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ADVANCED COPULA AND ITS IMPACT ON FATIGUE ANALYSIS OF OFFSHORE SYSTEMS

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The fatigue design of an offshore structure is highly dependent on environmental parameters. Since the long-term wave condition which is characterized by a realistic statistical model is required to be considered in the fatigue design, it is quite a challenging task to establish an appropriate statistical model. The most popular traditional statistical model is based on conditional joint distribution which only considers the linear relationship between pairs of random variables. In this paper, advanced multivariate copula models based on pair-copula construction (PCC) are employed to construct the joint statistical model of the random environmental variables. The PCC based multivariate copula models are able to capture the non-linear as well as linear relationships among random variables, which overcome the limitation of the traditional joint distribution based models. Moreover, the flexibility of PCC makes the advanced multivariate copula models more advantageous. This advanced copula model shows superiority compared with the traditional conditional joint distribution by some model selection methods as well as many statistical checking methodologies.

Keywords: fatigue, deep-water risers, long-term, pair-copula construction, environmental variables

1 Introduction

It is a great challenge to design offshore structures in an uncertain ocean environment, which requires broad engineering analysis and decision. An offshore structure is always exposed to a variety of sea states over its service life. Engineers have to ensure the safety of the structure during its design life. Ultimate and fatigue limit states are two important criteria during a global dynamic analysis. In this paper, we are interested in the fatigue limit state. For the fatigue limit state, the long-term approach is required for the reason that fatigue damage is the result of accumulation over a large amount of storms.

The long-term assessment of offshore structures requires the consideration of stochastic nature of environmental factors such as wave and wind. The fatigue design of an offshore structure should take into consideration a realistic statistical model which properly characterizes the complex offshore environment. The joint cumulative distribution function (CDF) or probability density function (PDF) of environmental random variables is required for the evaluation of long-term performance of offshore structures. Besides the marginal distribution of each environmental parameter, the dependency structure between different environmental parameters is also important in constructing an appropriate statistical model. The conventional way of constructing the joint CDF of environmental variables is by conditional distribution model. For instance, the two-dimensional statistical model with random variables significant wave height H_s and spectral peak period T_p recommended by Haver and Nyhus (1986) is widely used by many researchers in the offshore research (Low and Cheung, 2012; Gao and Low, 2016; Low and Huang, 2017). However, the conventional conditional distribution models only consider the linear relationship between two random variables at each time. Copula theory is widely applied in the construction of the joint probability distribution of multivariate data which may take into consideration of nonlinear relationship as well as linear relationship (Low et al, 2016; Zhang et al, 2015; Lin-Ye et al, 2016; Rashidi and Mohammadian, 2016). Zhang et al. (2015) constructed a copula-based multivariate probabilistic model to assess the long-term performance of structures but still restricted to only considering the dependence between two variables each time. Pair-copula construction (PCC) is employed in this paper to construct more advanced multivariate models to analyse the long-term fatigue damage of deep water risers. Furthermore, the impact of different statistical models in the fatigue analysis is also investigated.

2 Long-term riser fatigue analysis

Environmental loads, which are caused by environmental phenomena, contribute great amount to structural damage. The most important environmental phenomena for offshore structures are: waves, wind, current and tides. The environmental phenomena are always described by physical variables of statistical nature (DNV, 2012). Environmental phenomena are always described by joint distributions. The joint distribution for environmental variables is constructed on the basis of a large amount of empirical, statistical data for a sufficient long time period. Wave spectrum, also known as power spectral density (PSD) is a measure of ocean wave energy density. In this paper, JONSWAP spectrum is adopted to characterize the short-term wave environment, which can be expressed as a function of significant wave height H_s and spectral peak period T_n. In most cases, spectral peak period T_n is modelled together with significant wave height H_s in a joint distribution. Similar to random waves, short-term stationary wind conditions can be described by wind spectrums. In this study, wind speed V_w is simplified as 1- hour mean wind speed measured at 10 meters above the sea surface and is considered as the third important environmental random variable.

S-N curve is employed to calculate the fatigue life in this study. S-N curves are always derived from laboratory tests (DNV, 2011). In the meanwhile, the rainflow-counting algorithm is used to conduct the cycle counting task (Downing and Socie, 1982). The Miner-Palmgren's rule is employed to sum up all cumulative damage with the expression. In the long-term fatigue analysis, all possible sea states over the lifetime of the structure should be taken into the consideration. The expected long-term fatigue damage of offshore risers is expressed as a multidimensional integration of certain probability integral:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E[d \mid H_s, T_p, V_w] f(H_s, T_p, V_w) dH_s dT_p dV_w \tag{1}$$

where $f(H_s,T_p,V_w)$ is the joint probability density function (jpdf) of random variables H_s,T_p and V_w.

3 Approaches for multivariate statistical model

3.1 *Conditional joint distribution*

The most popular joint distribution model in offshore engineering field is the joint distribution of significant wave height H_s and spectral peak period T_p (DNV, 2012). Johannessen (2001) gives the most popular statistical model including three random variables H_s T_p and V_w which can be expressed as:

$$f(H_s,T_p,V_w)=f(V_w)f(H_s\,|\,V_w)f(T_p\,|\,H_s,V_w) \tag{2}$$

In the above equation, $f\,(V_w)$ is described by two-parameter Weibull distribution where the shape parameter and the scale parameter can be obtained through curve fitting from history database. The conditional distribution $f\,(H_s|V_w)$ is also described by two-parameter Weibull distribution with the shape parameter and the scale parameter obtained through regression analysis within each V_w range. The conditional distribution $f\,(T_p|H_s,V_w)$ can be described by a log-normal distribution as given in Johannessen et al. (2001) and Dong et al. (2011).

3.2 *Classical Copula Theory*

Sklar (1959) was the first one to use copula at mathematical or statistical sense. The basic function of copula is to describe the dependence among random variables. The popularity of copula in probability and statistics is due to two reasons as stated by Fisher (1997): scale free measures of dependence can be studied and bivariate as well as multivariate distributions can be constructed by one-dimensional distribution function with simulation views. For the construction of multivariate distributions, marginal distributions and specific dependence structures are two required key components. In copula theory, dependence structures are modelled by a copula function. In the general definition, the concept of copula can be interpreted as functions that link multivariate distribution functions to one-dimensional marginal distribution functions. The mathematical definition is given as follows: Suppose $(X_1, X_2, \ldots, X_d)$ is a random vector with the marginal distributions $F_i\,(x) = P[X_i \leq x]$, i=1, 2,…., d. Then the random vector $(U_1, U_2, \ldots, U_d) = (F_1\,(X_1), F_2\,(X_2), \ldots, F_d\,(X_d))$ will have a uniformly distributed marginal. The copula of $(X_1, X_2, \ldots, X_d)$ is defined as $C\,(u_1, u_2, \ldots, u_d)$ given as follows:

$$C(u_1,u_2,...,u_d)=P[U_1\leq u_1,U_2\leq u_2,...,U_d\leq u_d] \tag{3}$$

In the above equations, all the information in the dependence structure among $(X_1, X_2, \ldots, X_d)$ is included in the copula $C\,(u_1, u_2, \ldots, u_d)$ and all the marginal distribution related information is contained in $(F_1\,(X_1), F_2\,(X_2), \ldots, F_d\,(X_d))$. The most obvious advantage of copula model is the capacity of modelling nonlinear dependence. Classical copula methods are based on several families include Gaussian Copula, Archimedean Copula and so on. Interested readers can refer to Gao (2016).

3.3 *Advanced Copula Theory*

Advanced copula theory is based on pair-copula construction (PCC) which was first proposed by Joe (1996(a)~(b)) and further studied and explored in the literature, e.g., Bedford and Cooke (2001, 2002), Kurowicka and Cooke (2006) and Aas et al (2009). Different from traditional copula methods (Gao, 2016), this advanced copula method gives a new way of constructing multivariate highly dependent models by decomposing a multivariate density into a number of pair copulas, which in turn extend the bivariate copulas to higher dimensions. This probabilistic construction method can increase the number of higher dimensional copula selection by the large number of parametric bivariate copulas. The essence of PCC is to model dependency by simple local 2-D copula blocks which are based on conditional independence. The detail construction procedures are as follows. Consider a random vector (X_1, X_2, …, X_d) and the its joint probability density function (jpdf) $f(X_1, X_2, \ldots, X_d)$ can be expressed as:

$$f(x_1,x_2,...,x_d)=f(x_d)f(x_{d-1}\mid x_d)f(x_{d-2}\mid x_{d-1},x_d)\cdots f(x_1\mid x_2,...,x_d) \quad (4)$$

According to the property of copula, the jpdf $f(X_1, X_2, \ldots, X_d)$ can also be expressed as:

$$f(x_1,x_2,...,x_d)=c_{1...d}(F_1(x_1),F_2(x_2),...,F_d(x_d))f_1(x_1)f_2(x_2)\cdots f_d(x_d) \quad (5)$$

where $c_{1\ldots d}$ is the density of the copula $C_{1\ldots d}$. For the three dimensional problem in the case study, the jpdf $f_{123}\left(x_1,x_2,x_3\right)$ is factorized as:

$$f_{123}\left(x_1,x_2,x_3\right)=f_3\left(x_3\right)f_{2|3}\left(x_2\mid x_3\right)f_{1|23}\left(x_1\mid x_2,x_3\right) \quad (6)$$

where $f_{2|3}\left(x_2\mid x_3\right)$ is:

$$f_{2|3}\left(x_2\mid x_3\right)=\frac{f_{23}\left(x_2,x_3\right)}{f_3\left(x_3\right)}=\frac{c_{23}\left(F_2\left(x_2\right)F_3\left(x_3\right)\right)f_2\left(x_2\right)f_3\left(x_3\right)}{f_3\left(x_3\right)}=c_{23}\left(F_2\left(x_2\right),F_3\left(x_3\right)\right)f_2\left(x_2\right) \quad (7)$$

Moreover, we can have the $f_{1|23}\left(x_1\mid x_2,x_3\right)$

$$f_{1|23}\left(x_1\mid x_2,x_3\right)=\frac{f_{12|3}\left(x_1\mid x_2,x_3\right)}{f_{2|3}\left(x_2\mid x_3\right)}=\frac{c_{12|3}\left(F_{1|3}\left(x_1\mid x_3\right),F_{2|3}\left(x_2\mid x_3\right);x_3\right)f_{1|3}\left(x_1\mid x_3\right)f_{2|3}\left(x_2\mid x_3\right)}{f_{2|3}\left(x_2\mid x_3\right)}$$
$$=c_{12|3}\left(F_{1|3}\left(x_1\mid x_3\right),F_{2|3}\left(x_2\mid x_3\right);x_3\right)c_{13}\left(F_1\left(x_1\right),F_3\left(x_3\right)\right)f_1\left(x_1\right) \quad (8)$$

By inserting Eq. (8) into Eq. (6), we obtain

$$f_{123}\left(x_1,x_2,x_3\right)=$$
$$f_1\left(x_1\right)f_2\left(x_2\right)f_3\left(x_3\right)c_{13}\left(F_1\left(x_1\right),F_3\left(x_3\right)\right)c_{23}\left(F_2\left(x_2\right),F_3\left(x_3\right)\right)c_{12|3}\left(F_{1|3}\left(x_1\mid x_3\right),F_{2|3}\left(x_2\mid x_3\right);x_3\right) \tag{9}$$

Eq (9) is a full PPC expansion of a three-dimensional jpdf $f_{123}(x_1, x_2, x_3)$ and it is one out of the three possible decompositions in three dimensions.

4 Fatigue Assessment based on Different Environment Statistical Modelling

A turret floating, production, storage and offloading (FPSO) in a water depth of 1500m is considered for the current study in this paper. The floating system consists of three main components: one vessel, two flexible risers and four mooring lines. A turret is installed to connect all mooring lines and flexible risers. The wave environment is described by the JONSWAP spectrum. The environmental random variables significant wave height H_s, spectral peak period T_p and 1-hour mean wind speed at 10m above sea level V_w are considered as random variables in the proposed statistical models. The commercial software OrcaFlex Dynamics (Version 9.5a) is employed to perform dynamic analyses in the global coupled model in time domain and subsequent rain flow counting analyses.

The data collected for this research is from National Data Buoy Centre. The location of data collected is in the east coast of Brazil in the Atlantic (7.91°S, 30.49°W) and the water depth near the location is 1500m. The hourly measured data set covers 42 years' span from year 1973 to year 2014 with a few months' data missing. In this study, for the construction of an appropriate PCC model, the 3 possible options for selection of factorization shown in Table 1 are considered. 1, 2 and 3 in the subscripts in the factorization correspond to H_s, T_p and V_w, respectively. The choice of pair-copula types is decided for each pair-copula combination of any two random variables. Five well known copulas are considered: Gaussian copula, T copula, Clayton copula, Frank copula and Gumbel copula. Conditional pair-copula required in each of the factorization is constructed. In this work, a new algorithm for simulating samples from the jpdf constructed for the PCC model is developed. The details of the formula required for the above construction and the proposed sampling algorithm can be found in the journal version of this paper.

Table 1. Comparison of the statistical models and results of normalized fatigue damage

f (Hs,Tp,Vw)		No of parameter	AIC	BIC	Mean Fatigue Damage
traditional method		17	2064033	2064089	0.564
classical 3D copula	Gaussian	9	2024179	2024208	0.669
	T	10	2022300	2022333	0.703
advanced 3D copula	1st type $f_{123}=f_1f_2f_3*C_{13}C_{23}*C_{12\|3}$	26	2007720	2007805	0.875
	2nd type $f_{123}=f_1f_2f_3*C_{21}C_{31}*C_{23\|1}$	25	2004167	2004248	0.645
	3rd type* $f_{123}=f_1f_2f_3*C_{12}C_{32}*C_{13\|2}$	25	1996644	1996725	1

(Notes: The lowest AIC or BIC value indicates the best model, highlighted by "*".)

Table 1 gives the final results of the comparison of different statistical models and normalized long-term fatigue damage estimation. The normalization is conducted by dividing the fatigue damage at the best fit model which is 3rd type of the advanced 3D copula model. As shown in the table, all three advanced 3D copula models are superior to the two classical 3D copula models which in turn are better than the traditional model based on conditional joint distribution, according to both two model selection methods AIC and BIC statistics. In the application of long-term fatigue analysis, around 43.6% difference exists between the estimation constructed by the 3rd type of advanced copula statistical model and traditional conditional distribution model which is generalized used.

5 Conclusions

There are significant differences in the way traditional conditional distribution models, multivariate classical copula models and multivariate advanced copula models are constructed. Besides the capability of copula models which may model both the linear as well as nonlinear relationships among random variables. The result of model selection shows that the advanced models outperform the classical copula models and the traditional models in modelling the joint probability distribution of the environmental variables. The effect of statistical models in the assessment of the long-term fatigue damage of riser systems is also presented. The significant differences in the fatigue damage results among those obtained by considering multivariate classical copula models, advanced copula models and the popular traditional conditional models indicate that the statistical model for the joint probability distribution of the environmental variables can be improved by employing copula-based approach. Even though some publications on the PCC have appeared in financial field (Fisher et al., 2007; Chollete and Valdesogo, 2008), it has rarely been seen in the long-term fatigue application in offshore field in the statistical modelling of ocean parameters. This work shows great potentials of constructing statistical model which is more flexible and is able to capture the nonlinear relationship of different random variables. Ongoing works include improving the current PCC models and integrating the advantages of PCC and traditional models. The impact of the proposed PCC models on extreme response and system reliability is currently under investigation. Preliminary results show that the proposed PCC model leads to very different results in extreme response and system reliability compared with those given by the traditional model.

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