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Risk Assessment for Slope Stability with enhanced Bayesian Networks Methods

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A graphical model for risk assessment of slope stability is presented in this paper. A Bayesian Network is combined with methods for structural reliability assessment in order to achieve near-real-time analyses. This approach allows for a simultaneous consideration of continuous and discrete variables of slope parameters, and it utilizes the causal graph of slope stability for easy understanding among decision makers. The development is demonstrated for shallow, translational landslides, assumed as infinite slopes, and yields realistic results, with the advantage of simplicity. Its implementation allows to update expected results, based on additional information regarding the geotechnical properties. This update is realized through Bayesian Networks and is performed in real time. The developed enhanced Bayesian Network shows a large application potential for reliability analysis of slopes.

Keywords: slope stability, Bayesian networks, continuous random variables, information updating

1 Introduction

Shallow landslides might be analysed as infinite slopes, whose failure is induced by various geotechnical factors, external environment, and anthropogenic influence. To identify the main induced factors for the instability of slopes is of importance for geotechnical design, disaster prevention, and decision-makers. Bayesian Networks are causal graphical models for quantifying the uncertainty and have been applied for the analysis of slope stability with a limited explored field (Liang et al. 2012, Liu et al. 2013, Peng et al. 2013). The objective of this paper is to illustrate the feasibility of risk assessment of the slope by means of advanced Bayesian networks. Furthermore, it explains how to build the model of the slope to estimate the failure probabilities, and then new observations were inserted to update the model in order to identify its effect on the slope stability.

2 Methodology

2.1 *Bayesian Networks*

Traditional Bayesian networks (BNs) are precise probabilistic models by means of directed acyclic graphs, with no cycles. Each node represents an event with random variables, which is given by conditional probabilities tables (CPTs). The edges connecting the nodes reveal the conditional dependencies between adjacent nodes. Figure 1 presents a simple example of a Bayesian Network (BN) with 3 nodes, where the node X_3 is the child of X_1 and X_2, and all of them are discrete variables.

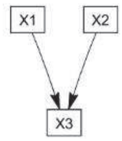


Figure 1. Example of a simple Bayesian network

According to the chain rule of BNs (Pearl 1988), the joint probability distribution is,

$$P(X_1, X_2, X_3) = P(X_1)P(X_2)P(X_3|X_1, X_2) \tag{1}$$

Exact inference algorithms (Pearl 1988) as a kind of precise methods are used to calculate marginal or joint probabilities of the variables of interest in a BN. Furthermore, BNs provide efficient algorithms for probability updating, allowing to take evidence as input. For instance, a case with given the observed node X_2=e, then this information will update through the prior probabilities to the posterior probabilities as follows,

$$P(X_1, X_3|e) = \frac{P(X_1,e,X_3)}{P(e)} = \frac{P(X_1)P(e)P(X_3|X_1,e)}{P(e)} \tag{2}$$

and $P(e)$ should be computed by marginalisation calculation when it fails to be obtained directly.

2.2 *Enhanced Bayesian Networks*

In the case continuous variables are involved, it is impractical to give discrete probabilities to all the nodes. Besides, exact inference is only available for discrete or Gaussian nodes in a very limited manner. These restrictions, hence, impede the application of BNs to study fields involving both continuous and discrete nodes. Enhanced BNs (eBNs), integrating Bayesian networks and structural reliability methods into a model, enable effective computation of continuous nodes with stochastic distributions (Straub and Kiureghian 2010).

The main concept of eBNs is to simplify the eBNs through removing all the continuous nodes from the original model, by means of structural reliability methods. Precisely, structural reliability methods erase the links between continuous nodes and their discrete children (the so-called deterministic nodes). Thus, they become barren nodes (a barren node has neither evidence nor children), enabling them to be removed without altering the CPTs of their offspring.

The process of node elimination is shown in Figure 2a, and then in light of Eq. (1), the joint probability for this network can be written as

$$P(X_1, X_2, X_3, X_4) = P(X_1)P(X_4|X_2, X_3)f(X_3)f(X_2|X_1) \tag{3}$$

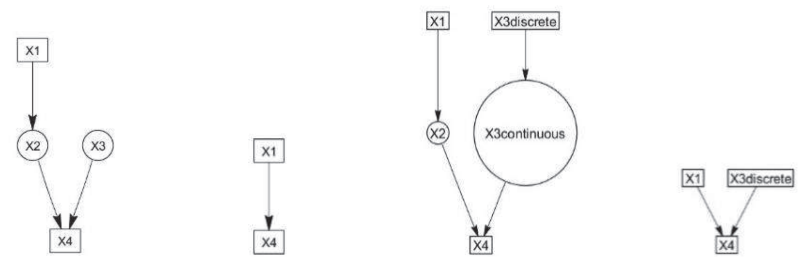
in which X_1 and X_4 are discrete nodes while $f(X_3)$ and $f(X_2|X_1)$ are probability density functions of the continuous nodes X_2 and X_3, respectively. In view of marginalization calculations, the joint probability of the discrete nodes can be achieved:

$$P(X_1, X_4) = \iint_{X_2,X_3} P(X_1)P(X_4|X_2, X_3)f(X_3)f(X_2|X_1)dX_2dX_3 \tag{4}$$

If the outcome space of node X_4 is determined by the parent nodes X_2 and X_3, then Eq. (4) can be rearranged as:

$$P(X_1, X_4) = P(X_1) \iint_{\Omega_{X_4}(X_2,X_3)} f(X_3)f(X_2|X_1)dX_2dX_3 \tag{5}$$

Here, $\Omega_{X_4}(X_{2,}X_3)$ is the domain that defines the event X_4 in the outcome space of variables X_2 and X_3. Apparently, the form of Eq. (5) coincides with the definition of a structural reliability problem, and hence can be computed with simulation approaches.



a. Reduction procedure b. Discretization procedure

Figure 2. An example of an eBN into BN

2.3 *Information Updating on Continuous Variables*

The most feasible technique for observing continuous nodes in hybrid BNs is discretization (Langseth et al. 2009). Continuous nodes should be discretized according to the following properties:

- given evidence on continuous nodes.
- continuous nodes are taken as query variables.

For the problem studied in this paper, a credible discretization approach especially for eBNs, which is proposed by Straub and Kiureghian (2010), is adopted. If the continuous node X_3 in Figure 2a is given a new observation, then a discrete variable $X_{3discrete}$ and a continuous variable $X_{3continuous}$ are introduced in order to substitute node X_3 (see Figure 2b).

The outcome space of $X_{3discrete}$ consists of some sub-domains from the divided initial continuous domain. The default number of sub-domains in this example is five with equivalent length. The node $X_{3continuous}$, as child of $X_{3discrete}$, inherits the properties of node X_3 as well as descendants. Eventually only the discrete node $X_{3discrete}$ is retained to facilitate new observations for updating the model.

3 Risk assessment of slope stability

3.1 *Formulation of an infinite slope failure*

Driving forces and resisting forces determine the stability of a slope and, as a result, the factor of safety (FOS) of a slope is the ratio of resisting and driving stresses along a potential slip surface. It is frequently computed to identify whether a slope is safe, which occurs in the case FOS≥1, or unsafe, which occurs if FOS<1. For an infinite slope, as the one shown in Figure 3, the equation for calculating the factor of safety in terms of effective stress analysis is given by,

$$FOS = \frac{c\prime + (\gamma_d Z_d + \gamma_{sat} Z_{sat} - \gamma_w Z_{sat}) * \cos\beta * \tan\phi\prime}{(\gamma_d Z_d + \gamma_{sat} Z_{sat}) * \sin\beta} \tag{6}$$

Here, the drained parameters of the soil are used, where cohesion (c') and friction angle (ϕ') are its strength parameters. Z_d , Z_{sat} are the unsaturated and saturated soil thickness, respectively, and γ_d, γ_{sat} are, respectively, the dry and saturated unit weight of the soil. Z is the total thickness of the unstable soil mass while the Greek symbol β stands for slope inclination. The unit weight of water γ_w is 9.81kN/m^3.

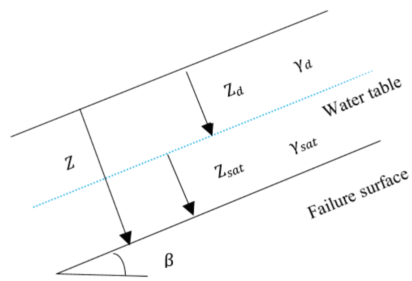


Figure 3. An infinite slope

In light of this, the failure model of slopes can be denoted by a limit state function G(X),

$$\mathrm{G(X)} = \mathrm{FOS} - 1 \tag{7}$$

Similarly, G(X)≥0 represents the safe state of the slope, otherwise, shows the unsafe state.

3.2 *The BN model of Slope Stability*

The BN structure of an infinite slope shown in Figure 4 is considered based on the slope failure model, and the relevant factors are linked by the causal reason.

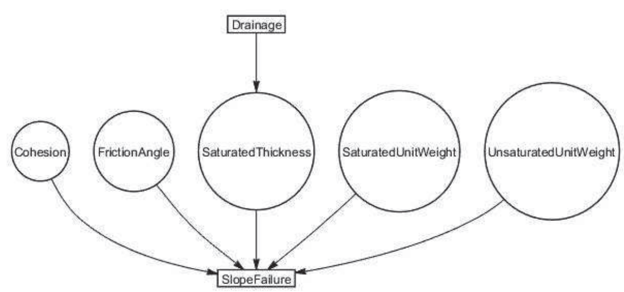


Figure 4. An enhanced Bayesian network

The occurrence of the infinite slope failure results from some potential unsafe factors. First and foremost, in view of the main properties of soil, *Cohesion* and *Friction Angle* 'resists' movement down the slope, playing a decisive role in stabilizing the slope, which is controlled by geotechnical characteristics. Meanwhile, the geometrical parameters of slope: slope inclination and slope's height are two important factors for slope stability. The angle of a slope defines what percentage of the driving force is distributed in the parallel direction along the slope surface. Small angle means small pulling force on the downslope movement, while large angle provides the large pulling force. In this work, for the sake of the simplicity, Z substituted the height of the slope, which can be obtained as $Z\,/\cos\beta$. For the infinite slope studied, Z and β are constant, and therefore, there is no need to involve these two parameters into the BN.

Water position can play a decisive role in the slope stability problems. The original source of water in the soil is from external rainfall and the percolation within the slope. Many researchers choose the change of pore water pressure to observe the stability of the slope influenced by rainfall, or the water table position. Some studies demonstrated that rainfall influences groundwater pressure associated with transient infiltration, causing variations in the water table and subsequent slope failure (Yubonchit et al. 2016).

In light of this, according to the effective stress principle, variable pore water pressure for a soil slope can be estimated by the product of unit weight of soil and saturated soil thickness. In most studies, the position of groundwater table is expressed by the depth of saturated soil, and hence, the node *Saturated Thickness* proposed is governed by the drainage condition. Specifically, if drainage takes place, water table is taken away from the critical slip surface, and therefore, Z_{sat} is equal to 0. It is rational to regard the event of *Drainage* as a parent node of *Saturated Thickness*.

In general, the model includes 6 factors and one failure event, where the event of slope failure is introduced as a target node.

3.3 *Example implementation*

A translational slip (Figure 3) is analysed in this work. It is assumed that the total thickness of the slope is 4 m at the inclination angle $\beta = 30^{\circ}$. The key parameters of the soil slope in the BN (Figure 4) are set as random variables with known probability distributions in reference to the changeable properties. The thickness of unsaturated soil can be expressed by $(4 - Z_{sat})$. The specific definition of variables can be seen in Table 1.

Table 1. Input parameters of the infinite slope in the BN

Parameters	c' (kPa)	ϕ' (°)	γ_d (kN/m^3)	γ_{sat} (kN/m^3)	Z_{sat}(m)	Drainage (D)	Slope Failure (SF)
Variable type	Continuous	Continuous	Continuous	Continuous	Continuous	Discrete	Discrete
CPD	logN(22,10)	N(35,3)	N(17, 0.4)	N(19, 0.5)	U(0, 4) or 0	[0.5, 0.5]	$[P_f,\ P_s]$

* N, logN, and U represent Normal, lognormal and uniform distribution with mean and standard deviation, respectively.

In light of the above-mentioned method, the CPT of the querying node can be computed by Eq. (3) to (5), which is expressed as follows,

$$P(SF) = P(D)\int_{\dots\dots}\int_{\Omega_{SF}(\gamma_d,\gamma_{sat,}Z_{sat},c',\varphi')} f(\gamma_d)f(\phi')f(c')f(\gamma_{sat})f(Z_{sat}|D)\,d\gamma_d d\phi' dc' d\gamma_{sat} \quad (8)$$

the outcome space $\Omega_{SF}(\gamma_d,\gamma_{sat,}Z_{sat},c',\phi')$ can be described by the limited state function defined
by Eq. (7), and hence:

$$\mathrm{P}(SF) = \begin{cases} P_f, & FOS - 1 < 0 \\ P_s, & FOS - 1 > 0 \end{cases}$$
(9)

where P_f denotes the failure probability of the node Slope Failure while the safe probability is P_s.

For inserting evidence into the continuous variables, Monte Carlo simulation is used to update the slope parameters.

3.4 Results

A real scenario problem can be solved by posing queries with respect to the BN. In this example, after removing continuous variables, only two discrete nodes: *Drainage* and *Slope Failure* are left. Then the reasoning in BN can be inferred with this reduced model, and the results are exhibited in Table 2. The failure probability of the node *SlopeFailure*, $\mathrm{P}(FS)$ is 2.74%. Compared with no evidence inserted, $\mathrm{P}(FS)$ with drainage state, is much lower than with no drainage, whose result is 5.13%, proving the importance of the water when analysing slope stability problems.

Table 2. The effect of Drainage on slope safety

Evidence	-	Drainage	No Drainage
$\mathrm{P}(FS)$	0.0274	0	0.0513

Table 3 the Failure probabilities given new observations on continuous nodes separately. $\mathrm{P}(FS)$ varies from 2.55% to 2.67%, which is close to the original result. Although just a small variation is observed, the clarification, in any case, is reduced because the range given for the observations is quite large. If the observations are narrower, the outcome will be evident.

Table 3. Slope failure probability updated with new information

Factors	c'	ϕ'	γ_d	γ_{sat}
Evidence	[0, 100]	[25, 45]	[16, 19]	[18, 21]
$\mathrm{P}(FS)$	0.0255	0.0267	0.0259	0.0256

4 Discussion

An attempt was made to analyse the risk of a slope with the eBNs approach. A discretization process is conducted to update the model with new evidence in continuous nodes. The example, although very simple, demonstrated that eBNs have present a useful capability to assess the risk for slope stability problems.

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