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TIME DEPENDENT RELIABILITY ASSESSMENT OF CORRODED STEEL PIPES WITH CIRCUMFERENTIAL CRACKS UNDER COMBINED LOADING

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Surface cracks have long been recognized as a major cause of potential failures for pipes. To ensure safe operation of deteriorated steel pipes, it is essential to accurately predict the failures. This paper presents a reliability based methodology for the assessment of corrosion affected steel pipes with circumferential cracks. Based on a recently derived elastic fracture toughness model for steel pipes with circumferential cracks, a failure criterion based on linear elastic fracture mechanics is employed. After development of the stochastic model of the load effect, the first passage probability theory is employed to quantify the probability of pipe failure. Then a case study is used to demonstrate the application of the proposed methodology, followed by sensitivity analysis to investigate the effects of key parameters on the probability of failure. Results show that the correlation length significantly affects the probability of failure of the corroded steel pipes and the coefficients *k* and *n* in the corrosion model influence the probability of failure for corroded steel pipes the most.

Keywords: Circumferential Crack, Elastic Fracture toughness, Steel pipe, Combined loading.

1 Introduction

Surface cracks have long been recognized as a major cause of potential failures for steel pipes. To ensure safe operation of deteriorated steel pipes, accurate failure prediction is essential. It has been known that most parameters associated with steel pipe failures, such as pipe geometry, material properties and corrosion process, exhibit various degrees of variations. To take into account the uncertainty in these parameters, a stochastic approach is essential for assessing pipe failures.

Literature review suggests that different criteria have been established for failure assessment of steel pipes and most previous research considered that a steel pipe fails by loss of strength, leakage or plastic collapse or a combination of the above (e.g. Ahammed and Melchers 1994, 1997, Zhou 2010). Little research has been carried out on steel pipes, considering ductile fracture as the failure mode. Lin et al. (2004) developed a probabilistic assessment methodology based on an engineering assessment procedure R6. Monte Carlo simulation was used to evaluate the structural reliability of pipes, without consideration of the autocorrelation between stochastic processes. Due to the time-variant nature of the load effect, time dependent reliability methods are more appropriate for assessment of corrosion affected steel pipe cracking failures.

In this paper, a time-dependent reliability method based on the first passage probability theory is proposed to predict the probability of failure of corrosion affected steel pipes with circumferential cracks. The criterion of stress intensity factor is employed based on the newly developed model of elastic fracture toughness by Li et al. (2017). The merit of the elastic fracture toughness model is that it allows the use of extensive results based on linear elastic fracture toughness for ductile metals. Stochastic model of the load effect is then developed. A case study is presented to illustrate the proposed methodology, followed by sensitivity analysis to investigate the effects of key variables on the probability of pipe failure.

2 Formulation of pipe reliability

In assessing the risk of failures for a pipe, a performance criterion should be established. In the theory of structural reliability, this criterion is expressed in the form of a limit state function as follows

$$G(R, L, t) = R(t) - L(t) \tag{1}$$

where $L(t)$ is the load or its effect at time t, $R(t)$ is the resistance. With the limit state function of Equation (1), the probability of pipe failure p_f can be determined by

$$p_f(t) = P[G(R, L, t) \leq 0] = P[L(t) \geq R(t)] \tag{2}$$

where P denotes the probability of an event.

Based on first passage probability theory and under the assumption of Poisson processes, the probability of pipe failure can be expressed as follows (Melchers 1999)

$$p_f(t) = 1 - [1 - p_f(0)]e^{-\int_0^t v d\tau} \tag{3}$$

where $p_f(0)$ is the probability of pipe failure at time $t = 0$ and υ is the mean rate for the response process $L(t)$ to up-cross the threshold $R(t)$. The up-crossing rate υ in Equation (3) can be determined by Rice formula (Rice 1944) as follows

$$v = v_R^+ = \int_{\dot{K}_C}^{\infty} (\dot{L} - \dot{R}) f_{L\dot{L}}(R, \dot{L}) d\dot{L} \tag{4}$$

where v_R^+ is the up-crossing rate of the stochastic process $L(t)$ relative to the threshold $R(t)$, $\dot{R}$ is the slope of R with respect to time, $\dot{L}$ is the time derivative process of L and $f_{L\dot{L}}$ is the joint probability density function for L and $\dot{L}$.

Squared exponential function is the most common autocorrelation function used in structural reliability analysis (Vanmarcke 1983), which can be expressed as follows

$$\rho_{LL}(t_i, t_j) = exp\left\{-\left(\frac{t_i - t_i}{\theta_L}\right)^2\right\} \tag{5}$$

where θ_L is the correlation length of the process L. Assuming $L(t)$ is a Gaussian process and the threshold R is deterministic, Firouzi et al. (2017) employed the squared exponential function and derived an analytical solution to the up-crossing rate as follows

$$v = \frac{\sqrt{2}}{\theta_L}\phi\left(\frac{R - \mu_L}{\sigma_L}\right)\left\{\phi\left[\frac{\left(\frac{R - \mu_L}{\sigma_L}\right) - \left(\frac{\dot{R} - \dot{\mu}_L}{\dot{\sigma}_L}\right)}{\frac{\sqrt{2}\sigma_L}{\theta_L\dot{\sigma}_L}}\right] + \frac{\left(\frac{R - \mu_L}{\sigma_L}\right) - \left(\frac{\dot{R} - \dot{\mu}_L}{\dot{\sigma}_L}\right)}{\frac{\sqrt{2}\sigma_L}{\theta_L\dot{\sigma}_L}}\Phi\left[\frac{\left(\frac{R - \mu_L}{\sigma_L}\right) - \left(\frac{\dot{R} - \dot{\mu}_L}{\dot{\sigma}_L}\right)}{\frac{\sqrt{2}\sigma_L}{\theta_L\dot{\sigma}_L}}\right]\right\} \tag{6}$$

The key to calculate the probability of failure $p_f(t)$ is to establish the limit state function based on a failure criterion. In the presence of corrosion induced sharp and narrow pits, pipe failure occurs when the relevant stress intensity factors exceed fracture toughness.

3 Failure Criterion

For steel, a model of elastic fracture toughness for pipes with circumferential cracks has been developed by Li et al. (2017). Using the developed model, the failure criteria used to assess the fracture conditions of ductile metal pipes can be expressed as follows

$$K_I \leq K_{IC}^e \tag{7}$$

where K_I is the Mode I stress intensity factor, which can be expressed as follows

$$K_I = \sigma\sqrt{\pi\, a/Q}\, F_I(a/d\,, a/c\,, d/R_i\,, \varphi) \tag{8}$$

where σ is the applied stress, which can be σ_a induced by axial tension N or σ_b induced by bending M, a is the crack depth, Q is the shape factor for an ellipse, c is half crack length, d and R_i are the thickness and internal radius of the pipe respectively, φ is used to define the position along the semi-elliptical crack and F_I is the influence coefficient for Mode I fracture, the expressions of which can be found in Fu et al. (2017) and Li et al. (2017).

$$K_{IC}^e = K_{IC} K_{rc} \tag{9}$$

where K_{IC} is the Mode I fracture toughness and K_{rc} is calculated as follows for circumferentially cracked pipes under constant axial tension N and varying bending moment M

$$K_{rc} = \left(1 + 0.5\left(\frac{K_{rc}K_{IC}\sqrt{\pi Q}(R_o^4 - R_i^4) - \sqrt{a}N(R_o^2 + R_i^2)F_I(N)}{4R_o\sqrt{a}F_I(M)\left\{4\sigma_y R_m^2 d\frac{M_L}{M_L'}\left\{\sin\left[\frac{\pi}{2}\left(1 - \frac{a}{d}\frac{\theta}{4} - \frac{N}{2\pi\sigma_y R_m d}\frac{N_L'}{N_L}\right)\right] - \frac{a}{d}\frac{f(\theta)}{2\theta}\right\}\right\}}\right)^2\right)^{-0.5} \cdot \left[0.3 + 0.7\exp\left(-0.65\left(\frac{K_{rc}K_{IC}\sqrt{\pi Q}(R_o^4 - R_i^4) - \sqrt{a}N(R_o^2 + R_i^2)F_I(N)}{4R_o\sqrt{a}F_I(M)\left\{4\sigma_y R_m^2 d\frac{M_L}{M_L'}\left\{\sin\left[\frac{\pi}{2}\left(1 - \frac{a}{d}\frac{\theta}{4} - \frac{N}{2\pi\sigma_y R_m d}\frac{N_L'}{N_L}\right)\right] - \frac{a}{d}\frac{f(\theta)}{2\theta}\right\}\right\}}\right)^6\right)\right] \tag{10}$$

where R_o and R_m are the external and mean radii respectively, σ_y is the yield strength,

$$f(\theta) = 0.7854\theta^2 - 0.0982\theta^4 + 0.0041\theta^6 - 0.000085\theta^8$$

$$\theta = \frac{c}{R_m}$$

$$N_L' = 2\pi\sigma_y R_m d\left[1 - \frac{a}{d}\frac{\theta}{4} - \frac{2\sin^{-1}\left(\frac{a}{d}\frac{f(\theta)}{2\theta}\right)}{\pi}\right]$$

$$M_L' = 4\sigma_y R_m^2 d\left[\cos\left(\frac{\pi}{8}\frac{a}{d}\theta\right) - \frac{a}{d}\frac{f(\theta)}{2\theta}\right]$$

$$N_L = 2\pi\sigma_y R_i d\left[1 + A_1\left(\frac{a}{d}\right) + A_2\left(\frac{a}{d}\right)^2\right]$$

$$A_1 = 0.066 - 0.038\left(\frac{\theta}{\pi}\right) - 0.960\left(\frac{\theta}{\pi}\right)^2$$

$$A_2 = -0.060 - 1.525\left(\frac{\theta}{\pi}\right) + 1.427\left(\frac{\theta}{\pi}\right)^2$$

$$M_L = 4\sigma_y R_i^2 d\left[1 + B_1\left(\frac{a}{d}\right) + B_2\left(\frac{a}{d}\right)^2\right]$$

$$B_1 = 0.074 - 0.169\left(\frac{\theta}{\pi}\right), \quad B_2 = -0.086 - 1.013\left(\frac{\theta}{\pi}\right)$$

$$\xi = \frac{\sigma_a}{\sigma_b} = \frac{N(R_o^2 + R_i^2)}{4MR_o}, \quad \left\{\left(1 + 0.5\left(\frac{\overline{\sigma}}{\sigma_y}\right)^2\right)^{-0.5}\left[0.3 + 0.7\exp\left(-0.65\left(\frac{\overline{\sigma}}{\sigma_y}\right)^6\right)\right]\right\} \leq K_{rc} \leq 1$$

Based on the failure criterion in Equation (7), the load effect $L(t)$ and resistance R can be defined and represented as follows

$$L(t) = K_I - K_{IC}^e \tag{11}$$

$$R = 0$$

4 Stochastic Model

Corrosion is considered as the deterioration mechanism in this paper. It has been known that the corrosion process is a very random phenomenon, depending on the localized conditions, such as soil type, moisture and oxygen contents, properties of the pipe materials etc. In this paper, the power law corrosion model, first postulated by Kucera and Mattsson (1987) for atmospheric corrosion, is employed for corrosion pit depth, which can be expressed as follows

$$a(t) = kt^n \tag{12}$$

where k and n are the multiplying and exponential constants respectively, which depend on pipe material and surrounding environments. This paper considered the localized pit induced by corrosion to be narrow and sharp, which can therefore be represented by a sharp crack. Ahammed and Melchers (1997) developed a model based on various sets of underground corrosion data presented in Schwerdtfeger (1971) with mean values of k and n 0.066 and 0.53, coefficients of variation 0.56 and 0.26 respectively. In the present paper, this model is adopted.

In addition to the coefficients k and n in the corrosion model, most parameter involved with pipe failures show fluctuations or variability during the pipe service life. Apart from variability, these parameters may change with time. Therefore, it is justifiable to consider the load effect $L(t)$ of the fracturing process for steel pipes as a stochastic process. It follows that the load effect $L(t)$ is a function of the basic variables X_i $(i=1, 2, \ldots, n_v)$ as well as time t and can be expressed as

$$L(t) = f(X_1, X_2, \cdots, X_{n_v}, t) \tag{13}$$

The statistical information of each basic random variable X_i is (presumed) available. With this treatment, the mean $\mu_L(t)$ and coefficient of variation $COV(t)$ of $L(t)$ can be obtained using Monte Carlo simulation.

5 Worked Example

The proposed methodology is applied to a cracked closed-ended steel pipe with a semi-elliptical crack placed in the circumferential direction on the external surface of the pipe. Axial tension N induced by internal pressure p and bending moment M are acted on both ends of the pipe. The aspect ratio a/c of the crack is assumed to be 0.4. As a result, only the deepest point along the crack front is considered, as the stress intensity factor at the deepest point for a/c=0.4 is the largest. The statistical information of the random parameters involved in the analyses is shown in Table 1.

Table 1. Statistical information for calculating the load effects

Variable		Mean	Coefficient of Variation
Symbol	Description		
Ri	Internal radius (mm)	225	0.05
d	Pipe thickness (mm)	7	0.05
p	Internal pressure (MPa)	5	0.15
M	Bending Moment (kNm)	150	0.20
k	Multiplying constant	0.066	0.56
n	Exponential constant	0.53	0.26
σ_y	Yield strength (MPa)	250	0.05
K_{IC}	Mode I fracture toughness (MPa $\sqrt{m}$)	50	0.10

5.1 *Probabilistic analysis.*

With the developed failure model Equation (7) for circumferntially cracked steel pipes, the load effect $L(t)$ and the barrier level, i.e., material resistence R can be determined by Equation (11). Using Monte Carlo simulation together with the corrosion model Equation (12), the mean funciton μ_L and coefficient of variation $COV(t)$ of load effect $L(t)$ can be calculated as a function of time t.

Then the up-crossing rate υ can be obtained from Equation (6) for a given correlation length, followed by the calculation of the probability of pipe failure p_f using Equation (3).

Figure 1 shows the effect of the correlation length (expressed in years) of the load effect process on the probability of failure. It can be seen that correlation length affects the steel cracking failure quite significantly. This makes sense as the corrosion affected steel pipe cracking failure depends on many factors, such as the pipe geometry, corrosion rate, external loads, etc. Since these factors are inter-related at different points in time, the load effect, which is the resultant effect, is correlated at different points in time. The considerable difference in probability of failure caused by the correlation length justifies the necessity to use a time-dependent reliability method based on the stochastic process theory. For a given load effect process, the correlation length can be estimated using statistical methods, e.g., maximum likelihood method, if sufficient time-history data of the load effect process is available.

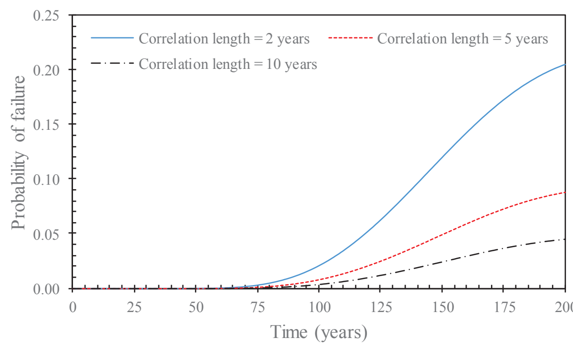

Figure 1. Probability of pipe failure for different values of correlation length

5.2 *Sensitivity analysis*

As may be appreciated, statistical information is essential to reliability analysis. Due to lack of full statistical data on some basic random variables, it is of interest to identify the most influential random variables so that more research can focus on those variables. This can be achieved by sensitivity analysis, using a probability sensitivity index (Nowak and Collins 2012) to demonstrate the contribution of each random variable to the probability of failure. The probability sensitivity index can be represented as follows

$$\alpha_i = \frac{-\frac{\partial G}{\partial X_i}\sigma(X_i)}{\sqrt{\sum_{k=1}^{n}\left(\frac{\partial G}{\partial X_k}\sigma(X_k)\right)^2}} \tag{14}$$

where $\sigma(X_i)$ is the standard deviation of the random variable X_i. The probability sensitivity index should be evaluated at the design point, which can be determined by an iterative procedure (Nowak and Collins 2012).

Based on the statistical information given in Table 1, the probability sensitivity indices of different random variables are calculated and plotted for the steel pipe with a circumferential crack as shown in Figure 2. Among the eight random variables, only the indices for pipe thickness, yield strength, pipe radius and fracture toughness are negative while those for the others are positive, which implies that the probability of pipe failure would increase with those parameters having positive indices, and vice versa. This makes sense both theoretically in terms of limit state function of Equation (1) and practical experience. From Figure 2, it can be seen that before around 110 years of pipe age, the probability of pipe failure is most sensitive to bending moment M. After that, the coefficients k and n become dominant. As pipes age, it has been found the coefficients k and n in the corrosion model have the most significant influence on the probability of failure. In addition, the external applied moment has significant contribution.

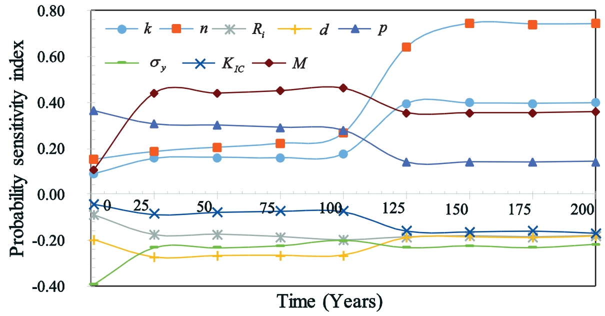

Figure 2. Change of probability sensitivity index with time for different input parameters

6 Conclusions

A reliability based methodology for the assessment of circumferentially cracked steel pipes due to corrosion under combined loading has been presented. The limit state function is established by adopting the simple criterion of stress intensity factor, based on the newly developed model of elastic fracture toughness. A stochastic model of the load effect has been developed and a time-dependent reliability method based on first passage probability has been employed to quantify the probability of failure. A case study has been carried out using the proposed methodology, followed by sensitivity analysis to investigate the effects of the random variables on the probability of failure. It has been found in the paper that the correlation length significantly affects the probability of failure of the corroded steel pipes. It has also been found that coefficients k and n in the corrosion model influence the probability of failure for corroded steel pipes the most.

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