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Optimization of a Cyclic Express Subway Service

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With rapid population growth and increasing demand for urban mobility, metropolitan areas such as Singapore, Tokyo, and Shanghai are increasingly dependent on public transport systems. Various strategies are proposed to improve the service quality and capacity of bus and subway systems. Express trains—i.e., trains that skip certain stations—are commonly used, because they can travel at higher speeds, potentially reduce travel time, and serve more passengers. In this paper, we study cyclic express subway service (CESS), in which express trains provide routine transport service with cyclic (periodic) station-skip patterns that can be used in daily service. We propose an exact Mixed Integer Programming (MIP) model to optimize cyclic station-skip patterns for express trains operating in a single-track subway system. The objective is to reduce passengers' total travel time—i.e., the sum of waiting time and riding time— while considering demand intensity and distribution and train headway, frequency, and capacity. We implement the model in a set of numerical experiments using real data from Singapore. We observe that the exact MIP model for CESS provides optimal cyclic express service patterns within a reasonable computational time, and demonstrate that passengers' average travel time could be significantly reduced compared to local train service. We also discuss the potential transfer of passengers between express trains and evaluate its effects using numerical experiments.

Keywords: Subway, express service, station-skip, single-track, mixed integer programming.

1 Introduction

With rapid population growth and increasing demand for urban mobility, metropolitans are increasingly dependent on public transport. For instance, Singapore's metro system serves over 2 million commutership daily for the 5.6 million population. To handle the increased subway service demand, other than long term infrastructural upgrades for subway capacity expansions, various operational strategies are proposed and implemented, such as rejecting passengers to board trains at or even to enter some stations. In this paper, we approach the problem from a different aspect – reducing average passenger travel time for boosting the ridership supply. We study the cyclic express subway service (CESS), which provides routine train service with cyclic and periodic station-skip patterns. Each CESS express train serves a subset of all of the stations along the subway line, thus experiences decelerations and accelerations at fewer stations and travels faster. Passengers can decide which train to board on for their destinations, with information displayed on the platform.

The main contribution of this paper is to propose an exact Mixed Integer Programing (MIP) model to optimize cyclic station-skip patterns for express trains operating in a single-track subway system. The problem is challenging. First, CESS provides routine express train services rather than temporary services. Thus, the combination of station-skip design patterns is critical. Second, headway between consecutive express trains is considered to avoid train overtaking under current single-track infrastructures. Third, complicated trade-offs between in-vehicle riding time and out-of-vehicle waiting time under cyclic station-skip patterns are considered.

2 Literature Review

Literature on the design and operation of express services generally focus on either bus systems or subway systems. For research on express bus services, Leiva et al. (2010) minimize social costs with limited-stop services in an urban bus corridor; Chiraphadhanakul and Barnhart (2013) maximize total user welfare with new limited-stop express services operating in parallel with existing bus services; Chen et al. (2015) optimizes bus stopping strategy given stochastic travel time and vehicle capacities. For express subway systems, research is rather limited. Among the earliest studies, Suh, Chon, and Rhee (2002) consider three typical train-stopping patterns for express subway systems. Freyss et al. (2013) focus on the skip-stop operation for rail transit line using single track, while only two type of skip patterns are considered. More recently, Gao et al. (2016) optimize express subway re-scheduling in the case of overcrowding after disruptions, outperforming existing services. Homero et al. (2017) present a framework for addressing the limited-stop service design problem in bus operation. They conside the transfers not only in deterministic and stochastic conditions.

Previous studies mainly focus on temporary express subway services under circumstances such as disruption or overcrowding, instead of routine services with cyclic station-skip patterns. In this paper, we focus on a single-track subway system, which imposes strong constraints on station-skip patterns due to the infeasibility of train overtaking. We also discuss the potential transfer of passengers—which is ignored in the literature—between express trains, and evaluate its effects using numerical experiments.

3 The Cyclic Express Subway Service Problem

We aim to optimize the cyclic station-skip patterns for express trains with the objective of minimizing passengers' average travel time. We consider a single-track subway system in which $\boldsymbol{S} = \{1, 2, \dots, S\}$ is the set of subway stations, and $\boldsymbol{K} = \{1, 2, \dots, K\}$ is the set of express trains with cyclic station-skip patterns. Given the number of station-skip patterns K in one service cycle and the number of skipped stations for each express train, the problem is to determine which stations $s \in \boldsymbol{S}$ should be skipped by each train k. Figure 1 presents an example of a CESS follow with 3 station-skip patterns in a periodic manner ($K = 3$). For each train $k \in \boldsymbol{K}, k = 1, 2, 3$, it serves a subset of all the stations along the subway line. Solid circles denote the stations served by a train, while empty circles denote the station skipped by the train. The CESS problem is a combinational optimization problem. Here we introduce the mathematical notation as follows:

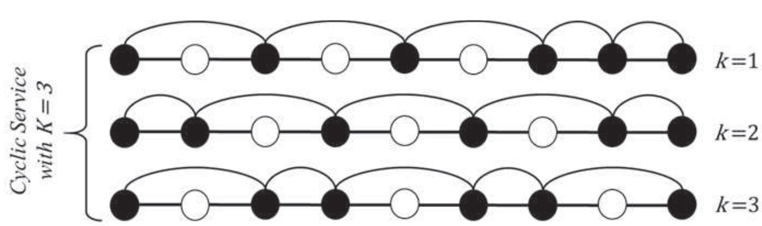

Figure 1. Example of Cyclic Express Subway Service

Table 1. Notations for the exact MIP model

$\boldsymbol{S}$	: $\{1, 2, 3, \ldots, S\}$, set of subway stations;
$\boldsymbol{K}$	: $\{1, 2, 3, \ldots, K\}$, set of express trains in one service cycle.
Parameters (Trains):	
t_i	: travel time for train from station i to station $i+1$, $i \in \boldsymbol{S} \backslash \{S\}$;
p	: service time for trains in each served station, i.e., time spent on train deceleration and acceleration and passenger alighting and boarding;
h_L	: headway between local service trains;
T_L	: total travel time for local service trains from station 1 to station S;
m	: number of stations skipped by each express service train;
h_S	: minimum allowed safety headway between express service trains;
C	: train capacity;
β	: coefficients of waiting time in the objective function.
Decision Variables (Trains):	
x_i^k	: $\in \{0,1\}$. 1 if station $i \in \boldsymbol{S}$ is served by train $k \in \boldsymbol{K}$; 0 otherwise;
y_{ij}^k	: $\in \{0,1\}$. 1 if both station i and j are served by train $k \in \boldsymbol{K}$; 0 otherwise.
Intermediate Variables (Trains):	
h_E	: headway between express service trains;
T_E	: total travel time for express service trains from station 1 to station S;
A_i^k	: arrival time of train k at station i in one service cycle;
c_i^k	: maximum number of passengers allowed to board at station i for train k due to train capacity.
Parameters (Passengers):	
λ_{ij}	: number of passengers entering origin station i with destination station j per unit of time.
Intermediate Variables (Passengers):	
d_{ij}	: number of passengers entering origin station i with destination station j during one headway between express service trains;
r_{ij}^k	: number of passengers at station i with destination j before the arrival of train k;
r_i^k	: total number of passengers at station i before the arrival of train k;
r'^k_{ij}	: number of passengers at station i with destination j before the arrival of train k with both the origin and destination stations served by train k ;
r'^k_i	: total number of passengers at station i before the arrival of train k with both the origin and destination stations served by train k;
b_{ij}^k	: number of boarding passengers at station i with destination j for train k;
b_i^k	: total number of boarding passengers at station i for train k;
w_{ij}^k	: number of waiting passengers at station i with destination j after the departure of train k from station i;
w_i^k	: total number of waiting passengers at station i after the departure of train k from station i;
n_i^k	: total number of onboard passengers on train k after its departure from station i.

We now propose an exact MIP model to optimize the cyclic station-skip patterns for express trains operating in a single-track subway system. To minimize passengers' total travel time, we choose an objective function and constraints as follows. The objective function contains two components: in-vehicle riding time ΔT_{riding} and out-of-vehicle waiting time $\Delta T_{\text{waiting}}$, variable ΔT denote the total passenger travel time saved in one cycle running of express service.

$$\text{Maximize } \Delta T = \Delta T_{\text{riding}} - \beta \cdot \Delta T_{\text{waiting}} = \sum_{k=1}^{K}\sum_{i=2}^{S-1} n_{i-1}^{k} \cdot p \cdot \left(1 - x_i^k\right) - \beta \cdot \sum_{k=1}^{K}\sum_{i=2}^{S-1} w_i^k \cdot h_E \tag{1}$$

Subject to:

Express service constraints:

$$m = S - \sum_{i=1}^{S} x_i^k \qquad \forall\, k \in \boldsymbol{K} \tag{2}$$

$$\sum_{k=1}^{K} x_i^k \geq 1 \qquad \forall\, i \in \boldsymbol{S} \tag{3}$$

$$x_i^k = 1 \qquad i = 1 \text{ and } S, \forall\, k \in \boldsymbol{K} \tag{4}$$

$$y_{ij}^k = x_i^k \cdot x_j^k \quad \forall\, i,j \in \boldsymbol{S} \text{ and } i \neq j, k \in \boldsymbol{K} \tag{5}$$

$$\sum_{k=1}^{K} y_{i,i+1}^k \geq 1 \qquad \forall\, i \in \boldsymbol{S} \tag{6}$$

$$x_i^k \in \{0,1\} \qquad \forall\, i \in \boldsymbol{S}\backslash\{1,S\}, k \in \boldsymbol{K} \tag{7}$$

$$y_{ij}^k \in \{0,1\} \quad \forall\, i,j \in \boldsymbol{S} \text{ and } i \neq j, k \in \boldsymbol{K} \tag{8}$$

Train constraints:

$$T_E = T_L - m \cdot p \tag{9}$$

$$h_E = \frac{T_E}{T_L} \cdot h_L \tag{10}$$

$$A_1^k = (k-1) \cdot h_E \qquad \forall\, k \in \boldsymbol{K} \tag{11}$$

$$A_i^k = (k-1) \cdot h_E + \sum_{1 \leq i' \leq i-1} \left(t_{i'} + p \cdot x_{i'}^k\right) \quad \forall\, i \in \boldsymbol{S}\backslash\{1\}, k \in \boldsymbol{K} \tag{12}$$

$$A_i^{k+1} - A_i^k \geq h_S \qquad \forall\, i \in \boldsymbol{S}, k \in \boldsymbol{K}\backslash\{K\} \tag{13}$$

$$c_i^k = \begin{cases} C & i = 1, \forall\, k \in \boldsymbol{K} \\ C - \sum_{i'<i}\sum_{j'>i} b_{i'j'}^k, & \forall\, i \in \boldsymbol{S}\backslash\{1,S\}, k \in \boldsymbol{K} \end{cases} \tag{14}$$

$$n_i^k = \begin{cases} b_i^k & \forall\, i \in \boldsymbol{S}, k = 1 \\ n_{i-1}^k - \sum_{j=1}^{i-1} b_{ji}^k + \sum_{j=i+1}^{S} b_{ij}^k & \forall\, i \in \boldsymbol{S}, k \in \boldsymbol{K}\backslash\{1\} \end{cases} \tag{15}$$

$$b_i^k = \min\{r'^k_i, c_i^k\} \qquad \forall\, i \in \boldsymbol{S}, k \in \boldsymbol{K} \tag{16}$$

Passenger boarding and alighting constraints:

$$d_{ij} = \lambda_{ij} h_E \qquad \forall\, i \in \boldsymbol{S}, k \in \boldsymbol{K} \tag{17}$$

$$r_{ij}^k = \begin{cases} d_{ij} + w_{ij}^K & \forall\, i,j \in \boldsymbol{S}, k = 1 \\ d_{ij} + w_{ij}^{k-1} & \forall\, i,j \in \boldsymbol{S}, k \in \boldsymbol{K}\backslash\{1\} \end{cases} \tag{18}$$

$$r_i^k = \sum_{j=i+1}^{S} r_{ij}^k \qquad \forall\, i \in \boldsymbol{S}, k \in \boldsymbol{K} \tag{19}$$

$$r'^k_{ij} = r_{ij}^k \cdot y_{ij}^k \qquad \forall\, i,j \in \boldsymbol{S}, k \in \boldsymbol{K} \tag{20}$$

$$r'^k_i = \sum_{j=i+1}^{S} r'^k_{ij} \qquad \forall\, i \in \boldsymbol{S}, k \in \boldsymbol{K} \tag{21}$$

$$b_{ij}^k \leq r'^k_{ij} \qquad \forall\, i,j \in \boldsymbol{S}, k \in \boldsymbol{K} \tag{22}$$

$$b_{ij}^k \geq \min\left\{r'^k_{ij}, \frac{\lambda_{ij}}{\sum_{j=i+1}^{S} \lambda_{ij}} \cdot b_i^k\right\} \quad \forall\, i,j \in \boldsymbol{S}, k \in \boldsymbol{K} \tag{23}$$

$$b_i^k = \sum_{j=i+1}^{S} b_{ij}^k \qquad \forall\, i \in \boldsymbol{S}, k \in \boldsymbol{K} \tag{24}$$

$$w_{ij}^k = r_{ij}^k - b_{ij}^k \qquad \forall\, i, j \in \boldsymbol{S}, k \in \boldsymbol{K} \tag{25}$$

$$w_i^k = \sum_{j=i+1}^{S} w_{ij}^k \qquad \forall\, i \in \boldsymbol{S}, k \in \boldsymbol{K} \tag{26}$$

4 Numerical Experiments Using Real Data

We implement the model proposed in a set of numerical experiments using smart-card data collected in Singapore metro system. A subway line of 15 stations is investigated during evening peak hours (6:00–7:00 pm). Demand distributions at stations are identified as unbalanced, from passenger demand λ_{ij} re-constructed using following Sun et al. (2013). To reduce computational time of solving the MIP model, model linearization techniques are used. In addition, a rolling horizon along the subway line is introduced, that is, we solve a sub-problem for the first half of stations, then fix the station-skip patterns on these stations and solve for all stations.

We evaluate the performance of the CESS, up to 4 station-skip patterns in one service cycle (denoted as K) and up to 5 skipped stations for each express train (denoted as m) in numerical experiments. Most experiments are solved reasonably within an hour. The average travel time reduction for passengers is shown in Table 1. Generally, with more stations skipped, the average time saved will increase and then decrease, due to the riding and waiting time trade-offs. No feasible solution may be provided if too many stations are skipped and some passengers cannot be served (i.e. $K = 2, m = 5$). Importantly, we find that a more flexible CESS (a larger K) is more likely to help passengers save total travel time.

Table 1. Average Passenger Travel Time Reduction

Patterns K / Skip Stations m	$K = 2$ sec (percent)	$K = 3$ sec (percent)	$K = 4$ sec (percent)
1	13.32 (1.59%)	13.64 (1.62%)	13.83 (1.65%)
2	21.13 (2.52%)	24.57 (2.93%)	24.74 (2.95%)
3	**24.39 (2.90%)**	**26.95 (3.21%)**	**26.53 (3.16%)**
4	-18.31 (-2.18%)	16.01 (1.91%)	21.72 (2.59%)
5	---	-5.70 (-0.68%)	4.66 (0.55%)

As part of the sensitivity analysis, we examine the model with changed passenger demand (i.e. passenger origin and destination distributions). We find that express train service brings comparatively more value to passengers with increased travel distances (Figure 2).

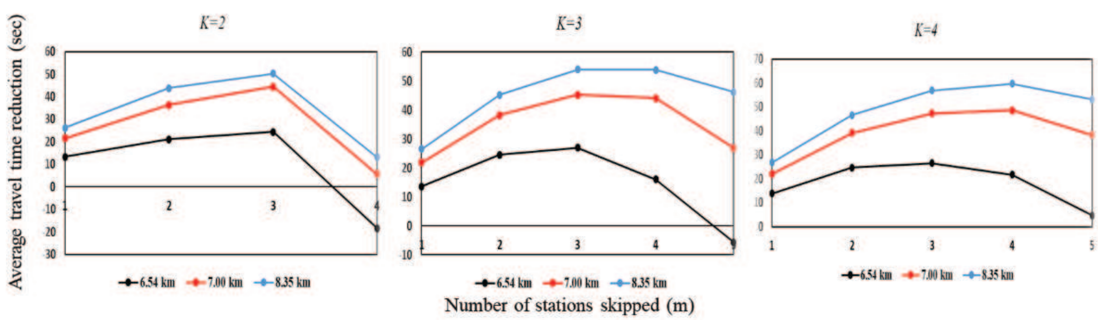

Figure 2. Comparison of Average Travel Time Reduction for Different O-D Distribution

We provide a simple approximated component to the objective function to capture the travel time reduction due to passenger transfer, and evaluate CESS performance with passenger transfer using numerical experiments. If passengers can transfer between express trains, waiting time will be further reduced compared to the case in which passengers are limited to using direct service.

Specifically, with passenger transfer, w_i^k, the number of waiting passengers at each station i after the departure of each train k will be reduced, which leads to less out-of-vehicle waiting time. Intuitively, with a larger K, passengers will have more feasible transfer routes, so out-of-vehicle waiting time will decrease; with a larger m, passengers will have fewer feasible transfer routes, so out-of-vehicle waiting time will increase. We tested different formulas through numerical experiments, and obtained with a simple approximated term $h_E K\left(\sum_{i\in S} w_i^k + \frac{1}{m}\right)$ as a third component in the objective function to capture waiting time reduction due to passenger transfer. We evaluate the approximated formula using numerical experiments and find that it works well when m is not too large. As shown in Figure 3, the average passenger travel time reduction in CESS with passenger transfer is greater compared to that in CESS without passenger transfer (Table 1).

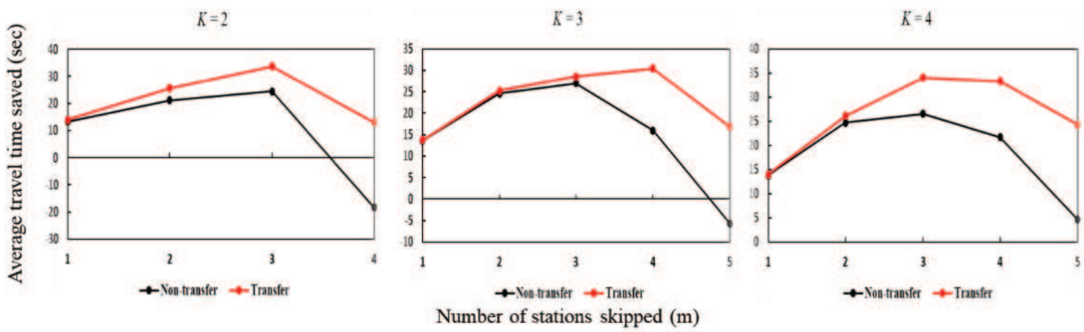

Figure 3: Comparison of Average Travel Time Reduction for CESS with and without Passenger Transfer and Average Travel Distance 6.54 km

5 Conclusion

The paper addresses the CESS problem. We study routine express subway service with cyclic station-skip patterns, and propose an optimization model to determine cyclic station-skip patterns for express trains operating in a single-track subway system. Given demand intensity and distribution, train headway, frequency and capacity, an exact Mixed Integer Programing (MIP) model is formulated to reduce passenger travel time. We implement the model in a set of numerical experiments using real data from Singapore, and find that average passenger travel time is significantly reduced compared to local train services. Within a reasonable computational time, the exact MIP model is able to provide optimal solutions for CESS. In addition, we discuss potential passenger transfers between express trains and evaluate their effects with numerical experiments.

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