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STEPS TOWARDS QUANTIFYING FRAGILITY IN LARGE-SCALE URBAN TRANSIT SYSTEMS

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Cities depend on public transit systems. However, the complexity and numerous uncertainties faced by these systems can make it seemingly intractable to cope with unforeseen disruptions. This work presents the analysis of the inherent variability and prediction uncertainty of passenger in- and outflows at stations in an urban rail system. We study data gathered from New York City's subway system over seven years and develop probabilistic models of the expected passenger flows. We determine the operating envelope over the course of a week, find critical flow levels during specific times of day, and establish fragility curves of the system using a Bayesian approach to estimate the probability of exceeding a critical flow level over the duration of a disruption. In addition, we present a prediction model based on Gaussian Process regression to determine future expected counts of passengers. This approach can be useful for planners and operators in improving the system based on expected platform capacities, assessing the risks associated with everyday operations, reliably forecasting whether the station in- and outflows are expected to remain within the operating envelope, and guiding the deployment of mitigation measures.

Keywords: urban rail networks, fragility, Gaussian Processes, uncertainty

1 Introduction

Public transit systems are indispensable to the functioning of cities. To keep pace with the development of urban spaces, their system architectures as well as operations need to be persistently adapted and expanded. However, given the rising complexity of these systems and despite many improvements, disruptions will be inevitable and potentially more catastrophic in the face of ever-growing urban population. In fact, uncertainties such as the variability of passenger demand, reliability of technical equipment, or limited knowledge about the effects of operational control actions render the prediction of the effects of disruptions a challenging problem. Therefore, planning future transit systems as well as embedding resilient management of disruption and recovery cycles inescapably demands assessing and quantifying the associated uncertainties.

Planning and controlling a system under uncertainty requires that necessary methods and models capture the system conditions and performance probabilistically. Most studies that assess transport systems and their susceptibility to disruptions within a stochastic perspective use frameworks associated with vulnerability analysis (Mattson 2015). Recent examples of such works are Rodríguez-Núñez 2014, Sun 2016, Sun 2016(2), Xing 2017, or LaniM'cleod 2017. Other works focus on optimizing system operations as well as understanding and augmenting the disruption response cycle to increase resilience (Cadarso 2013, Cacchiani 2014, Jin 2014, Cats 2015, van der Hurk 2015, Yin 2016, Adjetey-Bahun 2016, Altazin 2017, Gao 2017). However, inherent system uncertainties (e.g., demand fluctuations, operator control latency) and prediction inaccuracies (i.e., model prediction uncertainties) are often neglected. Particularly for system planners, discerning between inherent variabilities and prediction uncertainties is essential to assess the expected performance of a system and incorporate flexible design strategies. Only few have developed probabilistic approaches for transit systems that take into account demand variabilities (Soltani-Sobh 2016, Yin 2016, Nogal 2017) and even fewer have considered modelling errors validated against real-world data. For instance, Silva et al. (2015) demonstrate a statistical model that captures the effects of disruptions on passenger flows in the London underground. However, the model relies on detailed smart-card data and passenger surveys.

Here, we exemplify the proposed stochastic framework with reference to the daily and weekly passenger fluctuations of New York City's subway system. The results demonstrate our initial work to express both system variability and prediction uncertainty. This paradigm is the basis of our future work to develop simulation models that can propagate uncertainties into quantifiable probabilistic results and inform flexible engineering design strategies to create resilient public transport systems.

2 Approach

The condition of the transit system is assessed in two parts. First, we analyze the historical observations of passenger counts at subway stations. Next, we aim to predict future passenger flow-counts in a probabilistic way. All analyses are implemented in Python 3.5, unless explicitly specified.

2.1 *Quantifying system fragility*

This part aims to measure the empirical distribution of passenger entry and exit counts, illustrate the trace of passenger counts during a daily commute cycle, and determine the fragility of passenger inflow levels. To this aim, records are collected into empirical histograms and cumulative distributions for every recording interval, given by day and time. The resulting operating envelope stretches from the minimum to the maximum of the empirical counts and measures the frequency of passenger count observations during a particular recording interval.

Next, we determine kernel density estimates of the collected records for every recording interval and find the probability of exceeding a given entry count (i.e., station inflow). The resulting curves are used to assess critical threshold values within the operating envelope, which are important in determining the required passenger flow capacity at stations and guide the design of train frequency, available train capacity, and station platform capacity.

We define the fragility as the probability of exceeding a critical inflow threshold at a station for the entire duration of an event (e.g., a disruption). Fragility curves have been developed for seismic risk analysis of structural components and systems, and measure the probability of exceeding a certain level of damage given the intensity of seismic activity (Shinozuka 2000). This concept is adapted to the transit system by assuming that a disruption event occurs over a specified duration. Consequently, fragility measures the conditional probability of exceeding a critical passenger inflow n_c over the entire course of a disruption given its duration. The conditional probability is given by

$$P(X = x|Y = y) = \frac{F_Y(y|X = x)P(X = x)}{F_Y(y)} \tag{1}$$

where X is a Boolean random variable, denoted as either x^- if the passenger inflow n is smaller than the critical inflow $(n < n_c)$ or as x^+ if it is larger $(n \geq n_c)$. The random variable Y is continuous and corresponds to the duration over which the critical inflow is exceeded. The cumulative distribution function $F_Y(y|X = x)$ is determined from the kernel density estimate $f_Y(y|X = x)$, i.e., the probability density of the duration y, given that the critical inflow has either been exceeded or not. The probability $P(X = x)$ is determined from the empirical frequency of observing counts below or above the critical inflow. Since X is discrete (i.e., either x^- or x^+), $F_Y(y)$ is determined according to $\mathrm{F_Y(y) = F_Y(y|X = x^-)P(X = x^-) + F_Y(y|X = x^+)P(X = x^+)}$.

2.2 *Forecasting passenger counts*

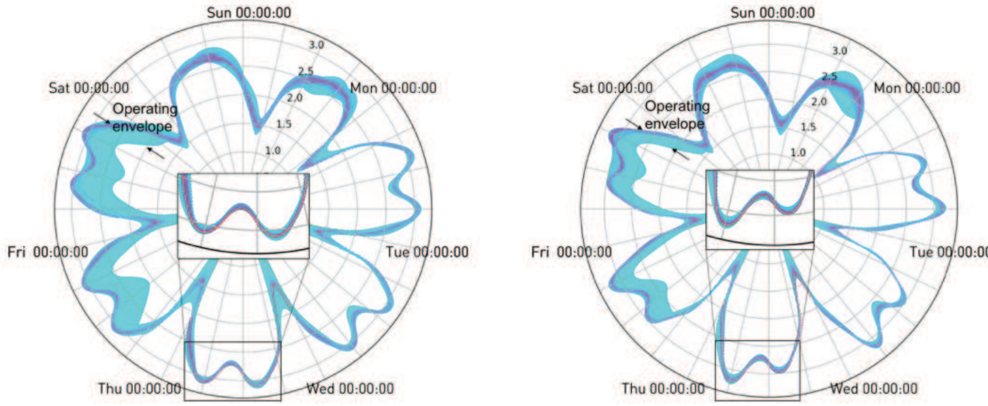
Forecasting intends to provide a stochastic estimate of the expected number of passengers entering and exiting at a station, given recent observations. The prediction needs to propagate and quantifiably express the inherent operational variability and prediction uncertainty about the expected passenger counts. To this end, we implement a Gaussian Process regression model.

Gaussian processes have been studied and used to various ends, and an exhaustive review would be beyond the scope of this work. However, a comprehensive introduction about the theory and some applications of Gaussian processes can be found in Rasmussen 2006. The strength of the Gaussian process model is that it discerns between the variability of observations and prediction uncertainty. In addition, the model can be tailored to include domain knowledge such as the seasonality of observations or the correspondence between observations at different spatial distances from each other.

Assumptions on the underlying likelihood functions (i.e., the distributions from which the measured data are sampled from) need to be included. Here, we assume that the likelihood function is a negative binomial distribution, because the passenger flows are count data and the historical measurements are over-dispersed (i.e., the variance of the observations is larger than the mean). We implement the Gaussian Process (GP) regression model in the GP for Machine Learning (GPML) toolbox in Matlab®. Due to the assumed negative binomial likelihood, an exact inference method cannot be used. We therefore choose Laplace approximation and an exponential link function to determine the latent variable mean and standard deviation as well as the dispersion parameter of the underlying likelihood (i.e., the expected value and variance of the intensity as well as the dispersion parameter). The regression model assumes a summed kernel covariance matrix, consisting of radial basis function (squared exponential), Matern, periodic, and rational quadratic kernels to account for different length scales and periodicities.

3 Case Study - New York City's subway

The developed approach is implemented on the records of the number of passengers who enter and exit at NYC's subway station "34th Street – Penn Station" (henceforth denoted as Penn Station) of the Independent Subway System (IND) Eighth Avenue Line, served by train lines A, C, and E. The data was gathered from the Metropolitan Transport Authority's (MTA) online open-access turnstile count data repository (MTA 2017). The collected and processed data set spans over approximately seven years from April 2010 to May 2017. The raw data includes the cumulative counts per every turnstile of each station in approximately four-hour intervals. In addition, the



(a) (b)

Figure 1. Number of passengers (a) entering and (b) exiting at 34th Street – Penn Station. The radius represents the counts per ten-minute interval – the scale is logarithmic. The color scale represents the relative frequency of the observed counts. The red lines in the insets represent the trace of the counts on Wednesday, May 7, 2015.

data are conservatively up-sampled to ten-minute interval counts. The resulting entry and exit counts are assumed to be the empirical records of passenger counts over the time and date range spanned by the original data set, yet given in ten-minute instead of four-hour intervals.

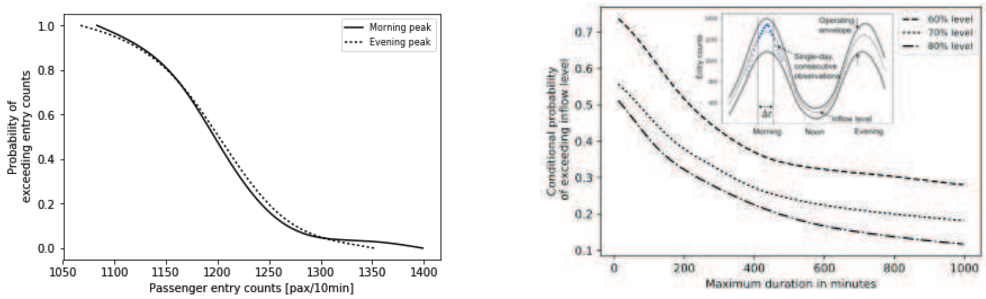
4 Results and Discussion

Fig. 1 shows the number of passengers entering (Fig. 1(a)) and exiting (Fig. 1(b)) per ten-minute interval at Penn Station over the course of a week during the months of April and May in the years 2010 to 2017. The plots depict how weekday counts follow the daily commute cycle of morning peak, midday off-peak, and evening peak times, whereas weekends only show midday-to-afternoon increases in passenger counts. Moreover, Thursdays and Fridays observe considerably larger variability as compared to the remaining week. The central insets show the entries and exits for a typical Wednesday, depicting how the observed counts are distributed within the boundaries of the operating envelope. The trace of a single day is shown as a red line.

4.1 *Assessing passenger demand and required flow capacity*

One key aim is to find useful statistical estimates of the passenger flow (i.e., entry and exit counts) at stations. To start, we measure the number of entries at Penn Station. As an example, we take records collected during every Wednesday in the months of April and May and determine the operating envelope for every ten-minute interval during the course of the day.

Fig. 2(a) plots the probability of exceeding a given entry count (i.e., inflow level) during the morning peak and evening peak times at Penn Station. Capturing the expected inflow levels is essential in designing the flow capacity and assessing known capacity limits. For instance, the probability of observing more than 1300 passengers per 10-minute interval during morning and evening peak hours is in the order of 5%, according to Fig. 2(a). While it may be desirable to adjust train schedules to accommodate these many passengers and limit excess crowding at the station, available rolling stock may be limited. Consequently, if the train schedule only allows for 1200 passengers serviced per every 10 minutes, the probability that more passengers can be expected and will be crowding at the station is about 55%.



(a) (b)

Figure 2. The curves in (a) plot the probability of exceeding a given level of passenger entry counts (inflow) during morning peak and evening peak intervals. The fragility curves in (b) plot the conditional probability of persistently exceeding a certain inflow level within the operating envelope over a given duration. The inset shows the operating envelope and generic inflow level. The dots represent single day, consecutive observations, with the filled dots signifying that the observations exceeded the inflow level. The corresponding duration Δt is recorded to infer the curves in (b) using Eq. (1). The plots are based on the passenger inflow counts recorded for a typical Wednesday during April and May between 2010 and 2017 at 34th Street – Penn Station.

4.2 *Assessing fragility*

Another key aim is to estimate the fragility of the operating conditions. Fig. 2(b) plots the fragility curves for three different inflow levels within the mid-week (Wednesday) operating envelope at Penn Station, based on Eq. (1). The inflow level is measured within the operating envelope as the margin between the minimum and maximum observed counts (i.e., the inflow level of z [%] corresponds to a value of $n_{min} + z(n_{max} - n_{min})$, where n_{min} and n_{max} correspond to the minimum and maximum observed entry counts, respectively).

The fragility curves measure the conditional probability of exceeding the 60%, 70%, and 80% inflow levels over the entire duration of an event – e.g., the duration of a disruption – given its expected maximum time span. For instance, the probability of persistently exceeding the 80% inflow level during a disruption that lasts up to 200 minutes is roughly 0.35. Such a disruption could for instance be a planned service disruption or station closure. Similarly, given that the expected duration of a sudden line closure lasts up to 20 minutes, the probability that the passenger count persistently exceeds the 70% threshold between the smallest and largest observed counts during the 20-minute interval is 0.55. In such cases, the fragility curves indicate the potential severity and corresponding probability of the number of passengers who would need to be diverted and cannot commence their journey at this station.

4.3 *Predicting passenger counts*

We assume that the distributions of the number of passengers entering and exiting at a station is described by a negative binomial likelihood. In order to reliably forecast future station in- and outflows, we first determine the two model parameters – the intensity and the dispersion parameter – of the underlying likelihood. Given the records generated from up- sampling the four-hour interval passenger counts, we determine the maximum likelihood estimate of the intensity and dispersion parameter of the negative binomial distribution defined at each ten- minute recording interval within a week. Next, in order to assess whether the proposed prediction is sufficiently accurate in estimating and predicting the statistics of the underlying likelihood, we generate samples over three consecutive weeks from the distribution determined in the previous step. The resulting samples are considered the actual observations (blue dots in Fig. 3).

The prediction based on the GP model for a one-week ahead time horizon is shown on the right-hand side of Fig. 3. The solid black line indicates the predicted mean intensity, whereas the shaded bands illustrate the prediction uncertainty. The lower and upper bounds of the shaded region are defined according to the variance of the mean intensity (i.e., one-standard deviation interval). The red curve, corresponding to the true intensity of the negative binomial distribution, indicates that

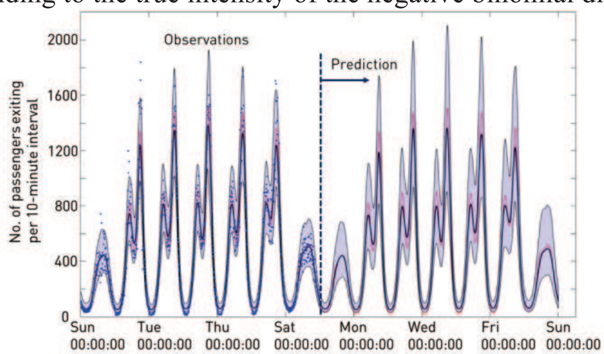


Figure 3. One-week ahead prediction of the mean intensity of exit counts at 34th Street – Penn Station. The shaded region represents the one-standard deviation interval of the prediction. The prediction is based on a GP regression model and observations gathered over three weeks (only the last week's observations are shown).

the prediction performs well in approximating the true intensity. Given the forecast of the variable demand together with the quantified prediction uncertainty, operators can for instance deploy flexible strategies and response steps that best trade-off additional cost and performance improvement under uncertainty. Future work will consider different models to comparatively assess and quantify the prediction accuracy.

5 Conclusion

This work links the fragility of the operations of an urban rail transit system and the inherent uncertainty of the number of passengers entering and exiting at a station during times of potential service disruptions. We define the operating envelope as the range between the minimum and maximum observed counts during a given time interval. We determine the empirical frequencies of observing count targets within the operating envelope and find kernel density estimates that help us to deduce the probabilities of exceeding a given station inflow or outflow level of passengers. In this way, we represent the operating envelope over the course of a week, determine critical inflow levels during specific times of day, and assess the fragility of the system by assuming an estimated duration of a disruption. Such information, which takes into account the variability of passenger in- and outflow, can be useful for planners and operators in improving the system or assessing the risks associated with everyday operations. In addition, we present a prediction approach based on GP regression models to determine future expected counts of passengers. The results presented will be extended to consider the full transit network. By quantifying both the inherent variabilities and prediction uncertainties, the aim is to devise flexible engineering design strategies that can optimally accommodate these uncertainties.

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