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PROBABILISTIC SEISMIC CAPACITY ANALYSIS OF REINFORCED CONCRETE FRAME STRUCTURES USING AN EFFICIENT RANDOM PUSHOVER APPROACH

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Probabilistic seismic capacity analysis (PSCA) is an important ingredient of seismic fragility and risk assessment. The key issue of PSCA is to determine the probabilistic model and its parameters of structural capacity considering the influences of the uncertain parameters. Motivated by this problem, this paper presents an efficient random pushover approach (RPA) which combines the conventional (deterministic) pushover analysis with an advanced point estimate method (APEM). The RPA is used for determining the probabilistic moments of structural capacity by adopting a specified ensemble of criteria, according to which the thresholds corresponding to the limit states of interest are identified from a pushover curve. Based on the first two moments of structural capacity, the probabilistic seismic capacity model (PSCM) is generated with the lognormal assumption for the model distribution. Finally, the presented method is applied to four typical low-to-mid-rise reinforced concrete frame structures designed according to Chinese codes. The PSCMs corresponding to the pre-determined limit states of slight damage, moderate damage, extensive damage and complete damage are derived.

Keywords: Probabilistic seismic capacity analysis; random pushover approach; advanced point estimate method; reinforced concrete structures; earthquake engineering

1 Introduction

A rational definition of limit state capacities plays an important role in seismic fragility and risk assessment of structures. In general, structural limit states (also referred to as "performance levels") are specified in seismic design codes and guidelines. However, the code-conforming thresholds are commonly limited to define the general capacity for specific structures. To overcome this shortcoming, the pushover curves, also called capacity curves, are naturally used to define the capacities for individual structures by incorporating the specific characteristics, e.g., geometry sizes, material strengths, capacities of elements, and construction level (Erberik and Elnashai 2004, Kwon and Elnashai 2006). In fact, the characteristics related to structural capacity are mostly random rather than deterministic, therefore probabilistic seismic capacity models (PSCMs) are necessary. In the SAC/FEMA methodology (Cornell *et al.*, 2002), structural capacity is customarily modeled as a lognormal variable, whose parameters, due to absence of data, are often assumed based on engineering experiences and expert judgments (Wen *et al*. 2004, Ellingwood *et al*. 2007). Rigorously speaking, such an empirical model is only feasible to define the probabilistic characteristics of the capacity for the generic building types rather than for the specific individual buildings.

Concerning the above considerations, it is beneficial to combine the conventional (deterministic) pushover approach together with some specified uncertainty analysis methods in order to perform probabilistic seismic capacity analyses (PSCA) for individual structures. Recently some probabilistic procedures to conduct pushover analysis have been studied. For example, Tomas and Trezos (2006) examined the influences of structural uncertainties on its static responses through Monte Carlo simulation (MCS). Barbato et al (2010) presented a probabilistic pushover procedure with the first order second moment method (FOSM). However, MCS is less attractive for practical engineers due to its large computational efforts. While FOSM predicts structural mean responses at the mean values of random variables, which may lead to inaccurate results in the case when the nonlinearity and uncertainty of structures cause a shift in the prediction of mean responses.

To avoid the above limitations, this study presents a random pushover approach (RPA) using an advanced point estimate method (APEM) (Yu and Lu, 2015), which can save favorable computational efforts while obtain the reasonable accurate statistics for structural static responses. As a beneficial complement, a sensitivity index is also proposed in the APEM. Consequently, the presented RPA can not only estimate the statistical moments for structural static responses, but also identify the sensitivities of structural uncertainty parameters. If a specified set of criteria to define the limit states of interest on a pushover curve is associated, then the RPA can be further used to perform PSCA for a given structure.

2 Probabilistic Seismic Capacity Model

The capacity of a given structure, C, is influenced by many parameters, most of which are random in nature. Therefore, C should be expressed by a random function, which is denoted by

$$C = g(\boldsymbol{X}) = g(X_1, X_2, ..., X_n) \tag{1}$$

where $\boldsymbol{X}=[X_1, X_2, \ldots, X_n]^{\mathrm{T}}$ is the collection of random variables of the structure, e.g., gravity loads, damping ratio, material strengths, geometric parameters, model uncertainty, and local capacities of structural elements.

To reflect the probabilistic characteristics of C, a probabilistic model (i.e., PSCM), $F_C(d)$, is required, which represents the conditional failure probability of the structure conditioned on the specified seismic demand parameter $D=d$

$$F_C(d) = P[C - D \leq 0 \mid D = d] \tag{2}$$

According to the SAC/FEMA methodology, a lognormal distribution is conveniently assumed for C by

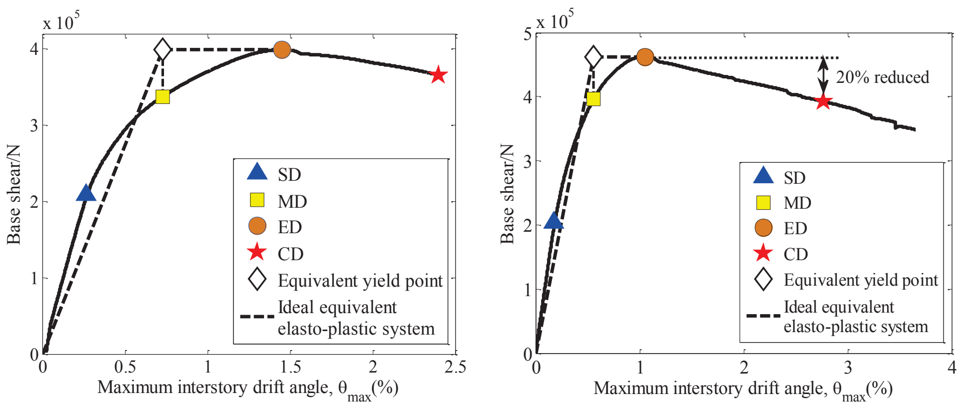
$$F_C(d) = \Phi\left(\frac{\ln(d / m_C)}{\beta_C}\right) \tag{3}$$

To establish a PSCM, the key task is to determine the first two moments of C, i.e., m_C and β_C. In this study, an efficient random pushover approach is presented to determine the capacity moments.

3 Probabilistic Seismic Capacity Analysis Using the Efficient Random Pushover Approach

For the conventional deterministic pushover analysis, it is performed for the deterministic structural model without considering the random properties. For a random pushover, the pushover analyses are performed for a set of random structural models including random structural properties. In this study, an APEM is combined with the pushover analysis. Due to space limitation, the mathematical details of APEM are not given herein. The step-by-step procedures of the presented random pushover approach are given in follows:

1. Select N_{var} structural random parameters and determine their probabilistic distributions.
2. Generate $N_{sam}=m\times N_{var}+1$ samples for the random parameters according to APEM.
3. Determine N_{sam} structural models by setting the uncertain parameters equal to the samples generated in step 2), while other parameters are taken as the medians.
4. Perform pushover analysis for each structural model and generate N_{sam} pushover curves
5. Identify the structural capacities for different limit states from each pushover curves
6. Determine the capacity parameter i.e., m_C and β_C, from the random structural capacities according to the APEM.
7. Examine the sensitivities for the considered uncertain parameters according to the APEM.



a) CD corresponds to the ultimate point of a building pushover curve

b) CD corresponds to the 20% reduced post-peak point on a building pushover curve.

Figure 1. Limit state identification on a building pushover curve

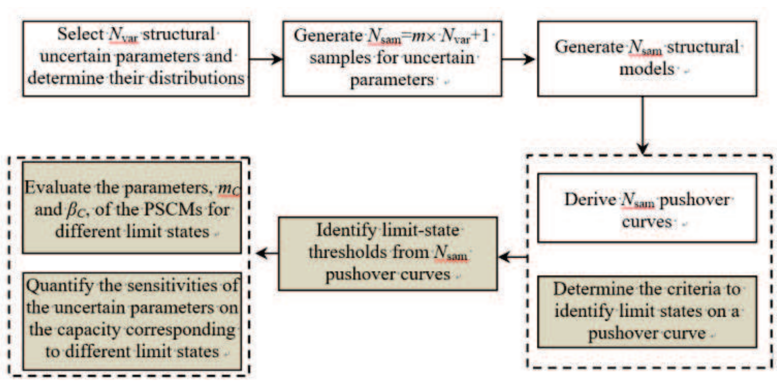


Figure 2. The main steps of PSCA by the RPA

In this study, the considered limit states are divided into slight damage (SD), moderate damage (MD), extensive damage (ED) and complete damage (CD). The first limit state, SD, is determined by the local damage behavior corresponding to the first longitude steel bar yielding in structural columns. The local limit state is then mapped onto the structural global pushover curve to obtain its threshold value. The other three limit states, MD, ED, and CD are totally defined directly from the structural global pushover curve. In particular, the MD state is identified as the equivalent yield point by the idealized equivalent elasto-plastic system with energy absorption equivalent to that of the original system, the ED state is assumed for the damage level of attaining the peak shear resistance, and the CD state corresponds to the 20% reduced post-peak capacity or the ultimate capacity. The considered limit states are shown in Fig. 1, in which the maximum inter-story drift angle, θ_{max}, is used to define the global deformation of the structure. Fig. 2 illustrates the main steps of PSCA by the RPA.

4 Case study: RC frame structures

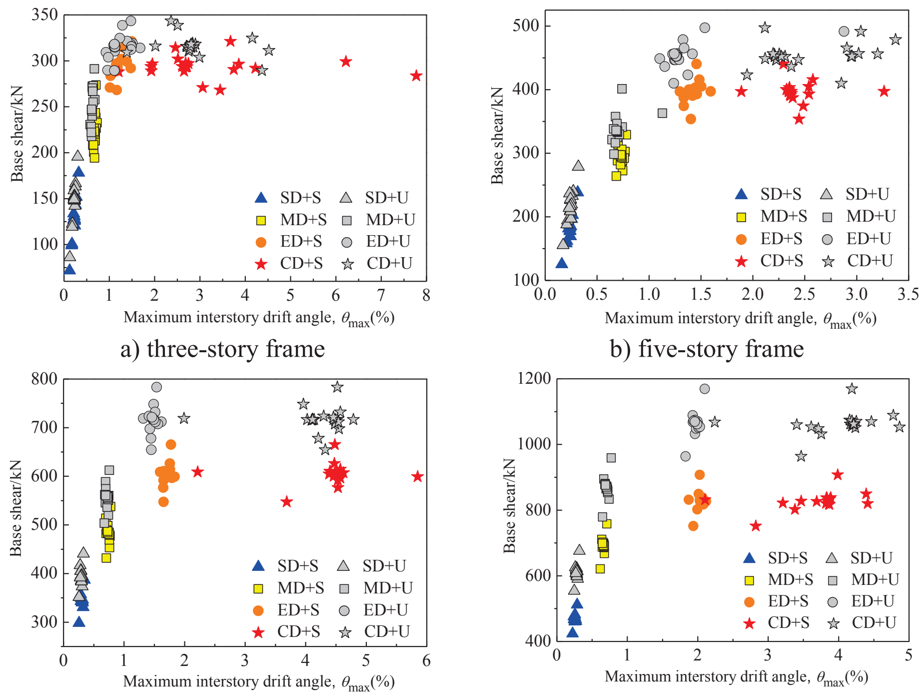
According to the current Chinese codes (GB 50010-2010), four RC frame buildings, with three, five, eight, and ten stories, are designed to represent the low-to-medium-rise frames typical of construction in China. The design and modelling details of the four case RC frames can be found in (Yu *et al*. 2016, Yu *et al*. 2017). A total of fourteen material constitute parameters are used to define the structural materials, i.e., four parameters for the unconfined concrete ($f_{cp,cover}$ = peak strength, $\varepsilon_{cp,cover}$ = the strain at peak strength, $f_{cu,cover}$= the residual strength, and $\varepsilon_{cu,cover}$=strain at which the residual strength is reached), four parameters for the confined concrete ($f_{cp,core}$, $\varepsilon_{cp,core}$, $f_{cu,core}$, and $\varepsilon_{cu,core}$), and six parameters for the reinforcing steel (E_s=initial stiffness, f_y=yield strength, α=post yield to initial stiffness ratio, and CR_1, CR_2, R_0=the parameters controlling the transition from elastic to plastic branches). Two concrete parameters and four steel parameters are treated as deterministic ($\varepsilon_{cp,cover}$=0.002, $f_{cu,cover}$=0, CR_1=0.925, CR_2=0.15, R_0=20, and α=0), while the other eight parameters are taken as random variables. Table 1 provides the distribution types, means and COV values of the random variables selected in this study. Only the correlations between some pairs of concrete parameters are considered: ρ=0.8 for (i) $f_{cp,cover}$ and $f_{cp,core}$; (ii) $f_{cp,core}$ and $f_{cu,core}$; (iii) $\varepsilon_{cu,cover}$ and $\varepsilon_{cu,core}$; (iv) $\varepsilon_{cp,core}$ and $\varepsilon_{cu,core}$; ρ=0.64 for (i) $f_{cp,cover}$ and $f_{cu,core}$; (ii) $\varepsilon_{cu,cover}$ and $\varepsilon_{cp,core}$; ρ=0 for all other pairs of parameters due to lack of knowledge. The random variables of concrete are assumed to be spatially fully correlated.

Table 1. The Distributions and the corresponding parameters of the random structural variables

Uncertainty sources	Random variables	Means	COVs	Distributions
Dead load	ρ_{Dead}	26.50kN/m^3	0.07	Normal
Live load	q_{Live}	0.98kN/m	0.41	Gamma
Grade C30 concrete	$f_{cp,cover}$	26.10MPa	0.14	Lognormal
	$\varepsilon_{cp,cover}$	0.004	0.20	
	$f_{cp,core}$	33.60MPa	0.21	
	$\varepsilon_{cp,core}$	0.0022	0.17	
	$f_{cu,core}$	22.20MPa	0.21	
	$\varepsilon_{cu,core}$	0.0113	0.52	
Grade C35 concrete	$f_{cp,cover}$	29.10MPa	0.13	Lognormal
	$\varepsilon_{cp,cover}$	0.004	0.20	
	$f_{cp,core}$	37.95MPa	0.20	
	$\varepsilon_{cp,core}$	0.0021	0.16	
	$f_{cu,core}$	28.70MPa	0.20	
	$\varepsilon_{cu,core}$	0.0110	0.52	
Grade HRB335 steel rebars	f_y	378MPa	0.07	Lognormal
	E_s	200000MPa	0.02	

5 *Probabilistic Seismic Capacity Models*

The random pushover curves are firstly generated by performing pushover analyses for the structural models, and then the limit-state thresholds are identified from these curves. Fig. 3 illustrates the captured limit-state thresholds, where "S" and "U" represents the lateral load patterns in terms of "SRSS" and "Uniform", respectively. As observed, the thresholds corresponding to the limit states of ED and CD are scattered in a wider scope than that relevant to the left two limit states. This observation is attributed to the more highly nonlinear behavior of the structure at the limit states of ED and CD. On the basis of the obtained thresholds, the parameters, m_C and β_C, required by the PSCM are further calculated in Table 2.



a) three-story frame b) five-story frame

c) eight-story frame d) ten-story frame

Figure 3. Limit-state thresholds identified from the random pushover curves

Table 2. PSCM parameters, m_C and β_C, derived by applying different lateral load patterns

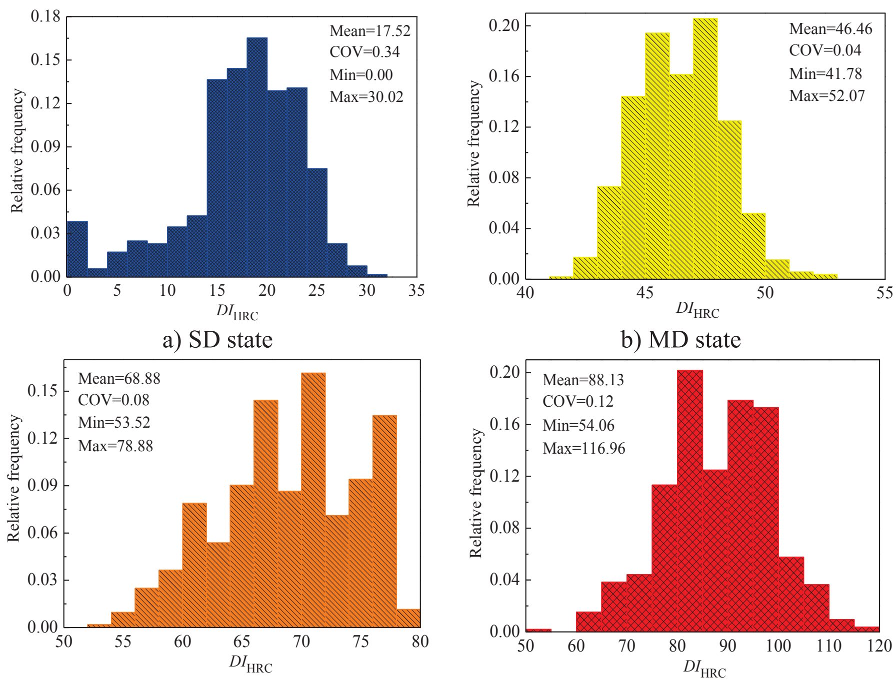
Case-study frames	Lateral load patterns	PSCM parameters	Limit states: θ_{max}(%)			
			SD	MD	ED	CD
Three-story	Uniform	m_C	0.21	0.64	1.52	6.46
		β_C	0.28	0.07	0.27	0.79
	SRSS	m_C	0.21	0.68	1.25	3.77
		β_C	0.31	0.05	0.18	0.56
Five-story	Uniform	m_C	0.24	0.74	1.38	3.21
		β_C	0.20	0.12	0.18	0.27
	SRSS	m_C	0.23	0.53	1.02	2.54
		β_C	0.21	0.06	0.08	0.17
Eight-story	Uniform	m_C	0.29	0.70	1.46	4.87
		β_C	0.09	0.06	0.07	0.31
	SRSS	m_C	0.30	0.75	1.70	4.07
		β_C	0.10	0.05	0.06	0.26
Ten-story	Uniform	m_C	0.28	0.70	1.95	3.45
		β_C	0.09	0.07	0.05	0.28
	SRSS	m_C	0.26	0.66	2.00	3.21
		β_C	0.09	0.05	0.05	0.27

6 *Damage Levels of the PSCMs*

According to the studies in (Olsson and Sandberg, 2002), the damage scale DI_{HRC} for the homogenized RC buildings (HRC), is used to assess the damage levels of the derived PSCMs for the case-study frames by employing the regression formula for general RC buildings,

$$DI_{\mathrm{HRC}} = 27.89 \ln\left[\theta_{\max,\%}\right] + 56.36 \tag{4}$$

Through Eq. (4), the identified limit-state thresholds for the case-study frames are re-calculated by DI_{HRC} (see Fig. 4). In Rossetto and Elnashai (2003), various damage scales have been correlated with each other by using DI_{HRC} as the unique measure. On the basis of such a result, the limit states defined in this study are further correlated with other damage scales in Table 3. Compared with the other damage levels, the limit states defined in this study have wide DI_{HRC} scopes referring the limit states of SD, ED and CD, while the MD state is defined by a narrow DI_{HRC} scope.



a) SD state b) MD state

c) ED state d) CD state

Figure 4. Histogram graphs of the limit-state thresholds re-defined by DIHRC

Table 3. Damage scale correlation table for ductile RC frame buildings

<table>
<tr><th>DI_{HRC}</th><th>HRC</th><th>HAZUS 1999</th><th>Vision 2000</th><th>FEMA 273</th><th>EMS 98</th><th>MSK</th><th>AIJ</th><th>ATC-13</th><th>This study</th></tr>
<tr><td>0</td><td>None</td><td colspan="8">No damage</td></tr>
<tr><td>10</td><td>Slight</td><td rowspan="4">Slight damage</td><td rowspan="2">Fully operational</td><td rowspan="3">Immediate occupancy</td><td rowspan="2">Grade1</td><td rowspan="2">D1</td><td>Light</td><td>Slight</td><td rowspan="4">SD</td></tr>
<tr><td>20</td><td rowspan="3">Light</td><td rowspan="3">Minor</td><td rowspan="3">Light</td></tr>
<tr><td>30</td><td rowspan="4">Operational</td><td rowspan="2">Grade2</td><td rowspan="2">D2</td></tr>
<tr><td>40</td><td rowspan="2">Damage control</td></tr>
<tr><td>50</td><td rowspan="3">Moderate</td><td rowspan="3">Moderate damage</td><td rowspan="3">Grade3</td><td rowspan="3">D3</td><td rowspan="4">Moderate</td><td rowspan="2">Moderate</td><td>MD</td></tr>
<tr><td>60</td><td rowspan="2">Life safety</td><td rowspan="3">ED</td></tr>
<tr><td>70</td><td rowspan="2">Life safety</td><td rowspan="2">Heavy</td></tr>
<tr><td>80</td><td rowspan="2">Extensive</td><td rowspan="3">Extensive damage</td><td>Limited safety</td><td rowspan="3">Grade4</td><td rowspan="3">D4</td></tr>
<tr><td>90</td><td>Near collapse</td><td rowspan="2">Collapse prevention</td><td>Major</td><td rowspan="2">Major</td><td rowspan="3">CD</td></tr>
<tr><td>100</td><td>Partial collapse</td><td>Collapse</td><td>Partial collapse</td></tr>
<tr><td>>100</td><td colspan="8">Collapse</td></tr>
</table>

5 Conclusions

This study presented a framework of probabilistic seismic capacity analysis using an efficient random pushover analysis approach. To demonstrate the application of the presented methodology, four typical Chinese low-to-mid-rise RC frame structures are selected as the case-study buildings. The presented methodology is proven to be effective for probabilistic seismic capacity analysis. Using the damage scale DI_{HRC} for the homogenized RC buildings, the obtained capacity thresholds for SD, MD, ED and CD are correlated to the other widely used damage scales.

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