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SPARSE KALMAN FILTER IMPLEMENTATION FOR INPUT AND STATE ESTIMATION VIA ACCELERATION MEASUREMENTS

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The application of interest in the paper is estimating the unknown dynamic input and state vector (displacements and velocities) by using partial noisy acceleration measurements for a structural system. A dual Kalman filter scheme is employed for state-parameter identification, but, in addition, we propose a sparse Bayesian learning framework to impose spatially-sparse input (e.g., impulse excitation), and also we would like our model to capture the evolution of the sparse input changes with shared "common sparseness", i.e., the input changes between two successive time instants are also sparse. To this end, we present a hierarchical Bayesian state-space model for computing the marginal posterior distributions of the state and input parameters, where the two sparseness constraints mentioned above are effectively incorporated for each time instant. The measurement and state prediction error parameters (noise parameters) are learned solely from the available data up to the current time, where Bayesian Ockham razor is automatically implemented. Finally, numerical investigation of the proposed algorithm is presented. It is shown that reasonable estimates of impulse and seismic input as well as structural state vector can be accomplished. It is also shown that the well-known drift problem in the estimated input commonly encountered by existing filter methods is effectively alleviated.

Keywords: Kalman filter, Sparse Bayesian learning, Input estimation, Bayesian Ockham razor.

1 Introduction

The advancement of filter theory (e.g., Kalman, 1960; Hoshiya and Saito,1984; Ching et al., 2006; Asif and Romberg, 2010; Sejdinovic et al., 2010), which allows the estimation of unknown variables recursively from incoming measurements observed over time, by using Bayesian inference and state-space models, has received significant interest over the past few decades. For Bayesian system identification (Beck, 2010) purposes, Bayes filters have been applied for simultaneous state (e.g., system displacements and velocities) and model parameter (e.g, structural input parameters) identification of a dynamical system (Ching et al., 2006), by finding the joint posterior probability distribution of hidden states and model parameters. In this paper, we investigate the dual Kalman filter for tracking the dynamic change of structural input, showing how our model of a dynamical system can be represented as two conditionally linear Gaussian state space models, leading to some interesting analytical properties, without linearization required.

In the Kalman filter formulation in the paper, we wish to incorporate sparse Bayesian learning (Tipping, 2001) to impose spatially-sparse input distribution. We would also like our model to capture the evolution of the sparse input changes with shared "common sparseness", i.e., the input changes between two successive time instants are also sparse. One can reasonably expect significant potential gain from this, since it uses the estimate at the previous time instant to aid reconstruction of the new estimate at the current time instant. The exploring of the two sparseness constraints involves a dynamic sparse estimation problem and is a nontrivial task. This problem has recently captured the attention of various researchers, though mostly with application to tracking the dynamically changing sparse signals in compressive sensing (Asif and Romberg, 2010; Mota et al., 2015). However, to our knowledge, no Bayesian method with analytical solutions for online tracking of the dynamically changing sparse models has been presented.

2 Dual Kalman filter algorithm for online tracking of time-varying spatially-sparse structural input

2.1 *Dynamic sparseness-inducing dual Kalman filter algorithm*

We define the system state vector as $\mathbf{x}_i = [\mathbf{d}_i^T, \mathbf{v}_i^T]^T \in \mathbb{R}^{2N_d}$, where $\mathbf{d}_i$ and $\mathbf{v}_i$ denote the structural displacement and velocity vectors, respectively, for the time instant $t_i = i\Delta t, i = 0,1, \ldots I$. To track the structural input change over time, the unknown input parameters $\mathbf{u}_i \in \mathbb{R}^{N_f}$ for each time instant t_i are also introduced, that are influencing some degrees of freedom (DOFs) on the structure as indicated via the influence matrix $\mathbf{S}_f \in \mathbb{R}^{N_d \times N_f}$.

It is assumed that only the accelerations $\hat{\mathbf{z}}_i \in \mathbb{R}^{N_o}$ of the structural response at each time instant t_i are measured. We let $\mathbf{S}_a \in \mathbb{R}^{N_o \times N_d}$ denote the selection matrix for the observed DOFs. According to the equation of motion for a linear dynamical system with the measurement and prediction errors taken into account, the measurements $\hat{\mathbf{z}}_i$ can be expressed binearly with respect to either $\mathbf{x}_i$ or $\mathbf{u}_i$:

$$\hat{\mathbf{z}}_i = \mathbf{C}_i \mathbf{x}_i + \mathbf{G}_i \mathbf{u}_i + \mathbf{e}_i \tag{1}$$

where: $\mathbf{C}_i = [-\mathbf{S}_a \mathbf{M}_s^{-1} \mathbf{K}_s, \quad -\mathbf{S}_a \mathbf{M}_s^{-1} \mathbf{C}_s]_{N_o \times 2N_d}, \mathbf{G}_i = \mathbf{S}_a \mathbf{M}_s^{-1} \mathbf{S}_f,$

$\mathbf{M}_s, \mathbf{K}_s$ and $\mathbf{C}_s \in \mathbb{R}^{N_d \times N_d}$ stand for the mass, stiffness and damping matrix, respectively; $\mathbf{e}_i \in \mathbb{R}^{N_o}$ is introduced as a combination of the measurement noise and output prediction error, which is modeled as $\mathbf{e}_i \sim \mathcal{N}(\mathbf{0}, r_i \mathbf{I}_{N_o})$.

From (1), one can derive a Gaussian likelihood function for $\mathbf{x}_i$ and $\mathbf{u}_i$, given the measurement $\hat{\mathbf{z}}_i$:

$$p(\hat{\mathbf{z}}_i | \mathbf{x}_i, \boldsymbol{\theta}_i, r_i) = \mathcal{N}(\hat{\mathbf{z}}_i | \mathbf{C}_i \mathbf{x}_i + \mathbf{G}_i \mathbf{u}_i, r_i \mathbf{I}_{N_o}) \tag{2}$$

By using numerical integration of the equation of structural motion, we get the following discrete state evolution equation for $\mathbf{x}_i$:

$$\mathbf{x}_i = \mathbf{P}_{i-1} \mathbf{x}_{i-1} + \mathbf{H}_{i-1} \mathbf{u}_{i-1} + \mathbf{w}_i \tag{3}$$

where: $\mathbf{P}_{i-1} = \exp(\mathbf{P}_c \Delta t),\ \mathbf{H}_{i-1} = \mathbf{P}_c^{-1}(\mathbf{P}_i - \mathbf{I}_{2N_d \times 2N_d})\mathbf{H}_c,$

$$\mathbf{P}_c = \begin{bmatrix} \mathbf{O}_{N_d \times N_d} & \mathbf{I}_{N_d \times N_d} \\ -\mathbf{M}_s^{-1} \mathbf{K}_s & -\mathbf{M}_s^{-1} \mathbf{C}_s \end{bmatrix}_{2N_d \times 2N_d}, \quad \mathbf{H}_c = \begin{bmatrix} \mathbf{O}_{N_d \times N_f} \\ -\mathbf{M}_s^{-1} \mathbf{S}_f \end{bmatrix}_{2N_d \times N_f},$$

and $\mathbf{w}_i \in \mathbb{R}^{2N_d}$ refers to the state prediction error for the system state vector, which is modeled as $\mathbf{w}_i \sim \mathcal{N}\left(\mathbf{0}, q_i \mathbf{I}_{2N_d}\right)$. The state transition PDF (probability density function) $p(\mathbf{x}_i|\boldsymbol{\theta}_{i-1}, \mathbf{x}_{i-1}, q_i)$ is then given by:

$$p(\mathbf{x}_i|\mathbf{x}_{i-1}, \boldsymbol{\theta}_{i-1}, q_i) = \mathcal{N}\left(\mathbf{x}_i|\mathbf{P}_{i-1}\mathbf{x}_{i-1} + \mathbf{H}_{i-1}\mathbf{u}_{i-1}, q_i \mathbf{I}_{2N_d}\right) \tag{4}$$

Some of the structural inputs (e.g., impulse excitation) undergo only small changes at limited locations over short times, so we introduce a sparse vector $\boldsymbol{\epsilon}_i \in \mathbb{R}^{N_\theta}$ to model this temporal dependence as:

$$\mathbf{u}_i = \mathbf{u}_{i-1} + \boldsymbol{\epsilon}_i \tag{5}$$

Inspired by the idea of automatic relevance determination and sparse Bayesian learning (Tipping, 2001), the state transition PDF for $\mathbf{u}_i$ is defined as:

$$p(\mathbf{u}_i|\mathbf{u}_{i-1}) = \prod_{n=1}^{N_\mathbf{u}} \mathcal{N}\left(u_{i,n}|u_{i-1,n}, \lambda_{i,n}\right) = \mathcal{N}(\mathbf{u}_i|\mathbf{u}_{i-1}, \boldsymbol{\Lambda}_i) \tag{6}$$

where $\boldsymbol{\Lambda}_i = \mathrm{diag}\left(\lambda_{i,1}, \dots, \lambda_{i,N_\theta}\right)$. To guarantee the spatial sparseness of structural input, we also want to induce sparseness in the accumulated input changes compared with an initial reference vector $\hat{\mathbf{u}}_i$, that is, $\Delta\mathbf{u}_i = \mathbf{u}_i - \hat{\mathbf{u}}_i$ should be sparse. We specify the reference vector $\hat{\mathbf{u}}_i$ as pseudo-data and use the automatic relevance determination idea again to define another likelihood function for $\mathbf{u}_i$ as:

$$p(\hat{\mathbf{u}}_i|\mathbf{u}_i, \boldsymbol{\alpha}_i) = \prod_{n=1}^{N_u} \mathcal{N}\left(\hat{u}_{i,n}|u_{i,n}, \alpha_{i,n}\right) = \mathcal{N}(\hat{\mathbf{u}}_i|\mathbf{u}_i, \mathbf{A}_i) \tag{7}$$

where $\mathbf{A}_i = \mathrm{diag}\left(\alpha_{i,1}, \dots, \alpha_{i,N_u}\right)$.

2.2 *Bayesian inference*

Because of the separate measurement and state prediction error parameters for $\mathbf{u}_i$ and $\mathbf{x}_i$, we employ the dual Kalman filter scheme to utilize the two coupled linear-in-the-parameter equations in Eq. (2) to perform Kalman filtering for $\mathbf{u}_i$ and $\mathbf{x}_i$ separately but with information exchange between them by each feeding its estimate to the other.

To proceed, we assume that the posterior PDFs for $\mathbf{u}_{i-1}$ and $\mathbf{x}_{i-1}$ conditional on the data up to the $(i-1)^{th}$ time instant are Gaussian PDFs:

$$p(\mathbf{u}_{i-1}|\hat{\mathbf{y}}_{1:i-1}) = \mathcal{N}\left(\mathbf{u}_{i-1}|\boldsymbol{\mu}_{\mathbf{u},i-1|i-1}, \boldsymbol{\Sigma}_{\mathbf{u},i-1|i-1}\right) \tag{8a}$$

$$p(\mathbf{x}_{i-1}|\hat{\mathbf{y}}_{1:i-1}) = \mathcal{N}\left(\mathbf{x}_{i-1}|\boldsymbol{\mu}_{\mathbf{x},i-1|i-1}, \boldsymbol{\Sigma}_{\mathbf{x},i-1|i-1}\right) \tag{8b}$$

It will be demonstrated later that the derived posterior PDFs $p(\mathbf{u}_i|\hat{\mathbf{y}}_{1:i})$ and $p(\mathbf{x}_i|\hat{\mathbf{y}}_{1:i})$ for the i^{th} time instant have the same Gaussian form as in Eqs. (8a) and (8b) when using some approximations, and thus the Gaussian assumption in Eqs. (8a) and (8b) is self-consistent.

In the following, recursive Bayesian estimation is employed to produce the posterior distribution of $\mathbf{u}_i$ conditioned on $\mathbf{x}_i$ and the measurements $\hat{\mathbf{y}}_{1:i}$ up to the current time step and the posterior for $\mathbf{x}_i$ conditional on $\mathbf{u}_i$ and $\hat{\mathbf{y}}_{1:i}$. Both the prediction and updating steps are written probabilistically.

2.2.1 *Kalman filter for model parameter vector $\mathbf{u}_i$ conditional on $\mathbf{x}_i$*

The predictive PDF for $\mathbf{u}_i$ is expressed as:

$$p(\mathbf{u}_i|\hat{\mathbf{y}}_{1:i-1},\mathbf{x}_i,\boldsymbol{\lambda}_i)=\mathcal{N}\left(\mathbf{u}_i|\boldsymbol{\mu}_{\mathbf{u},i|i-1},\boldsymbol{\Sigma}_{\mathbf{u},i|i-1}\right) \tag{9a}$$

where:
$$\boldsymbol{\mu}_{\mathbf{u},i|i-1}=\boldsymbol{\mu}_{\mathbf{u},i-1|i-1},\ \boldsymbol{\Sigma}_{\mathbf{u},i|i-1}=\boldsymbol{\Lambda}_i+\boldsymbol{\Sigma}_{\mathbf{u},i-1|i-1} \tag{9b}$$

When new data vector $\hat{\mathbf{y}}_i=[\hat{\mathbf{z}}_i^T,\hat{\mathbf{u}}_i^T]^T$ is available at the time instant t_i, the updated PDF $p(\mathbf{u}_i|\hat{\mathbf{y}}_{1:i},\mathbf{x}_i)$ is obtained first by using Laplace's asymptotic approximation (Beck and Katafygiotis, 1998) based on the assumption that the posterior $p\left(\boldsymbol{\lambda}_i,r_{\mathbf{u},i},\boldsymbol{\alpha}_i|\hat{\mathbf{y}}_{1:i},\mathbf{x}_i\right)$ has a unique maximum at $\{\tilde{\boldsymbol{\lambda}}_i,\tilde{r}_{\mathbf{u},i},\tilde{\boldsymbol{\alpha}}_i\}=\operatorname{argmax} p\left(\boldsymbol{\lambda}_i,r_{\mathbf{u},i},\boldsymbol{\alpha}_i|\hat{\mathbf{y}}_{1:i},\mathbf{x}_i\right)$, and then employing sequential Bayesian updating by treating the posterior having observed data $\hat{\mathbf{z}}_{0:i}$ as the `prior' for the pseudo data $\hat{\mathbf{u}}_i$ to compute the posterior PDF $p(\mathbf{u}_i|\hat{\mathbf{y}}_{1:i},\mathbf{x}_i)$:

$$p(\mathbf{u}_i|\hat{\mathbf{y}}_{1:i},\mathbf{x}_i)\approx p\left(\mathbf{u}_i|\hat{\mathbf{y}}_{1:i},\mathbf{x}_i,\tilde{\boldsymbol{\lambda}}_i,\tilde{r}_{\mathbf{u},i},\tilde{\boldsymbol{\alpha}}_i\right)$$

$$\propto p(\hat{\mathbf{u}}_i|\mathbf{u}_i,\tilde{\boldsymbol{\alpha}}_i)p\left(\hat{\mathbf{z}}_i|\mathbf{x}_i,\tilde{r}_{\mathbf{u},i}\right)p\left(\mathbf{u}_i|\hat{\mathbf{y}}_{0:i-1},\mathbf{x}_i,\tilde{\boldsymbol{\lambda}}_i\right)=\mathcal{N}\left(\mathbf{u}_i|\tilde{\boldsymbol{\mu}}_{\mathbf{u},i|i},\tilde{\boldsymbol{\Sigma}}_{\mathbf{u},i|i}\right) \tag{10a}$$

where:
$$\tilde{\boldsymbol{\Sigma}}_{\mathbf{u},i|i}=\boldsymbol{\Sigma}_{\mathbf{u},i|i}\left(\tilde{\boldsymbol{\lambda}}_i,\tilde{r}_{\mathbf{u},i},\tilde{\boldsymbol{\alpha}}_i\right);\ \boldsymbol{\Sigma}_{\mathbf{u},i|i}=\left(\boldsymbol{\Sigma}_{\mathbf{u},i|i-1}^{-1}+r_{\mathbf{u},i}^{-1}\mathbf{G}_i^T\mathbf{G}_i+\mathbf{A}_i^{-1}\right)^{-1} \tag{10b}$$

$$\tilde{\boldsymbol{\mu}}_{\mathbf{u},i|i}=\boldsymbol{\mu}_{\mathbf{u},i|i}\left(\tilde{\boldsymbol{\lambda}}_i,\tilde{r}_{\mathbf{u},i},\tilde{\boldsymbol{\alpha}}_i\right);\ \boldsymbol{\mu}_{\mathbf{u},i|i}=\boldsymbol{\Sigma}_{\boldsymbol{\theta},i|i}\left\{\boldsymbol{\Sigma}_{\mathbf{u},i|i-1}^{-1}\boldsymbol{\mu}_{\mathbf{u},i|i-1}+r_{\mathbf{u},i}^{-1}\mathbf{G}_i^T(\hat{\mathbf{z}}_i-\mathbf{C}_i\mathbf{x}_i)+\mathbf{A}_i^{-1}\hat{\mathbf{u}}_i\right\} \tag{10c}$$

2.2.2 *Kalman filter for system state vector x_i conditional on θ_i*

The predictive PDF $p(\mathbf{x}_i|\hat{\mathbf{y}}_{0:i-1},\mathbf{u}_i,q_i)$ is given by:

$$p(\mathbf{x}_i|\hat{\mathbf{y}}_{1:i-1},\mathbf{u}_i,q_i)=\mathcal{N}\left(\mathbf{x}_i|\boldsymbol{\mu}_{\mathbf{x},i|i-1},\boldsymbol{\Sigma}_{\mathbf{x},i|i-1}\right) \tag{11}$$

with mean and covariance matrix:

$$\boldsymbol{\mu}_{\mathbf{x},i|i-1}=\mathbf{P}_i\boldsymbol{\mu}_{\mathbf{x},i-1|i-1}+\mathbf{H}_i\mathbf{u}_{i-1},\ \boldsymbol{\Sigma}_{\mathbf{x},i|i-1}=q_i\mathbf{I}_{2N_d}+\mathbf{P}_i\boldsymbol{\Sigma}_{\mathbf{x},i-1|i-1}\mathbf{P}_i^T \tag{12}$$

When new data $\hat{\mathbf{y}}_i$ is available, the update PDF $p(\mathbf{x}_i|\hat{\mathbf{y}}_{0:i},\boldsymbol{\theta}_i)$ is computed as:

$$p(\mathbf{x}_i|\hat{\mathbf{y}}_{1:i},\mathbf{u}_i)\approx p\left(\mathbf{x}_i|\hat{\mathbf{y}}_{1:i},\mathbf{u}_i,\tilde{q}_i,\tilde{r}_{\mathbf{x},i}\right)=\mathcal{N}\left(\mathbf{x}_i|\tilde{\boldsymbol{\mu}}_{\mathbf{x},i|i},\tilde{\boldsymbol{\Sigma}}_{\mathbf{x},i|i}\right) \tag{13}$$

where the Laplace's asymptotic approximation (Beck and Katafygiotis, 1998) is used based on the assumption that the posterior $p\left(q_i,r_{\mathbf{x},i}|\hat{\mathbf{y}}_{1:i},\mathbf{u}_i\right)$ has a unique maximum at:

$$\{\tilde{q}_i,\tilde{r}_{\mathbf{x},i}\}=\operatorname{argmax} p\left(q_i,r_{\mathbf{x},i}|\boldsymbol{\theta}_i,\hat{\mathbf{y}}_{1:i}\right) \tag{14a}$$

$$\tilde{\boldsymbol{\Sigma}}_{\mathbf{x},i|i}=\boldsymbol{\Sigma}_{\mathbf{x},i|i}\left(\tilde{q}_i,\tilde{r}_{\mathbf{x},i}\right);\ \boldsymbol{\Sigma}_{\mathbf{x},i|i}=\boldsymbol{\Sigma}_{\mathbf{x},i|i-1}-\boldsymbol{\Sigma}_{\mathbf{x},i|i-1}\mathbf{C}_i^T\mathbf{S}_{\mathbf{x},i|i-1}^{-1}\mathbf{C}_i\boldsymbol{\Sigma}_{\mathbf{x},i|i-1} \tag{14b}$$

$$\tilde{\boldsymbol{\mu}}_{\mathbf{x},i|i}=\boldsymbol{\mu}_{\mathbf{x},i|i}\left(\tilde{q}_i,\tilde{r}_{\mathbf{x},i}\right);\ \boldsymbol{\mu}_{\mathbf{x},i|i}=\boldsymbol{\mu}_{\mathbf{x},i|i-1}+\boldsymbol{\Sigma}_{\mathbf{x},i|i-1}\mathbf{C}_i^T\mathbf{S}_{\mathbf{x},i|i-1}^{-1}\left(\hat{\mathbf{z}}_i-\mathbf{C}_i\boldsymbol{\mu}_{\mathbf{x},i|i-1}-\mathbf{G}_i\mathbf{u}_i\right) \tag{14c}$$

$$\mathbf{S}_{\mathbf{x},i|i-1}=\mathbf{C}_i\boldsymbol{\Sigma}_{\mathbf{x},i|i-1}\mathbf{C}_i^T+r_{\mathbf{x},i}\mathbf{I}_{N_o} \tag{14d}$$

Note that the pseudo-data $\hat{\mathbf{u}}_i$ is irrelevant to the system state vector $\mathbf{x}_i$ here. The authors can refer to Huang and Beck (2017) for detailed information for the computing of the maximum a posteriori values $\{\tilde{\boldsymbol{\lambda}}_i,\tilde{r}_{\boldsymbol{\theta},i},\tilde{\boldsymbol{\alpha}}_i\}$ and $\{\tilde{q}_i,\tilde{r}_{\mathbf{x},i}\}$ by maximizing their corresponding posterior PDFs.

For online implementation, we can use the dual Klaman filter procedure presented in Wan and Nelson (2001). To impose the sparse constraint of $\Delta\mathbf{u}_i=\mathbf{u}_i-\hat{\mathbf{u}}_i$, a pseudo-measurement stage should be further performed after the implementation of the dual Klaman filter for each time instant. A similar idea can be found in Asif and Romberg (2010), though it is a deterministic least-squares method regularized by using an l_1-norm.

3 Illustrative examples

Consider a 20-story shear building, which has uniformly spatially-distributed floor mass and inter-story stiffness. The mass per floor is taken to be 750 metric tons, while the inter-story stiffness is chosen to be 1000 MN/m. A Rayleigh damping model is adopted, so the damping matrix is given by $\mathbf{C}_s = a\mathbf{M}_s + b\mathbf{K}_s$, where $a = 0.0839\ \text{sec}^{-1}$ and $b = 0.0036\ \text{sec}$ (the damping ratios for the first two modes are 2%). The acceleration structural responses are measured at the 3rd, 5th, 7th, 8th, 10th, 13th, 15th, 17th, 18th, and 20th floors with a sampling frequency of 200 Hz, and these measurements are contaminated by zero-mean Gaussian noises with a standard deviation taken as be 10% RMS of the corresponding noise-free quantities.

The performance of the proposed method is first assessed for impulse excitation of the test structure. In reality, the initial reference vector $\hat{\mathbf{u}}_i$ is a zero vector for all time instants. The estimation results for the impulse input time history, hyper-parameters $\lambda_{i,n}$ and $\alpha_{i,n}$, and the Zoom in of the input estimation at the period of 1.9s-2.4s for DOF 20 are presented in Figure 1 (a)-(d), respectively. The red dash lines are the means of the identified input parameters and the blue lines represent the plus and minus three standard derivation confidence intervals which yield a probability of 99.7%. The actual input time histories is also indicated by the green line. It is shown that the impulse excitation time history estimation can be identified with reasonable confidence intervals and the results are stable and consistent with the actual values. Regarding hyper-parameters estimation, the hyper-parameters $\lambda_{i,n}$ and $\alpha_{i,n}$ corresponding to nonzero input have much larger values, indicating less confidence for the closeness to the reference values.

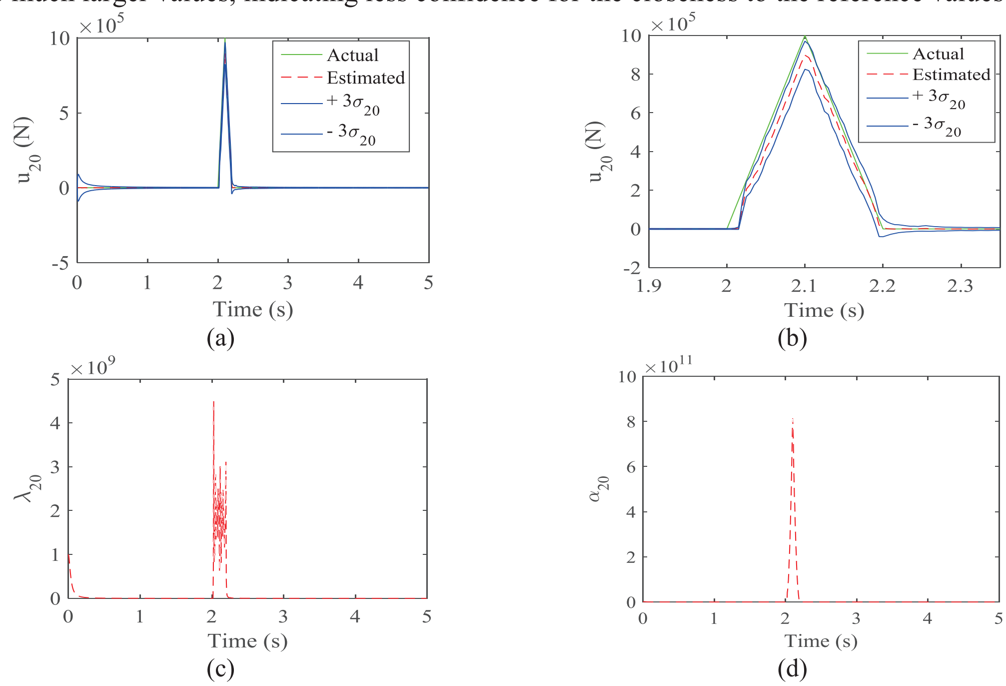
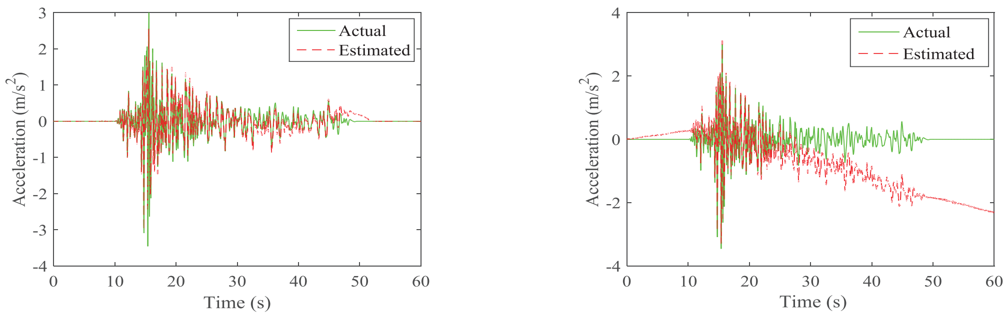


Figure 1. Identification of impulse input for DOF 20: (a) input estimation results; (b) zoom in at the period of 1.9s-2.4s for the input estimation. (c) $\lambda_{i,n}$ estimation results; (d) $\alpha_{i,n}$ estimation results;

In the remainder of this section the seismic excitation will be assessed as well. Figure 3 shows the estimated seismic acceleration time histories furnished by the proposed method. It is seen that, the proposed method provides reasonable estimates of inputs. We also investigate the performance of the method if there is no sparseness constraints imposed and it is seen that the method suffers from the low frequency drift, which is due to low frequency components stemming from double integration errors, while this problem is effectively suppressed by the "sparseness" constraints. Though the identified seismic input is just one-dimensional, the trade-off between data-fitting and model complexities is always present and the "sparseness" constraints are still useful to alleviate the drift problem.



(a) (b)

Figure 2. Identification of seismic excitation: (a) estimation with sparseness constraint (b) estimation results without sparseness constraint.

4 Conclusion

In the paper, we pursue a recursive Bayesian approach for sequential state and input estimations using a dual Kalman filter scheme, based on a series of incomplete noisy accelerations observed over time. The key contribution is to incorporate prior knowledge in a hierarchical Bayesian model that suggests sparse solutions for both the structural input relative to initial reference values and, especially, the change of input with time. In addition, the "noise" parameters in the dual Kalman filters are learned from the data.

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