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PROBABILISTIC SLOPE STABILITY ANALYSIS WITH SUBDOMAIN SAMPLING METHOD

WENPING GONG[1], C. HSEIN JUANG[2], HUIMING TANG[3] and LEI WANG[4]

[1]*Faculty of Engineering, China University of Geosciences, Wuhan, 430074, China.*
Email: wenpinggong@cug.edu.cn.
[2]*Glenn Department of Civil Engineering, Clemson University, Clemson, SC 29634, USA.*
Email: hsein@clemson.edu.
[3]*Faculty of Engineering, China University of Geosciences, Wuhan, 430074, China.*
Email: tanghm@cug.edu.cn
[4]*Department of Civil Engineering, University of the District of Columbia, Washington, DC 20008, USA.*
Email: lei.wang@udc.edu

Due to the inherent spatial variability of the geotechnical properties and the limited number of site investigation data, the geotechnical parameters at a site are usually characterized as random fields. The stability of a slope will be scattered and uncertain in the face of the spatial variability of the geotechnical parameters. Thus, the stability of the slope has to be studied probabilistically; and, the influence of the spatial variability of the geotechnical parameters on the slope stability can be explicitly considered using random numerical analysis. In which, potential realizations of the random fields of the geotechnical parameters are generated with the Monte Carlo simulation (MCS); then, for each realization of the random fields, the stability of the slope can be evaluated deterministically with numerical solutions. However, the required number of realizations of the random fields, and thusly the number of deterministic evaluations of the slope stability, might be too large to be computationally efficient, especially for slopes of low failure probability. In such a situation, the recently developed subdomain sampling method (SSM), in lieu of the brute MCS, is employed in this paper for generating the realizations of the random fields. Further, the failure probability of the slope is estimated utilizing the total probability theorem. The effectiveness of this probabilistic slope stability analysis approach is illustrated through one illustrative example.

Keywords: slope, spatial variability, failure probability, subdomain sampling method, random finite difference method.

1 Introduction

In that the geomaterials are natural materials, not artificial materials (e.g., concrete), the property of the geomaterials is dependent upon the natural deposit histories, which cannot be controlled by the engineer. However, due to the absence of the knowledge of the natural deposit histories, the geotechnical information at a site cannot be known prior to the site investigation; and, only a limited number of site investigation data can be acquired in a given project owing to the budget constraint. Thus, the geotechnical information can only be known at borehole locations; whereas, the geotechnical information at other positions cannot be known and has to be derived from that at borehole locations; and, the geotechnical parameters are oftentimes characterized as random variables or random fields (Phoon and Kulhawy 1999, Wang and Cao 2013, Gong et al. 2014, Tian et al. 2016). It is noted that the property of the geomaterials at various positions (at a site) is generally correlated to some extent in both horizontal and vertical directions, and this spatial correlation tends to decrease with the lagged distance. This feature of the geotechnical properties can best be simulated with the random field theory (Fenton 1999, Jiang et al. 2014).

With the random geotechnical parameters at the inputs, the stability of the slope will not be a fixed value; rather, it will be scattered and uncertain (Griffiths and Fenton 2004, Cho 2007). In the face of the uncertainty in the input parameters, the stability of the slope has to be studied in a probabilistic manner and the outcome of this probabilistic analysis will be a failure probability. Extensive studies have been undertaken on the probabilistic slope stability analysis, including probabilistic analysis of slope failure along a given slip surface (El-Ramly et al. 2002), searching for the most significant slip surface (Xue and Gavin 2007), system reliability analysis (Ching et al. 2009, Zhang et al. 2013), and probabilistic stability analysis with response surface methods (Li et al. 2015). Note that although these studies could be deemed meaningful and significant for the purposes of being practical applicable in real projects, the sampling-based probabilistic slope stability analysis, in terms of the random numerical analysis, is the most straightforward and which could yield an unbiased estimate of the failure probability, especially in situations where the inherent spatial variability of the geotechnical parameters must be considered (Griffiths and Fenton 2004, Xiao et al. 2016).

In the context of the random numerical analysis of the slope stability, the spatial variability of the input geotechnical parameters is characterized using the random field theory, and potential realizations of the random fields of the geotechnical parameters are sampled with the sampling method of Monte Carlo simulation (MCS). Then, for each and every potential realization of the random fields, the stability of the slope is studied deterministically using the numerical methods. From there, the failure probability of this slope is readily derived, through a statistical analysis of the results obtained from the deterministic analysis. In that no additional assumption is made, the random numerical analysis yields an unbiased estimate of the failure probability (P_f). Though conceptually sound, the required number of realizations of the random fields, and thusly that of deterministic evaluations of the slope stability, might be too large to be computationally efficient, especially for the slope of low failure probability. Thus, this paper presents a probabilistic slope stability analysis approach. In which, the recently developed subdomain sampling method (SSM) (Gong et al. 2016 & 2017, Juang et al. 2017), in lieu of the brute MCS, is adopted for sampling the realizations of the random fields of the input geotechnical parameters. A significant feature of this adopted SSM is that the realizations of the random fields can be "equally" sampled in the domain of the random fields, instead of being concentrated in the region of high joint probability density. Thus, a larger number of realizations of the random fields could be located in the failure domain and the failure probability can be estimated with higher efficiency. The rest of this paper is organized as follows. First, the sampling method of SSM is reviewed. Second, the framework for the probabilistic slope stability analysis is introduced. Third, an illustrative example, in terms of a one-layer earth slope problem, is studied, through which the effectiveness of the presented probabilistic slope stability analysis approach is demonstrated.

2 Subdomain Sampling Method (SSM)

The essence of the subdomain sampling method (SSM) (Gong et al. 2016 & 2017, Juang et al. 2017) is to partition the possible domain of uncertain variables into a set of subdomains and then to generate samples of uncertain variables in each and every subdomain separately. As a result, the samples of uncertain variables could be "equally" distributed in the domain of the uncertain variables, instead of being concentrated in the region of high joint probability density. Hence, a larger number of samples could be located in the failure domain and the failure probability could be estimated with higher accuracy and efficiency. In the context of the SSM, a distance index (d), which is utilized to partition the domain of the uncertain variables, is formulated based upon the Hasofer-Lind reliability index (Low and Tang 2007):

$$d = \sqrt{[\boldsymbol{n}]^{\mathrm{T}}[\boldsymbol{R}]^{-1}[\boldsymbol{n}]} \tag{1}$$

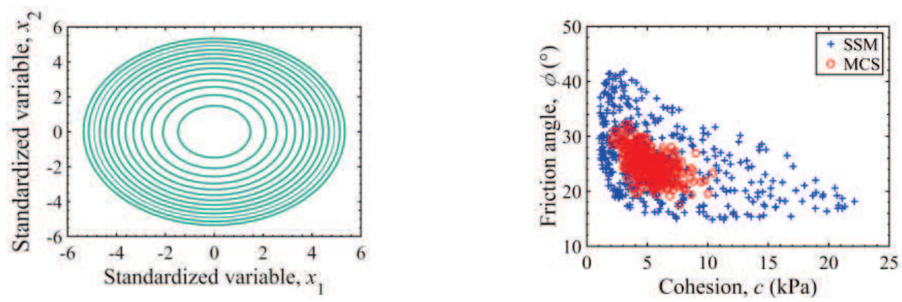
where $\boldsymbol{R}$ is the correlation matrix among the equivalent standard normal variables $\boldsymbol{n} = [n_1, n_2, \ldots, n_{n_x}]^{\mathrm{T}}$, where n_x is the number of uncertain variables $\boldsymbol{x}$; and, the component n_i in $\boldsymbol{n}$ is related to the uncertain variable x_i in $\boldsymbol{x}$ as follows.

$$n_i = \Phi^{-1}[F(x_i)] \tag{2}$$

where $F(x_i)$ is the cumulative distribution function (CDF) of the uncertain variable x_i, and $\Phi(\cdot)$ is the CDF of the standard normal variable. Note that d^2 is distributed as a chi-square distribution with n_x degrees of freedom (Gong et al. 2016 & 2017). With the distance index formulated in Eq. (1), the possible domain of the uncertain variables, denoted as $[0, d_{\max})$, can be identified.

$$\chi^2_{n_x}(d^2_{\max}) = \varepsilon \tag{3}$$

where $\chi^2_{n_x}(\cdot)$ is the chi-square CDF with n_x degrees of freedom, and ε is a probability value that is relatively small (e.g., ε = 1.0×10^{-6} is taken in this paper). The probability of the uncertain variables $\boldsymbol{x}$ being located in and outside this domain $[0, d_{\max})$ are $(1 - \varepsilon)$ and ε, respectively.



(a) Subdomain partition in standard normal space (b) Soil samples generated by SSM and MCS

Figure 1. Conceptual illustration of the subdomain sampling method (SSM)

This possible domain of uncertain variables $\boldsymbol{x}$, in terms of $[0, d_{\max})$, is readily partitioned into a set of subdomains with the distance index d, such as $[d_0, d_1)$, $[d_1, d_2)$, $[d_2, d_3)$, etc. For the purpose of being computationally efficient, the likelihoods of the uncertain variables $\boldsymbol{x}$ being located in the subdomains, denoted as $(p_{d1}, p_{d2}, p_{d3}, \ldots)$, can be taken as a decreasing sequence.

$$p_{di} = \Pr\left[d_{i-1} \le \sqrt{[\boldsymbol{n}]^{\mathrm{T}}[\boldsymbol{R}]^{-1}[\boldsymbol{n}]} < d_i\right] = \Pr\left[d^2_{i-1} \le d^2 < d^2_i\right] = \chi^2_{n_x}(d^2_i) - \chi^2_{n_x}(d^2_{i-1}) \tag{4}$$

where p_{di} is the likelihood of the uncertain variables being located in the i^{th} subdomain $[d_{i-1}, d_i)$. Next, the samples of uncertain variables could be generated in the subdomains with the sampling algorithm advanced in Gong et al. (2016); and, to include the cross correlation between different geotechnical parameters, the procedures outlined in Fenton and Griffiths (2003) are employed to update the geotechnical parameters generated by the SSM. For simplicity, same target number of samples, denoted as t_1, is adopted in all these subdomains and this target number is taken as: $t_1 = 10\, p_{di} / p_{d(i-1)}$. A parametric study indicates that the parameters setting of $p_{d1} = 1/3$, $p_{d2} = 1/3^2$, $p_{d3} = 1/3^3$, … yields the minimum total number of samples. Figure 1(a) depicts the partitioned subdomains in the standard normal space. Figure 1(b) depicts a comparison between the soil samples generated by the SSM and those by the MCS, in which the number of the soil samples is taken as 390. Here, the samples generated by the SSM are "equally" distributed in the possible domain while those by the MCS are concentrated in the region of high joint probability density. In that the sample domain is not partitioned into equally probable subdomains and the samples in each subdomain are sampled with the new sampling algorithm advanced in Gong et al. (2016), the SSM presented in this paper is different from the existing Latin hypercube sampling method.

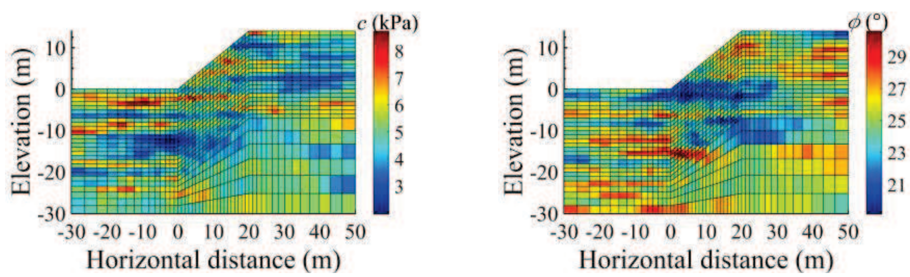
With the generated samples of uncertain variables, the deterministic analysis of the system performance is readily conducted. Then, the conditional failure probability in the subdomain $[d_{i\text{-}1}, d_i)$, denoted as p_{fi}, could be derived by counting the number of failure samples (t_{fi}).

$$p_{fi} = \frac{t_{fi}}{t_1} \tag{5}$$

In that the contribution of the i^{th} subdomain $[d_{i\text{-}1}, d_i)$ to the failure probability estimate P_f is only $(p_{di} \cdot p_{fi})$, the failure probability estimate P_f of the studied system can be approximated using the total probability theorem.

$$P_f = \sum_{i=1}^{i=n_s}\left(p_{di} \cdot p_{fi}\right) \Big/ \sum_{i=1}^{i=n_s} p_{di} = \sum_{i=1}^{i=n_s}\left(p_{di} \cdot p_{fi}\right) \Big/ (1\text{-}\varepsilon) \approx \sum_{i=1}^{i=n_s}\left(p_{di} \cdot p_{fi}\right) \tag{6}$$

where n_s is the number of subdomains.



(a) Cohesion of the soil (b) Friction angle of the soil

Figure 2. One realization of the random fields of the soil parameters in slope stability analysis

3 Framework for Probabilistic Slope Stability Analysis

In the numerical modelling of a slope, the geometrical domain of the slope is discretized into a set of smaller elements, thus different geotechnical parameters can be easily assigned to different elements. That is to say, the spatial variability of the geotechnical parameters might be directly simulated using the numerical modelling. To consider the spatial variability of the geotechnical parameters in the slope stability analysis, the random numerical analysis (Griffiths and Fenton 2004, Jiang et al. 2014, Xiao et al. 2016) could be applied.

In the probabilistic slope stability analysis approach presented in this paper, the framework of the existing random numerical analysis (Griffiths and Fenton 2004, Jiang et al. 2014, Xiao et al. 2016) will be followed; however, potential realizations of the random fields are sampled with the SSM, not the MCS. Further, the failure probability of the slope is estimated utilizing the total probability theorem (see Eq. 6). The 2-D explicit finite difference program FLAC version 7.0 (2011) is adopted as the deterministic solution model for evaluating the stability of the slope (in terms of the factor of safety, FS), in which the strength reduction method is utilized. It is noted that the geotechnical parameters within a given element, in the FLAC analysis, are captured by fixed parameters and no variation can be allowed. For example, Figure 2 depicts one realization of the random fields of the soil parameters in the slope stability analysis. Thus, the geotechnical parameters that are averaged over the element domain, rather than those at mesh grids, should be sampled (in the generation of the realizations of the random fields) and taken as the inputs to the deterministic numerical analysis. The statistics of the geotechnical parameters that are averaged over each element domain and the correlation coefficients between the averaged parameters may be derived with the equations in Xiao et al. (2016) and Gong et al. (2018), the derived statistics are then adopted in the generation of the input geotechnical parameters for the FLAC analysis.

4 Illustrative Example

A one-layer earth slope underlain by a rock layer, shown in Figure 3, is adopted as an illustrative example to demonstrate the presented probabilistic slope stability analysis approach. The width and height of the slope are 20.0 m and 14.0 m, respectively, and the depth of the underlying rock layer is taken as an infinitely large value. The surcharge on the top of the slope is not considered and the groundwater level is assumed to be far below the slope. The soil parameters are given in Figure 3. Here, the strength parameters of the cohesion and friction angle are treated as uncertain parameters, and the autocorrelation structure of the spatially varied soil strength is simulated by an anisotropic exponential model (Cho 2007, Xiao et al. 2016).

Uncertain parameter		Cohesion	Friction angle
Mean		5.0 kPa	25.0 °
COV		0.30	0.10
Scale of fluctuation	Horizontal	50.0 m	50.0 m
	Vertical	2.5 m	2.5 m
Distribution		Lognormal	Lognormal

Note: the correlation coefficient between cohesion and friction angle is -0.5.

Deterministic parameter	Value
Density	1,700 kg/m^3
Bulk modulus	330 MPa
Shear modulus	150 MPa

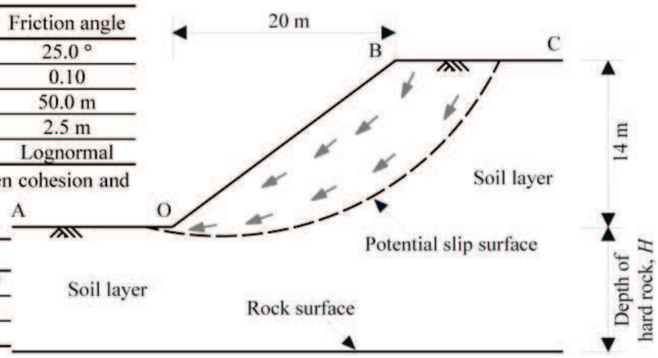
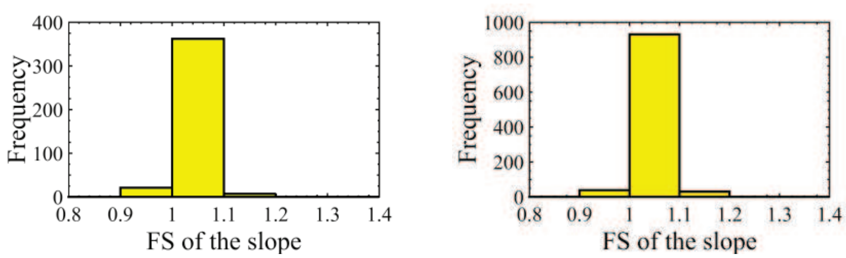


Figure 3. Schematic diagram of the illustrative example and essential soil parameters



(a) SSM (total number of samples = 390) (b) MCS (total number of samples = 1,000)

Figure 4. Obtained distributions of the FS of the example slope

With the aforementioned probabilistic slope stability analysis approach, the stability of this example slope is readily studied. The parameters setting of the adopted SSM is set up as: 1) $p_{d1} = 1/3$, $p_{d2} = 1/3^2$, $p_{d3} = 1/3^3$, … and the target number of samples in the subdomain is $t_1 = 30$; and 2) the number of subdomains is $n_s = 13$ and $\varepsilon = 1.0\times10^{-6}$. In short, a total of 390 realizations of the random fields of soil strength parameters will be generated and studied. Plotted in Figure 4(a) is the distribution of the FS of this example slope obtained with the SSM, and plotted in Figure 4(b) is that obtained with the MCS in which the number of realizations of the random fields is 1,000.

In Figure 4(a), FS ranges from 0.97 to 1.16 and the ratio of the number of failure samples (i.e., FS < 1.0) over the total number of samples is 0.054. In Figure 4(b), FS also ranges from 0.97 to 1.16 while the ratio of the number of failure samples over the total number of samples is 0.039. The plots in Figure 4 can be interpreted with the following fact. The samples generated by the SSM are "uniformly" distributed in the identified possible domain; whereas, those generated by the MCS are drawn directly from the joint probability density function (PDF), thus most samples are located in the region of high density values. It is known that a higher ratio of the number of failure samples over the total number of samples signals a lower COV of the failure probability estimate and thusly higher efficiency in estimating the failure probability. As such, the SSM can be more computationally efficient than the MCS. Here, the failure probability of this example slope estimated by the SSM and the MCS are 0.070 and 0.039, respectively. The comparison of the failure probability estimate demonstrated that the accuracy of the SSM and the effectiveness of the presented probabilistic slope stability analysis approach.

In a typical geotechnical practice, site-specific data is oftentimes quite limited due to budget constraints for site investigation; as such, it could be difficult to derive the statistical information of the intended geotechnical property with certainty. However, the failure probability obtained by the probabilistic analysis could be strongly affected by the input statistical information (Juang et al. 2013). Thus, the influence of the COVs of the soil strength parameters is further studied and the results are depicted in Figure 5. As expected, the performance of the slope degrades, as indicated by the increase of the failure probability (P_f), with the increase of the COVs of the soil strength parameters. That is to say, the significance of the statistical characterization of the input soil parameters on the probabilistic analysis is depicted.

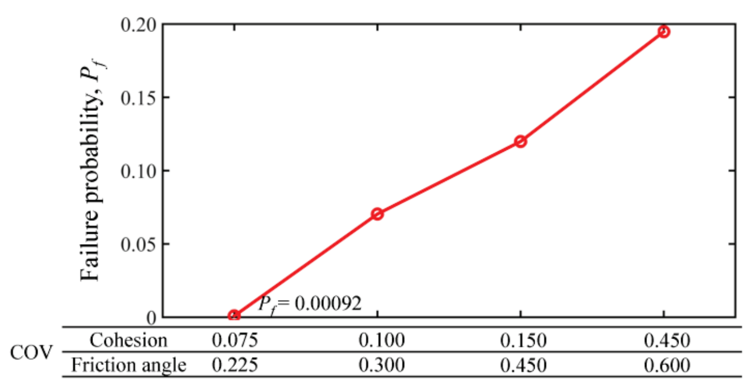


Figure 5. Influences of the COVs of soil strength parameters on the failure probability of the slope

5 Concluding Remarks

This paper presented a probabilistic slope stability analysis approach. Different from the current random numerical analysis methods, the recently developed subdomain sampling method (SSM), in lieu of the Monte Carlo simulation (MCS), was utilized for sampling the realizations of the random fields (of the geotechnical parameters), and then the failure probability was estimated with the total probability theorem. The effectiveness of the presented probabilistic slope stability analysis approach was demonstrated through a one-layer earth slope problem. The SSM and the MCS yielded a consistent failure probability estimate; however, less realizations of the random fields, and thusly the number of deterministic evaluations of slope stability, were needed with the SSM. Further, the parametric study indicated that the probabilistic analysis results (i.e., the failure probability) could be strongly influenced by the statistical characterization of the input geotechnical parameters; thus, the significance of the statistical characterization of the input soil parameters on the probabilistic analysis is demonstrated.

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