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Robust In-situ Instrument Noise Calibration for Field Vibration Testing

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Instrument noise calibration is an indispensable task in structural health monitoring, providing in-situ information and management for the quality of data in field vibration measurements. Different methods exist depending on the nature of information assumed, among which the three-channel method proposed by Sleeman and co-workers allows one to calibrate using only three collocated data channels. The method, however, is sensitive to sensor alignment error, which is inevitable especially in the field setting. This work proposes a method based on three collocated sensors that is applicable regardless of sensor orientation, thereby significantly enhances the robustness of the calibration procedure. A series of investigation including experimental studies verifies the method and demonstrates its applicability.

Keywords: Structural health monitoring, Instrument noise calibration, Sensor orientation, Signal-to-noise ratio.

1 Introduction

Structural health monitoring (SHM) aims at diagnosing the in-situ state of a structure. It has the promise of effectively assisting in risk management and resilience assessment of infrastructure systems using the information collected from in-situ measurements. Instruments including sensor, cable and data acquisition hardware are basic components in a SHM system. Their quality determines the accuracy of the system and influences the reliability of the results of other applications based on SHM.

One of the most crucial features that characterize instrument quality is its noise level, which must be calibrated when installing a SHM system. If the instrument noise is too high it can 'mask' the information carried in the original signal. Understanding the noise level of a data channel allows one to use it more properly. The information provided by specifications or calibration certificates is generally insufficient to evaluate the current state of the instruments (Brownjohn and Botfield 2009, Au 2017). It is therefore desirable to develop a method which can calibrate instrument noise robustly and conveniently in field tests. Instrument noise calibration is also an important task in seismology (Havskov and Alguacil 2016).

Currently, several methods based on different assumptions are available for sensor noise calibration. The three-channel method (Sleeman et al. 2006, Hutt et al. 2009) is a conventional choice that has been used to calibrate a variety of sensors (Ringle and Hutt 2010, Ringler et al. 2015). One advantage of the method is that it requires only three collocated data channels measuring the same motion and it does not require any other prior information such as the transfer function of a 'controlled' data channel whose noise characteristics is assumed to be reliably known. Alignment error is inevitable in practice, however, especially in the field situation. Previous studies (Ringler et al. 2011) show that the accuracy of noise estimates strongly depends on the alignment of the sensors; a small alignment error could cause a significant error in the instrument noise estimates. Besides, high SNR (signal-to-noise-ratio) can ironically significantly amplify the misalignment effect on noise estimates (Sleeman and Melichar 2012), rendering this method producing results with unstable behavior. Developing an explicit method that is robust to sensor orientation has been found to be non-trivial. Some attempts (Gerner and Bokelmann 2013, Tasič and Runovc 2012) based on the three-channel method have been made to mitigate misalignment effect. Despite these attempts, the fundamental issue is still unresolved.

Addressing a problem of significance, this work proposes an explicit method for instrument noise calibration based on three collocated sensors which is applicable regardless of sensor orientation and hence allows one to calibrate instruments robustly and conveniently in field vibration testing. A comprehensive experimental study is presented to verify the robustness of the new method and demonstrate its application.

2 Three-channel method

The three-channel method (Sleeman et al. 2006) is briefly introduced as it leads naturally to the need for the proposed method. It assumes three collocated and co-aligned sensors (say sensor *i*, *j* and *k*) recording the same motion. Assuming that the input-output relationship of the measurement system is linear and that the instrument noise and the input motion are uncorrelated, the noise of sensor *i* can be estimated by

$$S_{ei} = S_{ii} - S_{ji}\frac{S_{ik}}{S_{jk}} \tag{1}$$

where $S_{ik} = E(X_i X_k^*)$ is the cross PSD (Power Spectral Density) between sensor *i* and sensor *k*; X_i is the suitably scaled Fourier transform (FT) of the output signal obtained from sensor *i*; '*' denotes complex conjugation and $E(\bullet)$ denotes the expectation. Similar notations apply to other terms. For notational simplicity, the dependence of the PSDs on frequency has been omitted. As the theoretical PSDs are unknown in implementation, they are generally substituted by their sample estimates. To produce the sample estimates, the data obtained from the sensor is first divided into several non-overlapping segments with equal length. The sample PSD of each segment is then calculated and averaged. When the sample estimates are substituted into Eq. (1), the RHS (Right Hand Side) is generally complex-valued. In view of this, it can be shown that a legitimate way in implementation is to take only the real part of the RHS as the noise estimates.

The three-channel method is sensitive to alignment error. It can be shown mathematically (details omitted) that the method is applicable for the subject channel (*i* in Eq. (1)) if and only if there is another channel (*j* and/or *k*) measuring the exactly same motion (hence in exact alignment). Here we will use an experiment to illustrate this issue. Figure 1(a) shows a general view of the experiment performed under laboratory environment. Two sets of data were obtained from five sensors, i.e., A to E indicated in the figure. The two setups are specially designed with the sensors oriented differently to investigate conclusively when the three-channel method is applicable and when it is not. For setup 1, sensors E, C and D are oriented along the same direction while another two sensors (A and B) are along a different but common direction. The angle between these two groups of sensors is 60°, as shown in Figure 1(b). For setup 2 as shown in Figure 1(c), only sensor A is oriented differently from that in setup 1 and it has a clockwise angle offset of 15° with sensor B. Sensor E is the target sensor in the experiment and the noise of its North channel (parallel to handle) will be analyzed in different cases later. All sensors used here are triaxial servo-accelerometers. The data was recorded at 200Hz for 3 hours. It was divided into 1080 non-overlapping segments to produce an averaged sample PSD with a frequency interval of 0.1Hz and a c.o.v (coefficient of variation) of $1/\sqrt{1080} \approx 3.04\%$.

Four cases are considered in the analysis: 1) sensors E, C and D - the three sensors have the same direction; 2) sensors E, C and B - the target sensor E has the same direction with one another sensor (C); 3) sensors E, A and B in setup 1 - another two sensors (A and B) are oriented along the same direction while the direction of the target sensor is different from them; 4) sensors E, A and B in setup 2 - all the three sensors have different directions. Note that the sensors are inevitably placed at distinct locations (as in practice), which violates the basic assumption of collocated channel in the three-channel method, hence causing modelling error in the results. In view of this, the sensors are deliberately placed as close as possible, forming equilateral triangles so that spatial incoherence has a similar effect on different pairs of channels.

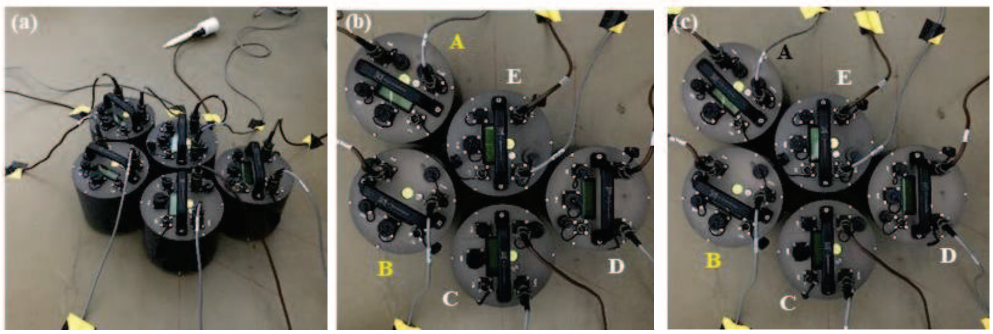

Figure 1. experimental setup. (a) General view; (b) Setup 1; (c) Setup 2

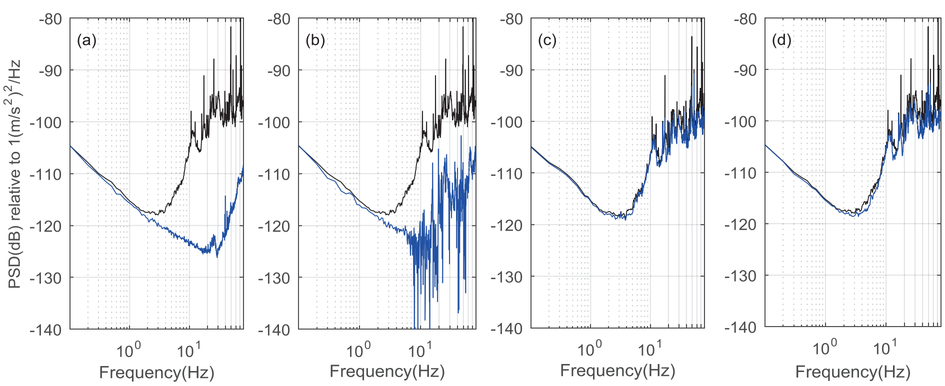

Figure 2. The data PSD (black line) and the noise PSD of the N-direction of sensor E estimated by the three-channel method (blue line)

Figure 2 shows the results of the N-channel of sensor E corresponding to each case introduced above. They can be explained by Eq. (2) below, which expresses the error of noise estimates induced by misalignment effect when considering the input motion as three-dimensional motion (derivation omitted here).

$$e_i = \frac{\hat{S}_{ei} - S_{ei}}{S_{ei}} = \frac{G_i G_i^*}{S_{ei}} (\mathbf{r}_i \mathbf{S}_Z \mathbf{r}_i^T - \mathbf{r}_j \mathbf{S}_Z \mathbf{r}_i^T \frac{\mathbf{r}_i \mathbf{S}_Z \mathbf{r}_k^T}{\mathbf{r}_j \mathbf{S}_Z \mathbf{r}_k^T}) \quad (2)$$

where e_i expresses the error term of this method caused by alignment error; $\hat{S}_{ei}$ is the instrument noise estimated by the three-channel method while S_{ei} is the theoretical one; G_i denotes the transfer function of sensor i; $\mathbf{S}_Z = E(\mathbf{Z}\mathbf{Z}^*)$ is the PSD matrix of the input motion while $\mathbf{Z}$ is the scaled FT of the three-dimensional input motion along the global coordinate axis; $\mathbf{r}_i=(r_{i1}, r_{i2}, r_{i3})$

is the direction cosines of sensor i and it satisfies $r_{i1}^2 + r_{i2}^2 + r_{i3}^2 = 1$. Similar notation applies to other terms. It can be seen that the error term is related to SNR and sensor orientations. If all the sensors are oriented along the same direction, the error term is identically equal to zero. When only one sensor is oriented along the same direction with the target sensor, i.e., $\mathbf{r}_i = \mathbf{r}_j$ or $\mathbf{r}_i = \mathbf{r}_k$ in Eq. (2), theoretically the error term also vanishes. This does not seem to agree with the results in Figure 2(b), where the noise estimates show high fluctuation when the SNR is high. Numerical investigation (details omitted here) also shows that the noise estimates vary highly when both the alignment error and the SNR are high. In view that the difference between the numerical and theoretical study is in the selection of data PSDs, the high fluctuation in Figure 2(b) is attributed to the statistical error when the averaged sample PSDs are utilized to substitute the theoretical counterparts. When no sensors have the same orientation with the target one, generally the error term is not equal to zero and it can be seen from Figure 2(c) and (d) that the three-channel method fails.

3 Three-sensor method

The experimental and theoretical results shown in the last section indicate that the three-channel method strongly depends on sensor alignment and the misalignment effect on noise estimates is determined by SNR. In the field situation, precise alignment of the sensors is difficult to be achieved, thus undermining the applicability or reliability of the three-channel method. In view of this, a method is proposed that can estimate the instrument noise explicitly and accurately regardless of sensor orientation, significantly enhancing robustness and convenience in implementation.

Consider three collocated sensors (say sensor i, j and k) with possibly different orientations driven by a common input motion (possibly multi-dimensional). For each sensor, it can measure d directions of the input motion at its location (d=1,2,3). This is the same setup in the three-channel method, where the three channels are assumed to measure exactly the same, i.e., one-dimensional, component of motion just described. Investigation on the mathematical structure of the calibration problem reveals that the shortcoming of the method stems from lower dimension (one) of the modelled motion compared to the actual one (d here). In view of this, as the key of the proposed method, the modelled motion is assumed to be fully d-dimensional. In the frequency domain, the scaled FT of the vibration with d Dofs (Degrees of freedom) measured by sensor i is modelled as:

$$\mathbf{X}_i(\omega) = \mathbf{G}_i(\omega)\mathbf{R}_i\mathbf{Z}(\omega) + \boldsymbol{\varepsilon}_i(\omega) \tag{3}$$

where $\mathbf{G}_i$ is a d-dimensional diagonal matrix with the m-th (m=1,…,d) diagonal entry being the transfer function of the m-th channel of sensor i; $\mathbf{Z}$ is a d-by-1 vector and it denotes the scaled FT of the modelled input motion along the global coordinate axis (common to all the sensors); $\mathbf{R}_i$ is a d-by-d rotation matrix and it transforms the input motion Dofs into the Dofs measured by sensor i; $\boldsymbol{\varepsilon}_i$ is a d-by-1vector and it denotes the scaled FT of the channel noise of sensor i. The dependence of the variables on frequency has been emphasized, but it will be omitted in the following for simplicity.

Similar to the three-channel method, the noise of different channels are assumed to be uncorrelated and they are also uncorrelated with the input motion. Let $\mathbf{H}_i$=$\mathbf{G}_i\mathbf{R}_i$ in Eq.(3), the PSDs related to the data obtained from sensor i can be given by,

$$\mathbf{S}_{ii} = \mathbf{H}_i\mathbf{S}_Z\mathbf{H}_i^* + \mathbf{S}_{ei} \tag{4}$$

$$\mathbf{S}_{ji} = \mathbf{H}_j\mathbf{S}_Z\mathbf{H}_i^* \tag{5}$$

$$\mathbf{S}_{jk} = \mathbf{H}_j\mathbf{S}_Z\mathbf{H}_k^* \tag{6}$$

$$\mathbf{S}_{ik} = \mathbf{H}_i \mathbf{S}_Z \mathbf{H}_k^* \tag{7}$$

where $\mathbf{S}_{ik}=E(\mathbf{X}_i\mathbf{X}_k^*)$ is the cross-PSD matrix between the data obtained by sensor i and sensor k. Similar notations apply to other matrices on the LHS (left hand side) of the above equations; $\mathbf{S}_{ei}$ is the noise PSD of sensor i , the target of the calibration procedure.

The noise PSD $\mathbf{S}_{ei}$ can be obtained algebraically from the PSD of the measured channels as follow. According to Eq. (5), Eq. (6) and Eq. (7),

$$\mathbf{S}_{ik} = \mathbf{H}_i \left(\mathbf{S}_Z \mathbf{H}_i^* \mathbf{S}_{ji}^{-1} \right) \mathbf{S}_{jk} \tag{8}$$

Post-multiplying both sides by $\mathbf{S}_{jk}^{-1}\mathbf{S}_{ji}$ and rearranging gives,

$$\mathbf{H}_i \mathbf{S}_Z \mathbf{H}_i^* = \mathbf{S}_{ik} \mathbf{S}_{jk}^{-1} \mathbf{S}_{ji} \tag{9}$$

According to Eq. (4) and Eq. (9), the noise PSD $\mathbf{S}_{ei}$ of sensor i can be obtained by

$$\mathbf{S}_{ei} = \mathbf{S}_{ii} - \mathbf{S}_{ik} \mathbf{S}_{jk}^{-1} \mathbf{S}_{ji} \tag{10}$$

Despite the non-trivial nature of the problem, this formula is remarkably simple and intuitive. It has a similar form to Eq. (1) and can be viewed as a multi-dimensional extension of the three-channel method. Eq. (10) indicates that the instrument noise can be explicitly determined solely from auto-PSD and cross-PSDs of the output signals measured by the three collocated sensors. Sensor orientation has no effect on the accuracy of the noise PSD estimates, thus improving significantly the robustness of calibration procedure.

Theoretically, the noise PSD calculated by Eq. (10) is a real diagonal matrix and can be shown to remain the same when j and k are swapped. However, this is not true in implementation when the theoretical PSDs are substituted by their sample counterparts, for the same reason in the three-channel method. It can be shown that the following is a legitimate estimation formula that always returns real values in the diagonal entries:

$$\mathbf{S}_{ei} = \mathbf{S}_{ii} - (\mathbf{S}_{ik} \mathbf{S}_{jk}^{-1} \mathbf{S}_{ji} + \mathbf{S}_{ij} \mathbf{S}_{kj}^{-1} \mathbf{S}_{ki}) / 2 \tag{11}$$

4 Experimental verification and application

Revisiting the experimental setup in section 2, here we will use the same data sets to verify the proposed method. Figure 3 shows the results of the N-channel of sensor E corresponding to each case in section 2. The noise PSDs calculated by the three-channel method are also plotted in the figure for comparison. As there is no 'exact' value of noise PSD for comparison, the extracted noise in case 1 is used as the benchmark to judge the accuracy of the instrument noise obtained in other cases. Figure 3(a) shows that the noise estimated by both methods almost coincide, except that slightly elevated noise estimated by the three-channel method can be observed in the band between 20Hz and 30Hz. This may be due to the small alignment error between the sensors in case 1. The error of noise estimates induced by small misalignment is not obvious in this example. Neither is this typical nor should it be taken for granted because generally the SNR is expected to be much higher in the field situation compared to that in the laboratory. Figure 3(b)-(d) show that the proposed method can still give reliable noise estimates when the three sensors are not oriented along the same direction. For all the cases, the noise PSDs extracted by the proposed method are consistent in view that the benchmark almost overlaps with the extracted noise. It is therefore verified that the proposed method can calibrate instrument noise robustly regardless of sensor orientation.

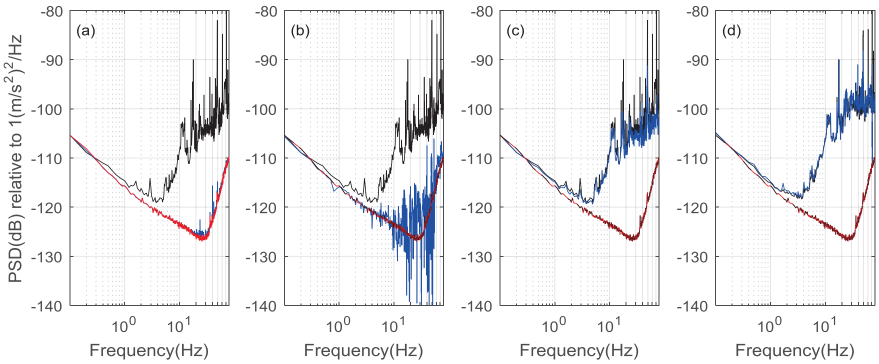

Figure 3. The data PSD (black line) and the noise PSD of the N-direction of sensor E estimated by the three-channel method (blue line) and the proposed method (red line) using experimental data; dash black line in (b)-(d): benchmark for noise estimates.

5 Conclusions

Addressing the important problem of instrument noise calibration, this paper has proposed a new calibration procedure that can be applied regardless of the orientation of sensors, hence significantly improving robustness in applicability compared to existing methods that rely critically on precisely aligned sensors. The proposed method has been verified and its applicability was demonstrated by a series of specially designed experiments.

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