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# Evaluating the Route Redundancy of Hong Kong Metro Network

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Abstract: Hong Kong Mass Transit Railway (MTR) is one of the most profitable urban rail transit systems in the world, providing a convenient way to access most corners of the city. Nevertheless, network vulnerability has drawn public attention due to the frequent disruptions. This paper aims to evaluate the route redundancy of Hong Kong metro network, which has been suggested in system analysis as a good characteristic to combat vulnerability. To carry out this analysis, definitions of alternative routes and a solution algorithm are described to calculate the travel alternative diversity and the top ten vulnerable stations are identified. In addition, the same approach is adopted to study the impact of a new metro line, the Shatin to Central Link (SCL), to be completed in 2021. Overall, this study aims to provide significant insights that can help understand the topology of Hong Kong Metro network for the pre-disaster transportation system evaluation and planning.

Keywords: Metro system; vulnerability; redundancy; alternative routes; reasonable routes

## 1 Introduction

Hong Kong is one of the most densely populated global cities, with over 7 million population and land area of about 1,100 square kilometres. A unique characteristic of Hong Kong is that over 90% of the 16 million daily motorized trips are made by public transport (Lam & Bell, 2003). Railway network is the backbone of Hong Kong transport system, accounting for over 5 million average daily passenger trips, over 40% of the 12 million public transport trips in Hong Kong. Although metro networks operate with large transport capacity and efficient energy consumption, they are vulnerable to destructions which could result in a disastrous consequence of stations and lines disruptions, which will, in turn, alter the efficiency of the whole network. Examples could be network failures, unexpected passenger flows, natural disasters and terrorist events. Recently, such failure incidents have occurred more frequently and drawn public attention to the problem on route redundancy. In case of disruptions, the inconvenience causes to travellers depends on the availability of alternative travel options, that is, the amount of redundancy in the network.

Some studies have tried to investigate the problems with graph and complex network indices to identify the vulnerable stations. Liu and Song (2010) studied the transit network based on the shortest paths and clustering coefficient, analyzing how the disruption of stations affects the network efficiency. Tu (2013) studied the urban rail transit network based on centrality characteristics and identified the most central stations. Wang et al. (2013) analyzed the vulnerability and identified the most vulnerable rail segments of urban rail systems in San Francisco and Boston. These studies improved our understanding on the vulnerability analysis on metro network. However, one of their common assumptions is that passengers always travel along the shortest paths between stations, neglecting the route choices behaviors of travelers.

Limitations to centrality measures have led to the development of new metrics by introducing transport engineering characteristics into network based methodology. Derrible and Kennedy (2010) suggested that robustness of subway systems, the ability to maintain functionality under service disruptions, corresponds to the number of alternative paths available in the network under disruptions. Xu et al. (2015) suggested using the concept of efficient/reasonable path to capture the route redundancy provided to travelers. The redundancy of a transport system can be revealed by the average number of reasonable paths among various origin-destination (O-D) pairs. Yang et al. (2017) applied the model of Xu et al. (2015) to Beijing metro network and identified the top ten vulnerable stations by comparing the route redundancy under each station disruption scenario.

To date, conversations around public transport network have extended to the planning and development of the network. Although the main reason for the investment in network improvement, especially in railway development project, is typically to save travel time and digest the increased transport demand in normal operations, the argument that the new lines will add redundancy of the transport system is also often heard. However, while the feasibility and benefits of the construction of new lines are often accessed with planning models, the claims of redundancy improvement are generally not supported by evidence from evaluation and real-world case study. From a network topological viewpoint, new expansions change the topologies of network and previous studies have suggested that network topology has effects on public transport robustness. The argument that new links increase robustness makes intuitive sense that it provides an alternative route for some travelers. Specifically, the metro system of Hong Kong is experiencing rapid expansion in recent years to meet the increase in daily transport needs. There will be more new lines from 2017 to 2031 and the total railway length will increase from 221 km in 2016 to over 300 km in 2031. A raised question is how each additional line changes the network topologies, affecting route redundancy of the network.

In this paper, we focus on discussing whether the new proposed metric, travel alternative diversity, provides theoretical support to the understanding and future planning in the case of Hong Kong Metro network. The objectives are to (1) evaluate the route redundancy of current Hong Kong metro network using the travel alternative diversity concept, and (2) assess the extended Hong Kong metro network to see how the SCL helps to improve the route redundancy of current Hong Kong metro network.

## 2 Model Descriptions

In this section, we explain how the urban metro network can be illustrated with a complex network model. We start with the graph theory, which is essentially a formal mathematical method, by which networks are transformed into graphs composed of links and nodes.

### *2.1 Assessment of route diversity*

Consider a metro network $M = (N, A)$, where *N* is a finite set of nodes (stations) and *A* is a finite set of links (connection between adjacent stations), and *N* and *A* also define the numbers of nodes and links, respectively. Any nodes in *N* can be an origin or a destination. We are interested in the route set of each O-D pair in normal operation and in disastrous events. Meanwhile, travellers do not necessarily choose shortest paths and the route choice factors normally relate to the travellers' level of knowledge to the route alternatives (e.g., in-vehicle travel time, waiting time, transfers time, etc.). For clarity, some definitions forming the routes sets are presented as follows:

- *Definition 1*: A reasonable route between O-D pair $(m, n)$ is defined as a route whose links is reasonable enough relative to the shortest path (Leurent, 1997; Xu et al., 2015). described mathematically as:

$$(1 + \tau_r^a)\big(l_r(head_a) - l_r(tail_a)\big) \geq l_a, \quad \forall a \in A_k \tag{1}$$

  where $head_a$ and $tail_a$ are the head and tail of link $a$; $l_r(head_a)$ and $l_r(tail_a)$ are respectively the shortest route cost from origin $r$ to the head and tail of link $a$; $\tau_r^a$ is an *acceptable elongation ratio* for link $a$ with respect to origin *r*, which is set to 1.5 for urban studies (Tagliacozzo & Pirzio, 1973; Xu et al., 2015); $A_k$ is the set of links in route $k$.
- *Definition 2*: For each O-D pair $(m, n)$, the number of reasonable routes $E_{mn}$ is obtained. The route overlapping issue is considered by 'penalizing' the link shared by multiple routes with *similarity coefficient* $(SC)$ (Russo & Vitetta, 2003), and subsequently the index of effective alternative route set is calculated as:

$$E'_{mn} = E_{mn} - \sum_{k \neq h \in K_{mn}} SC_{kh} \tag{2}$$

$$SC_{kh} = \frac{c_{kh}}{\sqrt{c_k c_h}} \tag{3}$$

  where $c_{kh}$ is the length of links common to routes $k$ and $h$, $c_k$ and $c_h$ are the lengths of routes $k$ and $h$, respectively; $K_{mn}$ is the set of reasonable routes for O-D pair $(m, n)$.
- *Definition 3*: In normal operation, all stations are connected with two-way links in metro networks, and the number of O-D pairs of the network is $|N|(|N| - 1)$. Route diversity $D$ of network $M$ is defined as the average of the index of effective route set between all O-D pairs, mathematically expressed as:

$$D = \frac{\sum E'_{mn}}{|N|(|N|-1)} \tag{4}$$

### *2.2 Importance of stations*

Urban transit network is vulnerable to disastrous events, which may lead to considerable reductions in the connectivity of the network. When a station is out of services, people may be redirected to a smaller set of alternatives, causing potentially overloading and further disruption of other stations. Thus, the importance of stations in the network is evaluated in terms of vulnerability, that is the overall impact on the reduction of available routes when the station is disrupted. The vulnerability is examined on each individual station *r*, based on the decreases in route diversity and the *vulnerability V(r)* of station *r* is defined as:

$$V(r) = \frac{D(O) - D(r')}{D(O)}, \quad \forall r \in N \tag{4}$$

where $D(O)$ is the travel alternative diversity of the network in normal operation mode and $D(r')$ is the travel alternative diversity after station *r* is disrupted.

Travel alternative diversity can be largely affected by the number of disconnected O-D pairs in disrupted scenarios according to the mathematical calculation. In this paper, when identifying the top vulnerable stations, if there are O-D pairs disconnected because of station disruptions, those stations are put into the '*most vulnerable*' category. Their disruptions cause disconnection of O-D pairs, making some areas isolated from the rest of network. Stations are marked as '*relatively vulnerable*' when their failures result only in decrease in route diversity. They are obviously considered to be vulnerable, whose disruptions will have a significant impact on network in terms of travel alternative diversity. This approach also allows us to extract some significant properties on the route redundancy of network apart from those without travel alternatives.

### *2.3 Value of network expansion*

In general, a network extension increases the number of route choices for some passengers under both normal and disturbed conditions, in addition to reducing impacts during disruptions for some passengers. For example, a new line that shortens travel times for many travelers under normal conditions typically does so also during disruptions of other lines, in particular if the disruption occurs far from the new line. The value of robustness encapsulates both these aspects of passenger benefits. The value of a network expansion on route redundancy *under normal operating conditions* is evaluated as the difference in route diversity in the extended network $M_+$ compared to the baseline network $M_0$:

$$U_{Redundancy}(\Delta \mathrm{M}) = \frac{\mathrm{D}(M_+) - \mathrm{D}(M_0)}{\mathrm{D}(M_0)} \tag{5}$$

Meanwhile, we are also interested in isolating the value of the network extension on individual stations specifically as a means to reduce the impacts of station disruptions. This paper introduces the route diversity value as the average number of the alternative routes with the station disrupted in turn. The value of a network expansion on individual stations *r under disturbed conditions* is evaluated as the difference between the disruption impact (i.e., station vulnerability) in the extended network $M_+$ compared to the baseline network $M_0$:

$$U_{station,r}(\Delta \mathrm{M}) = \mathrm{V}_r(M_+) - \mathrm{V}_r(M_0) \tag{6}$$

## 3 Results and Discussions

This section will take the Hong Kong metro network by July 2017 as an existing network to be analysed from the alternative route diversity perspective.

### *3.1 Route diversity and vulnerability analysis*

We attempted to present the Hong Kong Metro network in a ring-radial structure with the concept of circular metro map (Figure 3), for details of the method, refer to Roberts (2012). These maps are primarily used by the passengers of the transit network for navigation and route planning. With the presented algorithm, we use the travel alternative diversity index to address the route redundancy of Hong Kong metro network. It refers to the existence of effective routes available for travellers. More available routes correspond to more evacuation routes when encountering disastrous events. Hence, it is vital to provide multiple alternatives, particularly for an important O-D pair with a large amount of commuting trips and protecting vulnerable stations. There are 107 nodes in the Hong Kong metro network and we sort all nodes as $\{1, 2, \ldots, 107\}$. Figure 1 shows the cumulative distribution of alternative route index. The percentage of all O-D pairs connected by only 1 reasonable route is 46.55%, and the route diversity index is 1.11557. The O-D pair (24, 54) has the greatest reasonable routes index of 3.30, where node 24 denotes the Hong Kong Station (transfer station) and node 54 denotes the Kowloon Bay Station.

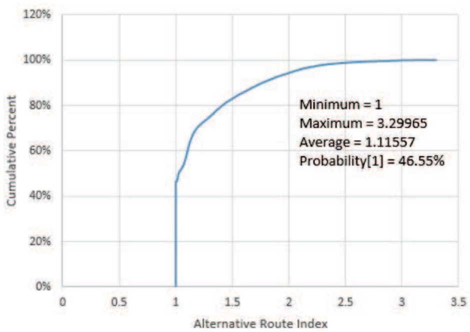


**Figure 1.** Cumulative distribution of alternative route index

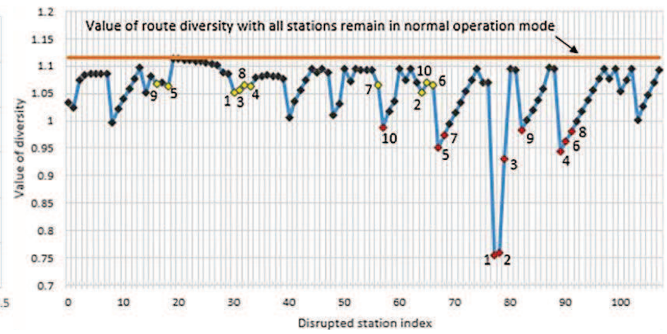


**Figure 2.** Value of the travel alternative diversity index with the station disrupted in turn.

We then assume that each node is disrupted in turn, and the route diversity index in each case is shown in Figure 2. The top ten most vulnerable stations in both 'most vulnerable' and 'relative vulnerable' category are marked in Figure 3. With the aid of the circle representation, we can easily identify the 'most vulnerable' stations, which are mainly located on the circumferential line and whose disruptions isolate radial lines from the remaining network. Meanwhile, the top ten relative vulnerable stations appear to be straightforward at a first glance, since stations in the center with more lines passing through are typically considered as important stations. However, as shown in Figure 3, not all vulnerable stations are the transfer hubs, according to the decrease in route diversity index. This suggests station locations may play a role in addition to connectivity in a metro network. Take Austin station, ranked 10th as relative vulnerable, as an example. It is an important node connecting to the center and the western part of circumferential line in the network. The overall result indicated that the diametrical line is an important line in the network structure.

**Table 1.** Value of the travel alternative diversity index with top ten vulnerable stations

| | 'Most Vulnerable' Category | | | 'Relative Vulnerable' Category | | |
|---|---|---|---|---|---|---|
| No. | Station | $D(r')$ | $V(r)$ | Station | $D(r')$ | $V(r)$ |
| 1 | Kowloon Tong ERL | 0.7556 | -32.27% | Admiralty TWL | 1.0508 | -5.81% |
| 2 | Tai Wai ERL | 0.7592 | -31.95% | East Tsim Sha Tsui | 1.0518 | -5.71% |
| 3 | Sha Tin | 0.9295 | -16.68% | Tsim Sha Tsui | 1.0571 | -5.24% |
| 4 | Tai Wai MOL | 0.9441 | -15.37% | Yau Ma Tei TWL | 1.0633 | -4.69% |
| 5 | Mei Foo WRL | 0.9503 | -14.81% | Kowloon Tong KTL | 1.0635 | -4.67% |
| 6 | Che Kung Temple | 0.9622 | -13.75% | Nam Cheong WRL | 1.0642 | -4.60% |
| 7 | Tsuen Wan West | 0.9740 | -12.69% | Nam Cheong TCL | 1.0648 | -4.55% |
| 8 | Sha Tin Wai | 0.9805 | -12.11% | Jordan | 1.0662 | -4.43% |
| 9 | University | 0.9828 | -11.90% | Prince Edward KTL | 1.0678 | -4.29% |
| 10 | Lai King TCL | 0.9873 | -11.50% | Austin | 1.0693 | -4.15% |

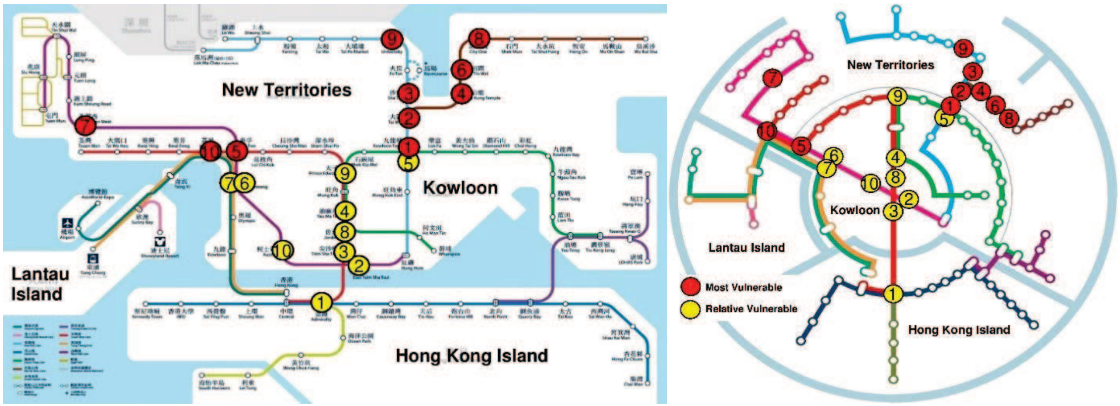


**Figure 3.** Top ten most vulnerable stations of existing network in display map (left) and in concentric circle map (right)

### *3.2 Value of the Shatin to Central link (SCL)*

By 2021, the Shatin to Central Link (SCL) will be launched in order to provide better connections between the southern and western inner suburbs (Figure 4, dotted blue and brown lines). The availability of alternative connections could be particularly valuable in case of service disruptions. The purpose of this application is to assess the impact of the SCL on network route redundancy. The value of the new SCL for network robustness is assessed by calculating the route diversity under normal operation and stations disrupted in turn. It is evident that the network extension results in higher route diversity not only under normal operations ($U_{Redundancy}(\Delta \mathrm{M})$ = (1.13666 - 1.11557) / 1.11557 = 1.89%) but also for all disruption scenarios. Thus, the results clearly suggest that the network extension increases route redundancy, providing one more cross-harbour alternatives and also redistributing the passengers and relieving the other railway lines in Hong Kong Island and Kowloon centre.

Figure 4 further shows that the SCL is particularly effective in increasing network robustness to disruptions on the trunk line (green dots) and the critical segment on the ERL (blue dots). In these two cases, the new connections are effectively reducing vulnerability of current weak points. Meanwhile, the implementation of SCL may increase vulnerability of stations on ISL which connects Central and the East Hong Kong Island and the segment in KTL near the two transfer hubs Diamond Hill and Kowloon Tong Station (orange dots). Finally, the top 10 most vulnerable stations after implementation of SCL are identified and shown in Figure 5. Metro planners should pay attention to the circumferential section of ISL for future development plans. The new top 10 vulnerable station form the new diameter truck in the circle. This indicated the importance of SCL to the structure of the metro network.

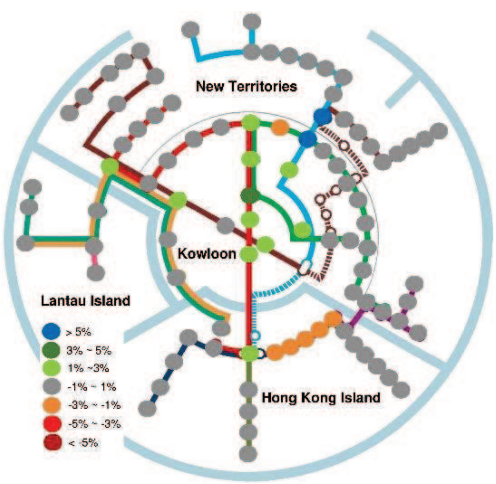


**Figure 4.** Value of vulnerable changes $U_{station}(\Delta \mathrm{M})$

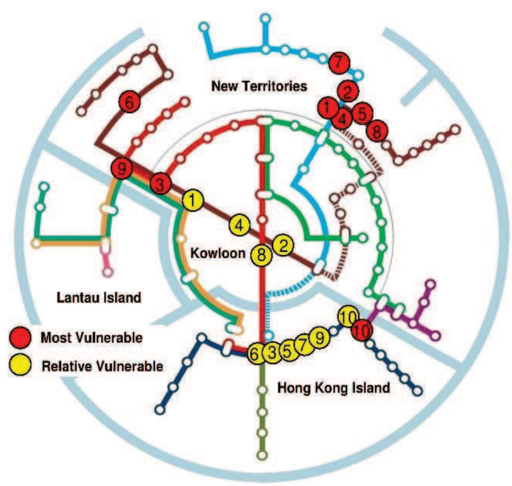


**Figure 5**. Top ten vulnerable stations in the extended network

**Table 2.** Value of the travel alternative diversity index with top ten vulnerable stations

| | 'Most Vulnerable' Category | | | 'Relative Vulnerable' Category | | |
|---|---|---|---|---|---|---|
| No. | Station | $D(r')$ | $V(r)$ | Station | $D(r')$ | $V(r)$ |
| 1 | Tai Wai ERL | 0.9471 | -16.68% | Nam Cheong WRL | 1.0803 | -4.96% |
| 2 | Sha Tin | 0.9662 | -15.00% | East Tsim Sha Tsui | 1.0814 | -4.86% |
| 3 | Mei Foo WRL | 0.9777 | -13.99% | Admiralty ISL | 1.0853 | -4.52% |
| 4 | Tai Wai MOL | 0.9813 | -13.67% | Austin | 1.0857 | -4.48% |
| 5 | Che Kung Temple | 0.9988 | -12.13% | Wan Chai | 1.0898 | -4.12% |
| 6 | Tsuen Wan West | 1.0019 | -11.86% | Admiralty TWL | 1.0905 | -4.07% |
| 7 | University | 1.0151 | -10.70% | Causeway Bay | 1.0913 | -4.00% |
| 8 | Sha Tin Wai | 1.0152 | -10.69% | Tsim Sha Tsui | 1.0915 | -3.97% |
| 9 | Lai King TCL | 1.0180 | -10.44% | Tin Hau | 1.0923 | -3.91% |
| 10 | Quarry Bay ISL | 1.0183 | -10.42% | North Point ISL | 1.0929 | -3.85% |

## 4 Conclusions

The paper introduces an approach to assess route redundancy of complex transit networks based on the reasonable route set and computation of route diversity. Traditional approaches, either in survey data driven or based on graph theory, cannot consider the impact of other key parameters in transit planning such as the ability to maintain functionality under service disruptions. By using a full-scan of stations disruptions in the network and the set of alternative routes of each scenario, the methodology can be used to identify which stations have the largest impact on available routes when they are disrupted. In long-term planning and management, knowing the weak points of the network is vital to make use of the limited human, material, and financial resources to protect these stations. The argument that new public transport links will increase the redundancy of the transport system has also drawn the attention of transport planners. The paper proposed an approach, where the value on redundancy is evaluated in terms of route diversity. By considering route diversity in both normal operation and disruption scenario, the methodology can be used to validate whether a new transport line will add to the redundancy of the system, and to help decision makers prioritize among alternative projects with redundancy effects in mind.

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