Proc. of the 6th Intl. Symposium on Reliability Engineering and Risk Management (6ISRERM)
31 May – 1 June 2018, Singapore
Editor(s) Xudong Qian, Sze Dai Pang, Ghim Ping Raymond Ong, Kok-Kwang Phoon

Probabilistic Distribution of Maximum Tsunami Height based on Stochastic Process of Water Depth

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Uncertainty is a key challenge in tsunami hazards analysis. The tsunami height has obvious stochastic nature in tsunami propagation process. In this paper a stochastic model of maximum tsunami height is studied. Firstly the physical process of tsunami propagation from deep sea to shallow water is illustrated, which indicates that tsunami propagation in shallow water areas is different from that in deep water areas. The tsunami spread velocity depends on water depth in shallow water areas. Then the water depth in shallow water area is considered random, and it is assumed that the random water depth follows the Wiener process, which is reasonable according to Wiener process's irregular characteristics. And according to the energy conservation principle, tsunami wave velocity affects wave height. So the tsunami height depends on water depth, namely the submarine topographies. Based on that a stochastic model of maximum tsunami height is established. Finally the probability distribution of maximum tsunami height is calculated using the stochastic model, and it is showed that the calculation results of the mean value of maximum tsunami height is very similar to the average value of 165 actual observation values of maximum tsunami heights during the recent twenty years from 1997 to 2017.

Keywords: tsunami, maximum wave height, probabilistic model, Wiener process, stochastic water depth

1 Introduction

Traditionally, tsunami hazard analysis was based on deterministic methods. Due to tsunami's stochastic nature efforts have been made recently toward the development of probabilistic estimates of tsunami hazards, often called "probabilistic tsunami hazard assessment" (PTHA) (Geist 2006, Thio 2007, Gonzalez 2014, Geist 2014). Currently, uncertainty remains a key challenge in tsunami hazards assessment. At present, probabilistic tsunami hazard is identified mostly by maximum tsunami height (Gonzalez 2013, Adams 2015). As is known, maximum tsunami height is of great importance in tsunami hazards analysis.

In fact, the tsunami hazards a site might be subject to are related to the seabed topography near the site, because the seabed topography of a shallow water zone affect mostly the tsunami propagation during the tsunami propagation process from deep sea to shallow water. It is thought in this paper that the submarine topographies of shallow water are the most important factor that affects the tsunami wave height and velocity. In this work based on the assumption of random water depth following the Wiener process, the maximum tsunami height near the shore probabilistic model is established, which is different from the maximum tsunami wave height deterministic model previously studied (Pacific Gas & Electric Company 2010). The maximum tsunami wave height is the key factor for the determination of tsunami inundation depth and tsunami flow velocities. Furtherly, the tsunami forces on structures can be computed based on maximum tsunami wave height (Yeh, 2014).

2 The probability distribution of maximum tsunami wave height

2.1 *The physical process of tsunami propagation*

Before studying maximum tsunami wave height, it's necessary to clarify the physical process of tsunami wave propagation from deep sea to shallow water, so that we can analyze the formation mechanism of the maximum height tsunami wave. As is known, a body of water which can be triggered to wave is called wave base, under the water base (water depth is larger than 1/2 wave length), the body of water is calm without wave motion. When the depth of the sea bottom is larger than 1/2 wave length, the waves are called deep water wave. The propagation velocity of deep water waves is the same as common mechanical waves, that is, the speed is equal to wave length divided by period

$$u=L/T.$$

When the water is shallow and the depth is less than 1/2 wave length, which is essential for water base, the body of water has not so much activity space, water molecules will move following a elliptical orbit. In this situation the wave is called shallow water wave. The propagation speed of a shallow water wave has nothing to do with its own wave period and wave length, it is only proportional to the square root of the water depth, the relationship between them is as below

$$u=\sqrt{gd}\ , \tag{1}$$

where g is gravitational acceleration. According to equation (1), the propagation speed will decrease as the water depth decreases.

Therefore, in the process of tsunami propagation from sea to land, in the deep water area the tsunami wave spreads to the shoal water zone as a form of deep water wave with propagation speed $u=L/T$; but when the tsunami wave reaches the shoal water zone the tsunami waves will change from deep water wave to shallow water wave, while the propagation speed changes to $u=\sqrt{gd}$. From the shoal water zone to the land the water is getting shallower and shallower and the propagation speed is getting slower and slower, the process of waves going ashore is called shoaling.

For earthquake-generated tsunami the frequency and period of the tsunami wave only depend on the hypocenter, so the tsunami wave's frequency and period is constant in the course of deep water wave changing to shallow water wave. According to "period = wavelength / speed", when the speed decrease, the wavelength will also decrease to keep the period constant. And the energy carried by the tsunami are almost constant, the decreasing of the frequency and period will cause a higher wave amplitude, that's the reason why the wave height will get larger when waves are closing to shore. As we know, things always have a stable state. As the wave length decreases and its amplitude increase, the wave is not as stable as before. When the wave amplitude is increasing to a certain extent, the wave's break point will be triggered and the break will happen next. In the course of shoaling, common sea waves will turn into wave splash, but for tsunami waves, a water wall with the height of a few meters to tens of meters will be generated because of its original high speed and the huge energy carried. The process of tsunami wave propagation from sea to land is showed in Figure 1.

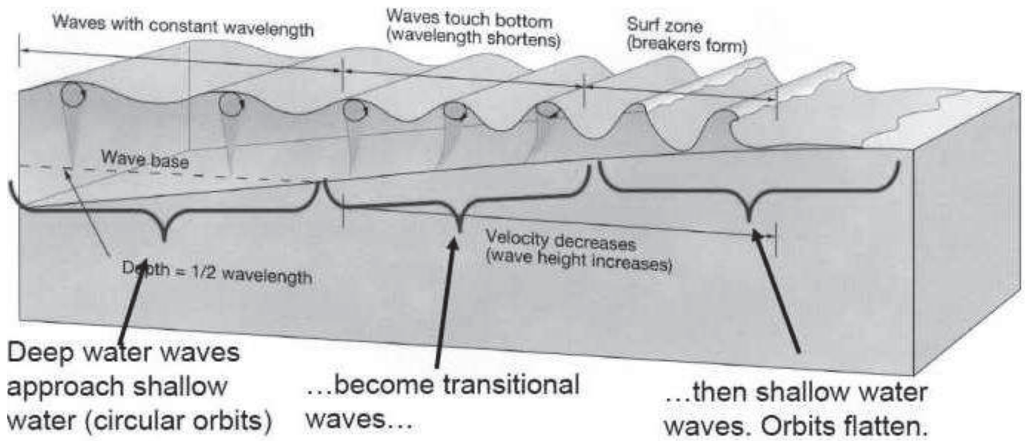

Figure 1. The process of tsunami wave propagation from sea to land.

2.2 *Tsunami wave height probability model bases on random water depth*

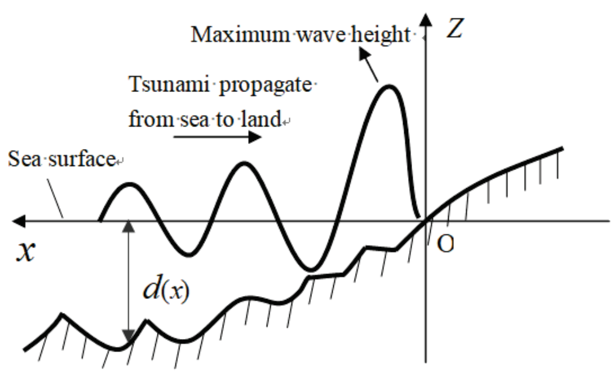

Figure 2. Assumption of water depth $d(x)$ following the Wiener process.

The tsunami wave's height gets higher instantly during tsunami wave propagation from sea to land, so the wave's height will increase to the maximum when the tsunami wave spreads near the shore, as showed in Figure 2.

As is known the landscape of the sea bottom is very rough, so the water depth from sea to land is decreasing in a non-monotonic way, as shown in Figure 2. According to the properties of the Wiener process, it can be used to describe the non-monotonic decreasing process. The method is as below.

Denote $d(x)$ as the sea depth at the location x. Different submarine topography produces different sea depth, therefore, regard $d(x)$ as random variable. When x changes from sea to land, $\{d(x), 0 \le x \le D\}$ (D represents the horizontal distance from seabed earthquake center) is a random process. Herein it is assumed that the random process $\{d(x), 0 \le x \le D\}$ (still abbreviated as $d(x)$ in the following text) is conform to a Wiener process. It should be pointed out that the independent variable in this random process is not time t but location x.

The tsunami propagation speed is determined by sea depth, substituting $d(x)$ into formula (1), and then we gain

$$u(x) = \sqrt{gd(x)}\ . \qquad (2)$$

Equation (2) means that the tsunami propagation speed $u(x)$ is also a random process.

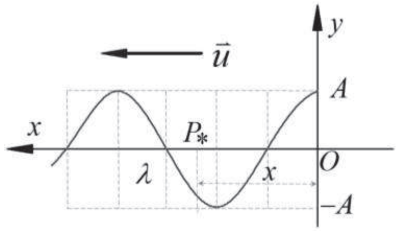

Figure 3. Harmonic wave.

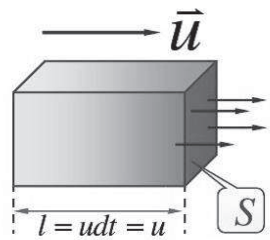

Figure 4. Energies passing through the area S.

In tsunami propagation processes, tsunami waves can be regard as harmonic waves (Figure 3). The wave equation of a harmonic wave is a function of time t and position x, that is

$$y(x,t) = A\cos\omega(t-\frac{x}{u})\ , \qquad (3)$$

where A is the amplitude of wave, ω is the circular frequency of wave, u is wave speed.

The tsunami wave carries huge energy. According to the wave theory, in the wave propagation the fluctuate of energy in a unit volume of medium is

$$w = \rho A^2\omega^2 \sin^2 \omega(t-\frac{x}{u})\ , \qquad (4)$$

where ρ is medium density. Energy passing vertically through the area S per unit time is (Figure 4)

$$P = w\cdot u\cdot S = uS\rho A^2\omega^2 \sin^2 \omega(t-\frac{x}{u})\ . \qquad (5)$$

So energy passing through area S within a period (i.e., the energy of a wave) can be calculated by

$$\begin{aligned} E &= \int_0^T uS\rho A^2\omega^2 \sin^2 \omega(t-\frac{x}{u})\ dt \\ &= uS\cdot\frac{1}{2}\rho A^2\omega^2 T \qquad (6) \\ &= 2\pi^2\rho A^2\frac{u}{T}S\ , \end{aligned}$$

where T represents the wave period. Without loss of generality, we consider the energy passing through unit area in one single wave period (i.e., wave's strength), the corresponding formula is

$$E_0 = \frac{E}{S} = 2\pi^2\rho A^2\frac{u}{T}\ . \qquad (7)$$

Then equation (7) can be solved for the amplitude A as follows

$$A = \sqrt{\frac{E_0 T}{2\pi^2 \rho u}} . \qquad (8)$$

According to the physical process of tsunami wave propagation, the wave height and wave speed change constantly and they are functions of horizontal position x, that is A=A(x) and u=u(x), then the equation (8) is

$$A(x) = \sqrt{\frac{E_0 T}{2\pi^2 \rho u(x)}} . \qquad (9)$$

The substitution of formula (2) into formula (9) yields

$$A(x) = \sqrt{\frac{E_0 T}{2\pi^2 \rho \sqrt{g d(x)}}} = \sqrt{\frac{E_0 T}{2\pi^2 \rho \sqrt{g}}} [d(x)]^{-\frac{1}{4}} . \qquad (10)$$

Tsunami height A(x) is a random process whose independent variable is horizontal position x. When the position x is fixed, then A(x) represents a random variable. In theory the maximum wave height would appear in positon x=0 just right on the shore. What we concern about is the probability distribution of maximum wave height. Generally, we can first analyze the probability distribution of A(x), its probability distribution function is

$$\begin{aligned} F_{A(x)}(y) &= P\{A(x) \le y\} \\ &= P\{\sqrt{\frac{E_0 T}{2\pi^2 \rho \sqrt{g}}} [d(x)]^{-\frac{1}{4}} \le y\} \\ &= P\{d(x) \ge \frac{E_0^2 T^2}{4\pi^4 \rho^2 g y^4}\} \\ &= F_{d(x)}(\infty) - F_{d(x)}(\frac{E_0^2 T^2}{4\pi^4 \rho^2 g y^4}) \\ &= 1 - F_{d(x)}(\frac{E_0^2 T^2}{4\pi^4 \rho^2 g y^4}). \end{aligned} \qquad (11)$$

Differentiate both sides of formula (11) and get the probability density function of A(x),

$$F'_{A(x)}(y) = -F'_{d(x)}(\frac{E_0^2 T^2}{4\pi^4 \rho^2 g y^4}) \frac{d}{dy}(\frac{E_0^2 T^2}{4\pi^4 \rho^2 g y^4}) , \qquad (12)$$

that is

$$f_{A(x)}(y) = f_{d(x)}(\frac{E_0^2 T^2}{4\pi^4 \rho^2 g y^4}) \cdot \frac{E_0^2 T^2}{\pi^4 \rho^2 g y^5} . \qquad (13)$$

The random process $\{d(x), 0 \le x \le D\}$ is a Wiener process. And a Wiener process is also a normal process, so we can have $d(x) \sim N(0, \sigma^2 x)$ for the random process's one-dimensional probability distribution (σ^2 is the parameter of Wiener process which can be determined by observation for many times). Thus the probability density function of random variable d(x) is

$$f_{d(x)}(z) = \frac{1}{\sqrt{2\pi\sigma^2 x}} e^{-\frac{z^2}{2\sigma^2 x}} . \qquad (14)$$

Hence, according to formula (14), formula (13) can be transformed into formula (15),

$$f_{A(x)}(y) = \frac{1}{\sqrt{2\pi\sigma^2 x}} \exp[-\frac{(\frac{E_0^2T^2}{4\pi^4\rho^2 gy^4})^2}{2\sigma^2 x}] \cdot \frac{E_0^2T^2}{\pi^4\rho^2 gy^5} \quad (y \geq 0) \qquad (15)$$
$$= \frac{E_0^2T^2}{\sqrt{2\pi\sigma^2 x}\pi^4\rho^2 gy^5} \exp(-\frac{E_0^4T^4}{32\pi^8\sigma^2\rho^4 g^2 xy^8}).$$

Formula (15) is just the probability density function of the random variable of tsunami height $A(x)$.

3 Conclusions

The probabilistic method for tsunamis has just been unfolding. In this paper a stochastic model of maximum tsunami height is studied. During the tsunami propagation process from deep sea to shallow water, the tsunami wave height increases randomly relying on the random submarine topographies along the path of propagation. The random water depth is thought to follow a Wiener process according to a Wiener process's characteristics. Based on that a stochastic model of maximum tsunami height is established. As is known water depth determines tsunami propagation velocity when a tsunami spreads to the shallow water area near land. And according to the energy conservation principle, the tsunami velocity affects wave height. So the tsunami height depends on water depth, namely the submarine topographies. The most important factor affecting tsunami height is taken into consideration in the stochastic model established in this paper.

Acknowledgments

This work was jointly supported by the Scientific Research Fund of Hunan Provincial Education Department (grant no. 15C0053), the National Natural Science Foundation of China (grant no.51108044), and the Systematic Project of Guangxi Key Laboratory of Disaster Prevention and Structural Safety (2016ZDK014).

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