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BEYOND THE INTERDEPENDENT NETWORK DESIGN PROBLEM

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Critical infrastructure systems such as power, gas, water, and telecommunications, are constantly exposed to natural and anthropogenic hazards, which combined with their expanding demands and high interdependency, increase their vulnerability and exacerbate human and economic losses. Thus, public and private stake-holders are giving priority to studying such critical interdependent systems, in order to improve their ability to withstand a hazard, contain its damage, and return to adequate performance levels. In order to determine the optimal recovery strategy of a partially destroyed system of interdependent networks, we have proposed the Interdependent Network Design Problem (INDP), concerned with finding the minimum-cost reconstruction strategy, while considering realistic operational constraints. In this paper, we propose novel analytical and computational methodologies that expand on the INDP capabilities, to perform analyses beyond the reconstruction of damaged systems of networks. In particular, we show how the proposed models can be extended to also study and optimize preparedness and retrofitting strategies, considering limited budget and resources. Moreover, this paper shows how the INDP models can be used to evaluate and quantify the tradeoffs between pre- and post-event decisions, their associated resource utilization, and their impact on the overall resilience of systems of interdependent networks.

Keywords: Interdependent networks, critical infrastructure, resilience, optimization.

1 Introduction

Maintaining adequate performance levels of critical infrastructure systems is imperative for the proper operation of society. However, these systems, which are increasingly interconnected and interdependent, are constantly affected by man-made and natural hazards. Thus, there is a need to develop methodologies that allow designing resilient systems of interdependent infrastructure networks, such that they can adequately withstand the occurrence of a disruptive event, as well as quickly recover from it (Hosseini et al. 2016). In order to address the problem of efficiently recovering a system of interdependent networks after a disruptive event, González, Dueñas-Osorio, Sánchez-Silva, et al. (2016) proposed the Interdependent Network Design Problem (INDP), which seeks the minimum-cost recovery strategy while considering operational constraints associated with resource availability and limited supply and transportation capacity, among others. Later on, González, Dueñas-Osorio, Medaglia, et al. (2016) proposed the time-dependent INDP (td-INDP), in order to solve this problem considering multiple recovery periods simultaneously. However, such mathematical formulation considered exclusively post-event decisions, and assumed that physical independencies occurred only between nodes in different networks. This paper presents an extended reformulation of the td-INDP, which allows: (i) modeling functional interdependencies between any pair of elements in the system, both within and between different infrastructure networks, and (ii) optimizing pre-event decisions associated with increasing the supply capacity and the availability of resources.

2 Time-dependent Interdependent Network Design Problem (td-INDP) Reformulation

Assume the existence of the sets, parameters, and variables described below:

- Sets: $\mathcal{K}$, set of infrastructure networks; $\mathcal{L}$, set of commodities; R, set of limited resources to be used in the reconstruction process; T, set of time periods in the recovery time horizon, $\mathcal{S}$, set of geographical spaces (spatial distribution of the area that contains the infrastructure networks); $\mathcal{N}$, set of nodes before a destructive event; $\mathcal{N}_k$, set of nodes in network $k \in \mathcal{K}$ before a destructive event; $\mathcal{N}_k'$, set of nodes destroyed in network $k \in \mathcal{K}$; $\mathcal{A}$, set of arcs before a destructive event; $\mathcal{A}_k$, set of arcs in network $k \in \mathcal{K}$ before a destructive event; $\mathcal{A}_k'$, set of arcs destroyed in network $k \in \mathcal{K}$; E, set of elements in the system of networks, i.e., $\mathcal{A} \cup \mathcal{N}$; E_k, set of elements in network k, i.e., $\mathcal{A}_k \cup \mathcal{N}_k$; E_k', set of destroyed elements in network $k \in \mathcal{K}$, i.e., $\mathcal{A}_k' \cup \mathcal{N}_k'$.
- Parameters: f_{et}, reconstruction cost of element $e \in E$ at time $t \in T$; c_{elt}, flow unitary cost of commodity $l \in L$ through arc $e \in \mathcal{A}$ at time $t \in T$; M_{elt}^{+}, cost of oversupply in node $e \in \mathcal{N}$ associated with commodity $l \in L$ at time $t \in T$; M_{elt}^{-}, cost of unsatisfied demand in node $e \in \mathcal{N}$ associated with commodity $l \in L$ at time $t \in T$; g_{st}, cost of preparing geographical space $s \in S$ at time $t \in T$; u_{et}, capacity of arc $e \in \mathcal{A}$ at time $t \in T$; v_{rt}, total availability of resource $r \in R$ at time $t \in T$; h_{ert}, amount of resource $r \in R$ used to reconstruct element $e \in E$ at time $t \in T$; β_{es}, binary (zero-one) parameter indicating whether or not space $s \in S$ has to be prepared when reconstructing element $e \in E$; $\gamma_{e\tilde{e}}$, binary (zero-one) parameter indicating whether or not element $e \in E$ depends physically on element $\tilde{e} \in E$, where $e \neq \tilde{e}$; b_{et}, demand/supply in node $e \in \mathcal{N}$ at time $t \in T$.
- Variables: x_{elt}, flow of commodity $l \in L$ through arc $e \in \mathcal{A}$ at time $t \in T$; δ_{elt}^{+}, oversupply in node $e \in \mathcal{N}$ associated with commodity $l \in L$ at time $t \in T$; δ_{elt}^{-}, unsatisfied demand in node $e \in \mathcal{N}$ associated with commodity $l \in L$ at time $t \in T$; y_{et}, binary variable indicating whether or not element $e \in E$ becomes functional at time $t \in T$; $\tilde{y}_{et}$, binary variable indicating whether or not element $e \in E$ is recovered at time $t \in T$; z_{st}, binary variable indicating whether or not geographical space $s \in S$ is prepared at time $t \in T$.

Considering these, the proposed td-INDP mathematical reformulation is described as follows.

Minimize

$$\sum_{t\in T}\left(\sum_{k\in\mathcal{K}}\left(\sum_{l\in\mathcal{L}_k}\left(\sum_{e\in\mathcal{A}_k} c_{elt}x_{elt}\right)\right)+\sum_{k\in\mathcal{K}}\left(\sum_{e\in E_k'} f_{et}\tilde{y}_{et}\right)+\right.$$
$$\left.\sum_{k\in\mathcal{K}}\left(\sum_{l\in\mathcal{L}_k}\left(\sum_{e\in E_k}\left(M_{elt}^{+}\delta_{elt}^{+}+M_{elt}^{-}\delta_{elt}^{-}\right)\right)\right)+\sum_{s\in\mathcal{S}} g_{st}\tilde{z}_{st}\right) \tag{1}$$

Subject to,

$$\sum_{j:(i,j)\in\mathcal{A}_k} x_{(i,j)lt} - \sum_{j:(j,i)\in\mathcal{A}_k} x_{(j,i)lt} = b_{ilt} - \delta^+_{ilt} + \delta^-_{ilt}, \quad \forall k \in \mathcal{K},\ \forall i \in \mathcal{N}_k, \forall l \in \mathcal{L}_k, \forall t \in T, \tag{2}$$

$$\sum_{l\in\mathcal{L}_k} x_{(i,j)lt} \leq u_{(i,j)t} y_{ekt}, \quad \forall k \in \mathcal{K}, \forall (i,j) \in \mathcal{A}_k, \forall e \in \{i, j, (i,j)\}, \forall t \in T, \tag{3}$$

$$\sum_{e\in E} \gamma_{e\tilde{e}}\, y_{et} \geq y_{\tilde{e}t}, \quad \forall \tilde{e} \in E, \forall t \in T, \tag{4}$$

$$y_{e0} = 0, \quad \forall k \in \mathcal{K}, \forall e \in E'_k, \tag{5}$$

$$y_{et} \leq \sum_{\tilde{t}=1}^{t} \tilde{y}_{e\tilde{t}}, \quad \forall k \in \mathcal{K}, \forall e \in E'_k, \forall t \in T \mid t > 0 \tag{6}$$

$$\sum_{k\in\mathcal{K}} \sum_{e\in E'_k} h_{ert} \tilde{y}_{et} \leq v_{rt}, \quad \forall r \in \mathcal{R}, \forall t \in T, \tag{7}$$

$$\tilde{y}_{et} \beta_{est} \leq \tilde{z}_{st}, \quad \forall k \in \mathcal{K}, \forall e \in E'_k, \forall s \in \mathcal{S}, \forall t \in T, \tag{8}$$

$$\delta^+_{ilt} \geq 0, \quad \forall k \in \mathcal{K}, \forall i \in \mathcal{N}_k, \forall l \in \mathcal{L}_k, \forall t \in T, \tag{9}$$

$$\delta^-_{ilt} \geq 0, \quad \forall k \in \mathcal{K}, \forall i \in \mathcal{N}_k, \forall l \in \mathcal{L}_k, \forall t \in T, \tag{10}$$

$$x_{elt} \geq 0, \quad \forall k \in \mathcal{K}, \forall e \in \mathcal{A}_k, \forall l \in \mathcal{L}_k, \forall t \in T, \tag{11}$$

$$y_{et} \in \{0,1\}, \quad \forall k \in \mathcal{K}, \forall e \in E_k, \forall t \in T, \tag{12}$$

$$\tilde{y}_{et} \in \{0,1\}, \quad \forall k \in \mathcal{K}, \forall e \in E'_k, \forall t \in T, \tag{13}$$

$$\tilde{z}_{st} \in \{0,1\}, \quad \forall s \in \mathcal{S}, \forall t \in T. \tag{14}$$

Eq. (1) describes the objective function to be minimized, which is composed of four different terms: (i) the flow or transportation costs, (ii) the recovery costs, (iii) the costs associated with excess or insufficient supply of commodities, and (iv) the costs related to preparing the geographical spaces for the recovery process. Eq. (2) represents the balance constraints related to the flow of commodities. Eq. (3) represents the capacity constraints in the system, such the maximum flow through each arc is limited, and dependent on its damage state. Eq. (4) models the physical interdependencies present in the system of networks, where the functionality of a given element may require the functionality of one or more additional elements. Eq. (5) guarantees that all elements that were damaged by the disruption event are not functional immediately after the occurrence of the event, while Eq. (6) guarantees that each element that was initially damaged cannot be functional until it has been recovered. Eq. (7) guarantees that the resources used for the recovery process at each period do not surpass the available resources. Eq. (8) guarantees that whenever any given element is being recovered, its surrounding geographical space is adequately prepared. Finally, Eq. (9) to Eq. (14) describe the nature of the decision variables used.

The described td-INDP reformulation so far focuses primarily on a post-event context, where the decisions taken will only take place and impact the operation and performance of the system after it has been damaged. However, the proposed td-INDP reformulation can be extended to also consider relevant pre-event decisions as well, focused on reducing the vulnerability of the system.

2.1 *Extending the td-INDP to include both pre- and post-event decisions*

In order to improve the resilience of any given infrastructure system, it is imperative to guarantee that the system's performance can recover fast after a disruptive event. However, it is also important to ensure that the initial reduction of performance caused by a disruptive event is minimal. While the proposed td-INDP reformulation can be used to determine the optimal recovery strategy of a system after it has been affected by a disruptive event, it also can be extended to consider pre-event actions. In particular, there are two possible pre-event actions that we consider in this paper: (i) increasing the supply capacity of critical elements, and (ii) increasing the availability of key resources. In order to model these, let us define α_{ilt} as the decision variable that indicates the percentage in which node i increases its capacity of supplying commodity l at the beginning of time t, and ρ_{rt} as the decision variable that indicates the percentage in which the availability of resource r increases at the beginning of time t. Note that, based on these definitions, in fact the two variables model both pre-event (using $t = 0$) and post-event (using $t \geq 1$) decisions. Considering these variables, Eq. (15) represents the new associated balance constraints of the system (which replace Eq. (2)), and Eq. (16) represents the new constraints associated with the use of limited resources (instead of Eq. (7)).

$$\sum_{j:(i,j)\in\mathcal{A}_k} x_{(i,j)lt} - \sum_{j:(j,i)\in\mathcal{A}_k} x_{(j,i)lt} = (1+\alpha_{ilt})b_{ilt} - \delta^+_{ilt} + \delta^-_{ilt}, \quad \forall k \in \mathcal{K},\ \forall i \in \mathcal{N}_k, \forall l \in \mathcal{L}_k, \forall t \in T, \tag{15}$$

$$\sum_{k\in\mathcal{K}} \sum_{e\in E'_k} h_{ert}\tilde{y}_{et} \leq (1+\rho_{rt})v_{rt}, \quad \forall r \in \mathcal{R}, \forall t \in T, \tag{16}$$

However, in general, increasing supply capacity and resource availability has an associated cost. To model these costs, let us define a_{ilt} as the cost of increasing one hundred percent the supply capacity in node i for commodity l at time t, and p_{rt} as the cost of increasing one hundred percent the availability of resource r at time t. Thus, the new total cost to be minimized, which considers both the post-event costs associated with the recovery of the system and the pre-event investment made, is given by Eq. (17) (which updates the objective function described by Eq. (1)).

$$\begin{aligned}&\left\{\sum_{t\in T}\left(\sum_{k\in\mathcal{K}}\left(\sum_{l\in\mathcal{L}_k}\left(\sum_{e\in\mathcal{A}_k} c_{elt}x_{elt}\right)\right) + \sum_{k\in\mathcal{K}}\left(\sum_{e\in E'_k} f_{et}\tilde{y}_{et}\right) + \right.\right.\\ &\left.\left.\sum_{k\in\mathcal{K}}\left(\sum_{l\in\mathcal{L}_k}\left(\sum_{e\in E_k}\left(M^+_{elt}\delta^+_{elt} + M^-_{elt}\delta^-_{elt}\right)\right)\right) + \sum_{s\in\mathcal{S}} g_{st}\tilde{z}_{st}\right)\right\} + \\ &\left\{\sum_{t\in T}\sum_{k\in\mathcal{K}}\left(\sum_{i\in\mathcal{N}_k}\sum_{l\in\mathcal{L}_k} a_{ilt}\alpha_{ilt} + \sum_{r\in R} p_{rt}\rho_{rt}\right)\right\}\end{aligned} \tag{17}$$

Thereby, the extended td-INDP formulation can be used to determine the optimal pre- and post-event decisions that result in minimum cost (or equivalently, in minimum average unsupplied demand, if M^-_{elt} is large enough with respect to the other costs). Thus, assuming performance is measured as the fraction of supplied demand, the extended td-INDP can be used to study the trade-offs between pre- and post-event investments, and their impact over the system's resilience.

3 Illustrative example (system of water, gas, and power networks in Shelby County, TN, USA)

In order to illustrate the capabilities associated with the proposed mathematical formulation, we performed computational experiments over a system of three interdependent infrastructure networks. In particular, we studied the system of water, gas, and power networks in Shelby County, TN, which are constantly under earthquake hazard due to their proximity to the New Madrid Seismic Zone (NMSZ). The disruption level modeled in this study is consistent with an earthquake with magnitude $M_w = 7$, while the system topologies, capacities, costs, and other relevant parameters used, are taken from González, Dueñas-Osorio, Sánchez-Silva, et al. (2016).

Figure 1 shows the evolution of the overall performance of the system for different increment levels of supply capacity and resource availability. For illustrative purposes, this example assumes that all supply nodes increase their capacity in the same percentage, and that resource availability is increased equally for all periods (i.e., $\alpha_{ilt} = \alpha, \forall i \in \mathcal{N}, \forall\, l \in \mathcal{L}, \forall t \in T$, and $\rho_{rt} = \rho, \forall r \in R, \forall t \in T$). As expected, it can be seen that the larger ρ is, the higher the performance at all times. However, note that, for any given period, the rate at which the performance improves is marginally decreasing. On the other hand, note that increasing the supply capacity of the system seems to have little effect on the initial performance reduction caused by the disruptive event. However, increasing the supply capacity does substantially improve the performance of the system during the recovery process.

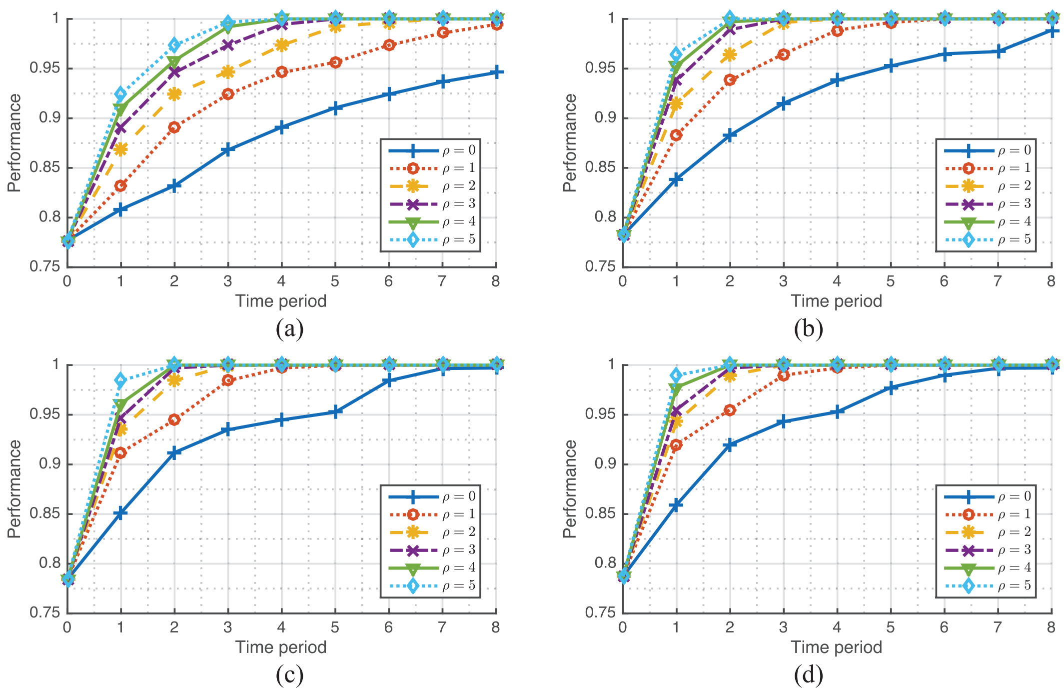

Figure 1. Overall performance of the system of interdependent infrastructure networks (water, power, and gas) in Shelby County, TN, as a function of the recovery period, for α values of (a) 0, (b) 0.1, (c) 0.2, and (d) 0.3.

In order to evaluate the impact of the modeled decisions over the resilience of the system, Figure 2a shows how resilience (as defined by Ouyang & Dueñas-Osorio (2012)) increases as a function of ρ, for different values of α. As expected, resilience grows with both α and ρ; however, the rate in which resilience increases is marginally decreasing. In order to illustrate the trade-offs between investing on increasing the supply capacity and investing on increasing the available resources, Figure 2b shows a contour plot that indicates the optimal resilience achievable for each possible combination of α and ρ. Among others, this Figure shows that the larger the values of α and ρ, the less sensitive the resilience is.

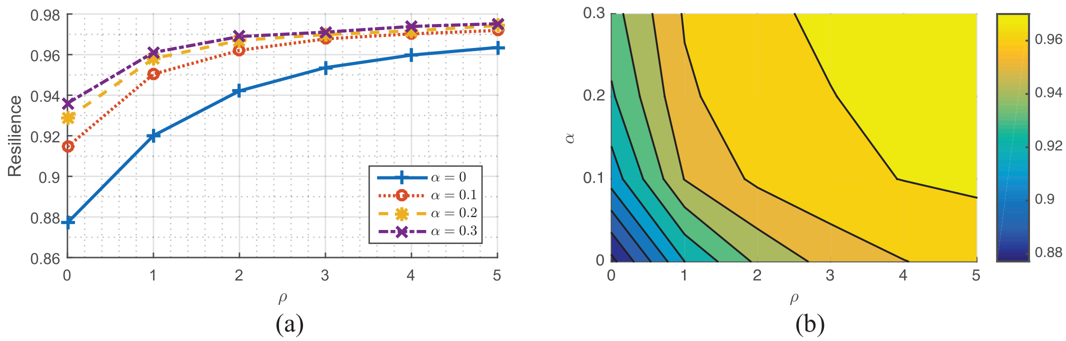

Figure 2. Measured resilience of the system of interdependent infrastructure networks (water, power, and gas) in Shelby County, TN, as a function of ρ, for different α values (a), and optimal resilience achievable for possible combinations of α and ρ (b).

4 Conclusions

This paper introduced a novel mathematical model based on the time-dependent Interdependent Network Design Problem (td-INDP), which enables determining the pre- and post-event decisions that optimize the resilience of a system of interdependent networks. In particular, this new mathematical model enables studying the trade-offs between investing on increasing supply capacity and investing on increasing resource availability. Using the system of gas, water, and power networks in Shelby County, TN, we show that resilience increases with both types of investment; however, the observed rate of increase in resilience is marginally decreasing for both types of investments as well. The proposed formulation models the increment of supply capacity and resource availability with time-indexed variables that include both pre-event and post-event domains. Thereby, the proposed model also presents a powerful tool to study the trade-offs between pre- and post-event investments.

The present study assumes that the observed disruption and system's operational parameters are fully known. However, for future work, we would like to explore extensions of the proposed model that allows for uncertainty in the damaging event and the properties of the system.

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