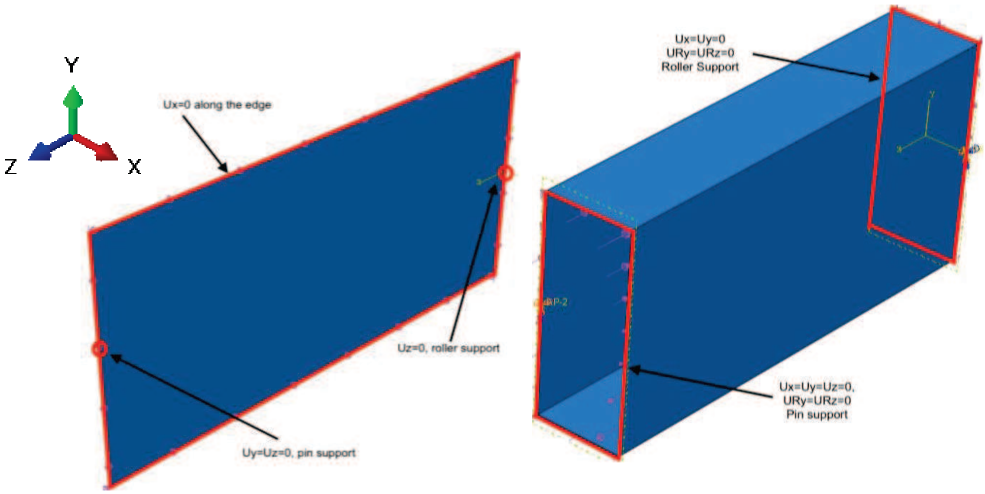

Figure 3. RHS and equivalent plate models with boundary conditions

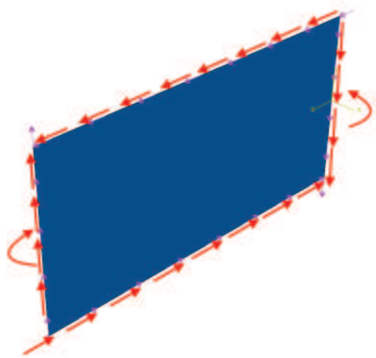

Figure 4: Plate Loading

3.2 *Results*

Plate buckling results were compared against well known elastic solutions for the fundamental cases of pure compression (k = 4), pure bending (k = 23.9) and pure shear ($k = 5.34+4/\alpha^2$), and it was established that our modelling matched these elastic values to within 0.1% (Full details in

Tang 2018 and Yau 2018). A set of typical results, for a given slenderness, varying the plate aspect ratio, and the relative amounts of shear and bending, is given in Figure 5. It can be seen how the buckled shape changes from a bending buckle to a shear buckle inclined at 45°. The interaction shape as the ratio of bending to shear changes matches the well established elliptical interaction equation (Timoshenko 1934).

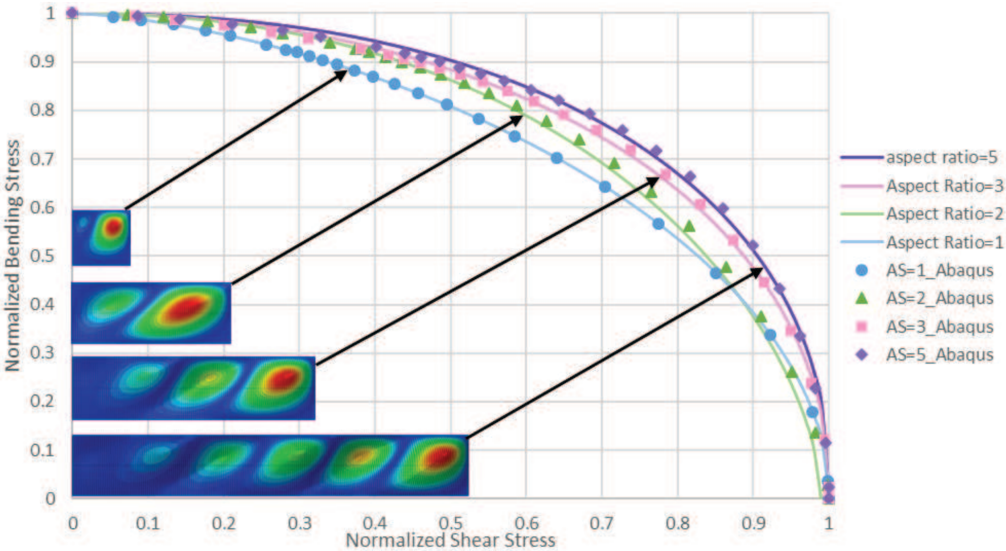

Figure 5: Normalised Bending vs. Shear Local Buckling Stresses at various Aspect Ratio

4 Analysis of complete RHS beams

4.1 *Modelling*

Since adding shear stresses directly on RHS is difficult to achieve, an alternative method was applied to obtain the equivalent effect of controllable combined bending and shear. A moment was applied on one end of a simply-supported beam, and thus the BMD and SFD were as shown in Figure 5 below. It can be observed that bending moment along the section varies linearly, and shear force remains constant along the section. Moreover, magnitudes of the bending moment and shear force can be controlled by changing the length of the section. This loading type was adopted for modelling RHS with different geometries. A typical RHS model under combined bending and shear is included in Figure 6.

Buckling analysis was conducted initially for each section to obtain the stress when buckle occurs. Afterwards, static analysis was performed to collect data of the stress distribution along the cross-section of the member. With the restraint of the rigid body, stress distribution at the edge of the section can have significant discrepancy. Thus the cross-section with stress-distribution data recorded was taken as the first plane with a consistent shear stress distribution.

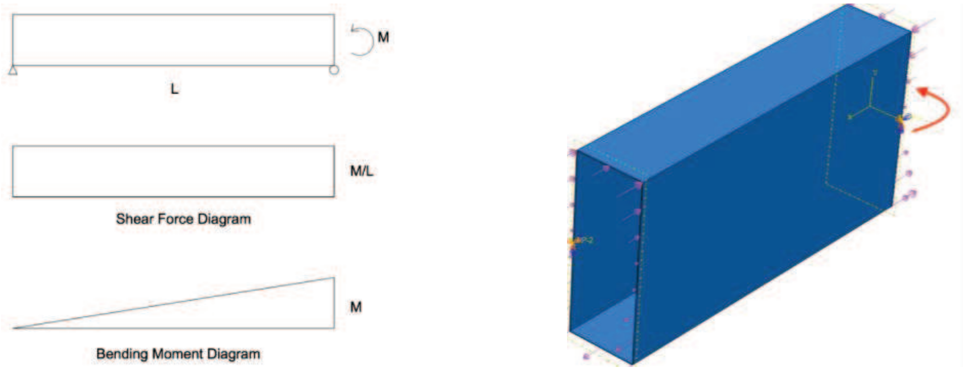

Figure 6. Loading of RHS beam to obtain a specific shear-moment ratio

4.2 *Results*

The primary aim was to examine if the reduction in bending buckling stress with increasing shear stress was similar for an RHS compared to a simple plate model. The first complexity in this issue is the difference in shear stress distributions for a plate compared to an RHS web (whereas the bending stress shapes are the same). This is shown in Figure 7. Hence the average value of shear stress vertically along the RHS or the plate was used in the following comparison of Figure 8.

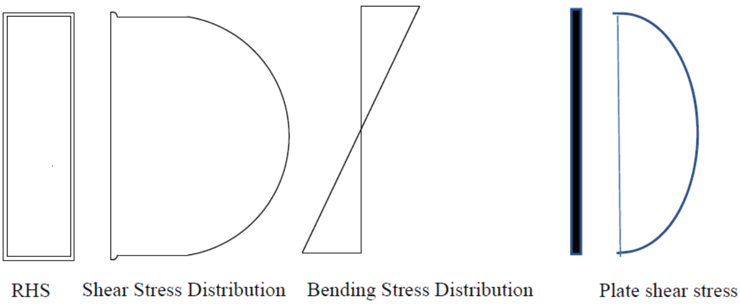

Figure 7. Comparison of stress distributions in an RHS of a plate

Figure 8 indicates rather unexpected results that, especially for plates, the bending stress at the onset of buckling continues to increase as the shear increases. However, the critical stresses reported in Figure 8, are the stresses at the end of the beam, where the bending moment is highest. When you consider the buckled shapes (Figure 9), the local buckles form over a finite length, and the bending stresses varying from the beginning of the buckle to the end of the buckle. It was therefore decided to report the bending stresses at the cross section location where the amplitude of the buckled shape was a maximum. Figure 9, shows that for short beams (high shear) this location is halfway along the beam, so the bending moment at that point is approximately $M/2$ (with a corresponding set of stresses that are 50% less). For a long beam, with only a slight moment gradient, there is only minor change in the moment and hence the bending stresses between the very end and where the buckling displacements were a maximum. Figure 10 shows the relationship when this second method is used. Figure 10 is also expressed different in terms of normalized stresses rather than absolute values to remove dimensional effects.

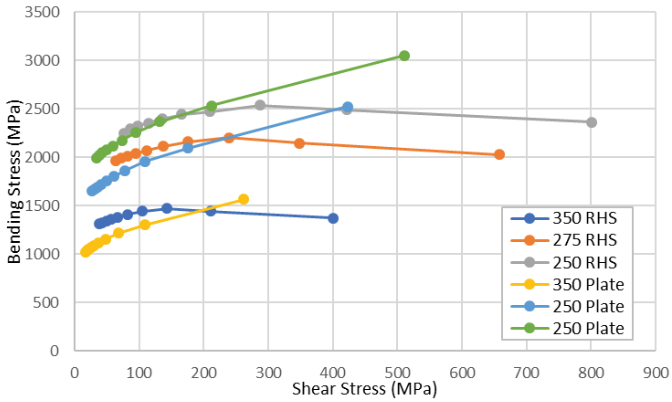

Figure 8: Bending Stresses vs. Shear Stresses of RHS and Plate at varying Aspect Ratios (method 1)

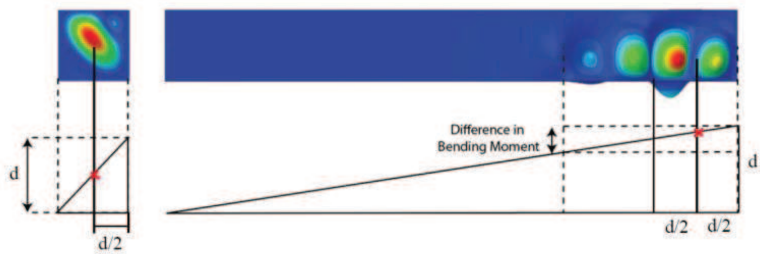

Figure 9: Buckled shape of short and long beams

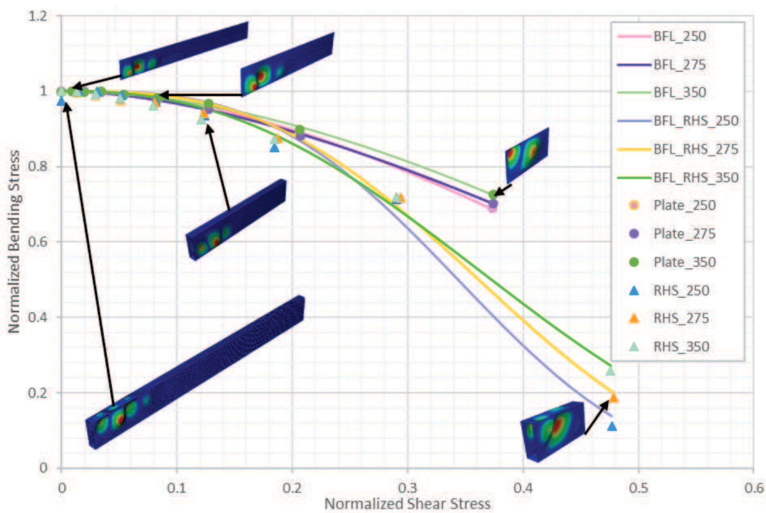

Figure 10: Normalized Bending vs. Shear Stresses for RHS and Plate under Combined Bending and Shear (method 2))

For small values of shear stress (less than 20% of elastic buckling stress under pure shear), the reduction in the critical bending stress follows an almost identical trend for both simple plate models and more complex and complete RHS models. The two methods show divergence in predictions once the relative shear stress exceeds 20%. However in most practical situations, it is common to see "long beam" behavior where the shear is relatively low.

For relative shear stress values of less than 10%, the reduction in critical bending stress in almost insignificant. This would suggest there is little need to reduce slenderness limits in the presence of low shear. Reductions may be necessary for higher values of shear.

5 Conclusions

This paper describes a small scale research project using finite element analysis investigating the impact of increasing shear on the local buckling of beams, compared to the local buckling behaviour under bending alone. A wide range of shear to moment ratio values were considered.

Analysing beams in shear presents in how to load the member (either in real life or via finite element simulation), and due to the moment of gradient it is awkward to report clearly the critical bending stress at buckling. This paper has taken the approach of reporting stresses at the cross section where the magnitude of the buckling displacements in the first mode is greatest.

Even though the shear stress distributions in an RHS web and a plate are different, by reporting the average shear stress, it was found that reduction in the critical bending stress followed a similar pattern for a plate and an RHS web for low shear. This suggests a simplified approach might be suitable in further, more detailed, studies.

It is recommended that extending further finite element analyses to incorporate material and geometric non-linearity be undertaken before adopting this process.

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