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Classical system reliability analysis by response surface method

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The response surface method (RSM), which is characterized by its understandability and efficiency, has been widely applied to reliability with implicit performance function. Nevertheless, due to multiple failure modes, it is difficult to apply the RSM directly to system reliability analysis. Further, because of the existence of multiple design points, the design point based RSM is unfeasible for system reliability. On this basis, this paper apply a RSM for classical system reliability by combining the complete system failure process with the high order response surface method. Firstly, following the idea of complete system failure process, a single equivalent performance function for system reliability is derived. Hence the application of RSM can possibly be applied. Secondly, by introducing high order modification, the stochastic response surface method is used to fit the equivalent performance function. In this way, the introduction of design points iteration is avoided. Thirdly, Monte Carlo method is adopted to evaluate the failure probability of the response surface function. Finally, several examples are investigated to verify the accuracy and efficiency of the method.

Keywords: System reliability analysis, Complete system failure process, High order response surface method, limit state function

1 Introduction

System reliability evaluation of structures has long been a challenging problem in the field of structural engineering although great endeavors have been devoted to it in the past over 30 years. A number of probabilistic analysis tools have been proposed to quantify classical system reliability analysis: In system reliability analysis, Ang et al. (1975) first introduced the idea of fault tree analysis (FTA) into structural system reliability analysis and proposed the probabilistic network evaluation technique (PENT) algorithm for evaluating the comprehensive failure probability of structural system. Then Moses (1977) combined the incremental load approach and limit state analysis method for identifying and expressing the main collapse modes in structural system reliability analysis. Thoft-Christensen & Sorensen (1982) presented β-unzipping method for calculating the probability failure of series and parallel structural systems, which the lower-upper bound method is applied. However, the major difficulties encountered in all these system reliability methods are how to solve the combinatorial explosion problems in multiple failure modes and how to tackle the use of correlation information of different random events and two-dimensional joint probability density function (PDF). To avoid the restrictions of traditional system reliability methods, Chen and Li (2007) proposed the development process of nonlinearity, which is also defined as complete system failure process (Chen and Li (2007)). Based on this idea, the classical system reliability is effectively described by a single equivalent performance function. Though the generalized density evolution equation (GDEE) provides an effective numerical method for the single function, the process of algorithm is complex. On the other hand, the chosen of moment methods and the comparison of each moment method will also be complicated.

Response surface method (RSM) is a helpful technique in structural reliability analysis where limit state function is implicit and numerical methods are needed. In RSM, the limit state function is approximated by mathematical expression with undetermined coefficients. By fitting the response surface to a number of sample points on the limit state, the response surface function (RSF) is constructed and applied in reliability analysis, such as first-order reliability method (FORM) (Hasofer & Lind 1974) and second-order reliability method (SORM) (Breitung 1984). The selection of the form of RSF has great influence on the efficiency and accuracy of RSM, and the polynomial form of RSF is commonly used and studied (Faravelli 1989, Bucher & Bourgund 1990, Rajashekhar & Ellingwood 1993). However, the RSF based on the iteration of design points, has a problem of non-convergence. Gavin and Yau (2007) proposed a high order stochastic response surface method (HO-SRSM), which suggested the use of higher order polynomials, in order to solve strongly-nonlinear problems and avoid form iteration.

In this paper, high order response surface method with the development process of nonlinearity is adopted for classical system reliability analysis. It is organized as follows. In Section 2, an equivalent performance function for system reliability of structure is formulated based on the complete system failure process, and High order response surface method is presented based on the equivalent performance function. Then in Section 3, an example is investigated to verify the proposed method. At last some conclusion are drawn in Section 4.

2 High order response stochastic surface method for system reliability analysis

2.1 *Equivalent performance function based on the complete system failure process*

The classical system reliability focus on collapse of the perfectly elastoplastic or elastic-brittle structures. the structure failure can be considered if the overall or local structure reached their yield points. And if the load increased in proportion, the failure load is regarded as bearing capacity. The failure mode can be defined as

$$F \cdot \boldsymbol{r} \geq F_{\max} \cdot \boldsymbol{r} \Leftrightarrow F \geq F_{\max} \tag{1}$$

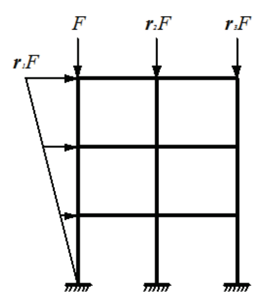

Fig.1 Figure of proportion load of frame

where $\boldsymbol{r}$ is distribution vector; F is the load coefficient; F_{max} is bearing capacity coefficient. And equation (1) can be also written as

$$P_f = \Pr\{F \geq F_{\max}\} \tag{2}$$

where P_f is failure probability of sample points; P_r represents probability.

If variable loads are random but not perfect correlation, the failure event of q^{th} sample can be defined as

$$F_q \cdot \boldsymbol{r}_q \geq F_{\max,q} \cdot \boldsymbol{r}_q \Big| \boldsymbol{r} = \boldsymbol{r}_q \tag{3}$$

in which $\boldsymbol{r}_q$ represent distribution vector of q^{th} sample, F_q and $F_{max,q}$ are the load coefficient and bearing capacity coefficient of q^{th} sample respectively. The failure probability of q^{th} sample is

$$P_{f,q} = \Pr\left\{F_q \cdot \boldsymbol{r}_q \geq F_{\max,q} \cdot \boldsymbol{r}_q \middle| \boldsymbol{r} = \boldsymbol{r}_q\right\} \cdot \Pr\left\{\boldsymbol{r} = \boldsymbol{r}_q\right\} = \Pr\left\{F_q \geq F_{\max,q} \bigcap \boldsymbol{r} = \boldsymbol{r}_q\right\} \tag{4}$$

It is easy to obtain the overall failure probability from above, namely

$$P_f = \sum_{q=1}^{N} P_{f,q} = \Pr\left\{\bigcup_{q=1}^{N} F_q \geq F_{\max,q} \bigcap \boldsymbol{r} = \boldsymbol{r}_q\right\} \tag{5}$$

Formula (5) can be rewritten as

$$P_f = \sum_{q=1}^{N} P_{f,q} = \Pr\left\{F \geq F_{\max} \bigcap \left[\bigcup_{q=1}^{N} \boldsymbol{r} = \boldsymbol{r}_q\right]\right\} = \Pr\left\{F \geq F_{\max}\right\} \tag{6}$$

Apparently, the corresponding function is

$$Z = F_{\max} - F \tag{7}$$

Hence, the multiple limit state functions in classical system reliability are translated into one equivalent limit state function, which can be approximated by HORSM efficiently.

2.2 *High order stochastic response surface method*

According to the high order stochastic response surface method (Gavin and Yau (2007)), the equivalent limit state function Z can be approximated by

$$Z \approx \bar{Z} = a + \sum_{i=1}^{n}\sum_{j=1}^{k_i} b_{ij} X_i^j + \sum_{q=1}^{m} c_q \prod_{i=1}^{n} X_i^{p_{iq}} \tag{8}$$

where the coefficients b_{ij} correspond to terms involving only one random variable, and the coefficients c_q correspond to mixed terms, involving the product of two or more random variables. The k_i, m, p_{iq} are the polynomial order, the total number of mixed terms and the order of a random variable in a mixed term respectively.

2.2.1 Polynomial Orders k_i

The polynomial orders, k_i, are determined by statistically and the mixed terms are neglected and numerically testing the significance of polynomial coefficients in the first stage of the method. Chebyshev polynomials of degree M in λ are adopted for order decision as follows:

$$T_M(\lambda) = \cos(M \arccos \lambda) \tag{9}$$

Where $\min(T_M(\lambda))=-1$, and $\max(T_M(\lambda))=1$, for all λ such that $-1 \leqslant \lambda \leqslant 1$. The polynomial $T_M(\lambda)$ has M roots in the interval [-1,1] at

$$\lambda = \cos\left(\frac{\pi(m-\frac{1}{2})}{M}\right) \quad \text{where } m=1,\ldots,M \tag{10}$$

The discrete orthogonality relation for Chebyshev polynomials is given by:

$$\sum_{m=1}^{M} T_i(\lambda_m) T_j(\lambda_m) = \begin{cases} 0 : i \neq j \\ M/2 : i = j \neq 0 \\ M : i = j = 0 \end{cases} \tag{11}$$

where λ_m (m = 1,…,M) are the M roots of $T_M(\lambda)$ given by equation (10).The orders of the variables k_i in equation (8) are estimated one-by-one along dimension X_i using one-dimensional Chebyshev polynomials,

$$\hat{g}_i(X_i) = d_0T_0(\lambda_i) + d_1T_1(\lambda_i) + d_2T_2(\lambda_i) + ... + d_{k_i}T_{k_i}(\lambda_i), \tag{12}$$

where λ_i is the interpolated values of X_i from interval $[\mu_i - h_{ord}\,\sigma_i, \mu_i + h_{ord}\,\sigma_i]$ to [-1,1], i.e.

$$X_i = \mu_i + h_{ord}\,\lambda\sigma_i \tag{13}$$

Where h_{ord} is the domain of the sampling points used to determine the polynomial degree of the approximation. The Chebyshev polynomial coefficients, d_j, are determined by the least squares method.

$$\mathrm{d} = [T_m^T T_m]^{-1} T_m^T g_i(x_i) \tag{14}$$

where the $T_{jk} = T_j(\lambda_k)$, λ_k is the *k-th* root of $T_K(\lambda)$, and $g_i(x_i)$ is a vector of the values of true limit state function evaluated with discrete values of random variable X_i set to

$$x_{ik} = \mu_i + h_{ord}\,\lambda_k\,\sigma_i \quad \text{where } k=1,\ldots,K, \tag{15}$$

and with all other elements of X set to their mean values. The coefficient covariance matrix,

$$V_{\mathrm{d}} = [T_m^T T_m]^{-1} \sum_{k=1}^{K} (\hat{g}_i(x_{ik}) - g_i(x_{ik}))^2 / (K - k_i) \tag{16}$$

is diagonal, due to the discrete orthogonality relationship of the Chebyshev polynomials, given in equation (11). In fact, every diagonal term of V_d, except the first, is

$$\sigma_{\mathrm{d}}^{\,2} = \frac{K}{2} \sum_{k=1}^{K} (\hat{g}_i(x_{ik}) - g_i(x_{ik}))^2 / (K - k_i) \tag{17}$$

The test of statistical significance of an individual term d_j in equation (12) involves the test of the null hypothesis, H0: the true coefficient of the term is 0. The test is performed by calculating values of the t-statistics [6],

$$\mathrm{t}_j = \frac{d_j}{\sigma_d} \tag{18}$$

Using a two-sided test and 90% confidence intervals, if the absolute value of t_j is smaller than the value of $t_{0.05} = 3.499$, the null hypothesis cannot be rejected and the $T_j(\lambda_i)$ term is determined to be statistically insignificant.

2.2.2 Mixed Terms m& p_{iq}

In general, a mixed term can be expressed as $X_1^{P_1} X_2^{P_2} ... X_n^{P_n}$. There are two criteria for a valid mixed term: (1) the power of a variable in a mixed term should not be larger than he estimated order of the variable alone, i.e., $p_i \le k_i$ and (2) the total order of the mixed term, $\sum_i p_i$, should not be larger than the highest order term, i.e., $\sum_i p_i \le \max(k_i)$.

2.2.3 Response Surface Approximation

Once the response surface has been formulated, the coefficients are estimated via singular value decomposition using sample points from the true limit state function. “A full factorial design” has P sample points where

$$P = \prod_{i=1}^{n} (k_i + 1) \tag{19}$$

Where n is the number of variables.

In problems with $n > 3$, the value of P, even $3n$, is much larger than the number of coefficients. Thus uniformly distributed random sample points are taken within the domain [$\mu + h_{reg}\sigma; \mu - h_{reg}\sigma$], where μ is a vector of the mean values of X and σ is a vector containing the standard deviation of X. The parameter h_{reg} here indicates the size of the domain of the sample points used in the regression for the polynomial coefficients.

2.2.4 Monte Carlo Simulation

In the fourth stage, a full scale MCS on the approximated limit state is carried out to determine the reliability index, β.

3 Numerical example

In this example (Zhao and Ang (2003)), an one-story one-bay elastoplastic frame is considered as shown in Fig.1. The geometric dimensions of this truss are l_0=6.0m, and h=4.5m; the section areas of all bars are $0.01m^2$. There are four random variables included, namely the load F_p and flexural bearing capacity of the bars M_i (i=1,2,3). M_i and F are independent and follows the log-normal distribution. The statistical information of t flexural bearing capacity M_i and the load F_p are listed in Table 1.

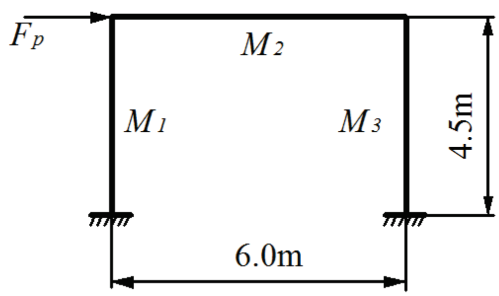

Fig.2 one-story one-bay frame

Table 1 The statistical information of yielding stresses of bars

random variables	Mean value	Standard deviation	distribution
M_1	2000kN·m	300kN·m	log-normal
M_2	2000kN·m	300kN·m	log-normal
M_3	2000kN·m	300kN·m	log-normal
F_0	500kN	200kN	log-normal

According to the complete system failure process, the system reliability of this truss can be expressed equivalently by

$$Z = F_{\max}\left(M_1, M_2, M_3, F_p\right) - F\left(F_p\right) \tag{20}$$

Finally, the failure probability are calculated according to the high order response surface method and the parameter h_{reg} and h_{ord} here are 3 respectively. The reliability index is 3.381. Using Monte Carlo simulations with 1 million samples, the reliability index is 3.274. The Number of Limit State Evaluations is 154. This result agrees well with the result of the Monte Carlo method.

4 Conclusions

In this work, by introducing the complete system failure process to obtain the equivalent performance function, and then adopted the high order response surface method to calculate the failure probability, an improved method for classical system reliability analysis is adopted. A numerical example is presented to verify the effectiveness of the proposed method.

The following conclusions can be drawn:

(1) The HORSM allows an indication of the accuracy of the estimated failure probability;

(2) The HORSM checks the accuracy of the response surface using a goodness-of-fit criteria and checks the failure probability by a comparison to the size of the domain of sample points.

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