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A NOVEL BAYESIAN OPERATIONAL MODAL ANALYSIS METHOD BASED ON MODAL-COMPONENT SAMPLING

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This paper presents a novel Bayesian operational method with emphasis on practical applications. The mathematical model of the dynamic system is first constructed using the modal parameters (i.e., modal frequencies, modal damping ratios, mode shapes and modal initial conditions). Conditional on the measured accelerations, the posterior probability density function (PDF) of the modal parameters is then derived following Bayes theorem. Bayesian modal analysis is thus to identify the posterior PDF. Because the posterior PDF cannot be analytically normalized, Markov chain Monte Carlo (MCMC) is applied to sample from the posterior PDF. Considering that the number of uncertain parameters to be identified is large, modal component sampling is developed. The idea is that instead of directly sampling from the posterior PDF, sampling is iterated among the PDFs of different modal components that consist of modal parameters of each mode. The efficiency of the proposed method is illustrated on a full-scale structure. The identified modal parameters reveal interesting dynamic behaviors of the structure and they are helpful for structural health monitoring.

Keywords: Bayesian modal analysis, modal-component sampling, MCMC, field test.

1 Introduction

Reliable mathematical models are important for accurately characterizing dynamics of real structures so activities such as structural response prediction under different loads, reliability analysis and structural health monitoring can be conducted efficiently. Finite element methods have been used almost exclusively for developing mathematical models of structures. The mathematical models built by finite element methods work fairly well for structural design. However, it is known that finite element models without updating using measured dynamic data cannot accurately represent real structural systems, limiting their practical applications. System identification is a process that identifies a mathematical model of a real system based on measured output and possibly input so that the identified model accurately predicts the output, and it is thus suitable to approximate actual structural systems. In civil Engineering, modal model is often used for system identification to identify modal parameters such as modal frequencies, modal dampings and mode shapes. This process is called modal analysis and it finds applications in many fields. Modal parameters can be used to continuously monitor performances of long bridges (Magalhães et al. 2008) and tall buildings (Zhang et al. 2017). Mass and stiffness distributions of structures are empirically evaluated based on the identified modal parameters for assessing the structural status (Michel et al. 2008). Dynamic wind loads under strong winds can be validated using modal parameters (Au and To 2012). Modal parameters are also successfully applied in the mechanics community. To assess the performance of airplanes under operational conditions, modal parameters are measured during the flight (James 2003). Modal parameters of wind turbines are measured at different rotation rates for health monitoring (Carne and James 2010). Considering the importance of modal analysis, it is necessary to develop efficient algorithms.

There are mainly two problems for modal analysis. For large scale structures the number of degrees of freedom (DOFs) to be measured is usually large. The number of uncertain parameters associated with mode shapes at all DOFs to be identified is thus very large. If not properly handled, it will be time-consuming to identify so many parameters at the same time. Even worse, it is very likely that the parameters cannot be identified at all because high-dimension identification problems are difficult to converge. Moreover, uncertainty always exists when one tries to predict a system using a mathematical model given measured data. In this case, uncertainty reflects possible inaccuracy of the mathematical model used for predicting the unknown "true" system given measured data. For modal analysis of civil engineering structures, usually identification has to be done using only measured output because input is expensive or impossible to measure. In this case, uncertainties of identified modal parameters are farily large. Appropriately considering uncertainties for modal analysis is thus important. The fast Bayesian FFT method can efficiently identify modal parameters and quantify their uncertainties following the Bayesian framework (Au 2011, Au and Zhang 2012, Au 2017). The application of this method in full-scale structures such as a tall building (Zhang et al. 2017), a footbridge (Ni and Zhang 2016), a coupled-slab system (Au et al. 2012, Lam et al. 2015) and a tall building with dampers (Ni et al. 2017) has validated its efficiency in practice. Considering the importance of the problems mentioned above, this paper also tries to develop an Bayesian modal analysis method (Yang et al. 2017) that can efficiently identify modal parameters of structures with a large number of measured DOFs and their associated uncertainties.

2 Methodology

Following the Bayesian framework, Bayesian modal analysis is to identify the posterior PDF of modal parameters given measured data, $p(\boldsymbol{\theta}|\mathcal{D})$, where $\boldsymbol{\theta}$ is a vector containing the modal parameters to be identified and $\mathcal{D}$ is the measured data. According to the Bayes' theorem, the posterior PDF is formulated as

$$p(\boldsymbol{\theta}|\mathcal{D}) = cp(\mathcal{D}|\boldsymbol{\theta}) \tag{1}$$

where c is a normalizing constant. One of the difficulties of identifying the posterior PDF is that there is no close-form formula for it. This problem could be solved by using Monte Carlo methods. The idea is to generate samples from the posterior PDF and these samples are then used to approximate the posterior PDF. However, the region with significant probability is small compared to the vast parameter space, so standard Monte Carlo methods that sample directly from the posterior PDF will not work. The reason is that most of the samples lie in the region with small probability. Furthermore, the dimension of $\boldsymbol{\theta}$ is high for full-scale structures, so sampling for all the modal parameters will be inefficient. In this paper, a modal-component sampling algorithm is proposed to sample from the high-dimension parameter space. It is a Markov chain Monte Carlo method based on the idea of Gibbs sampling. $\boldsymbol{\theta}$ is divided into different modal components, i.e.,

$$\boldsymbol{\theta} = \left\{ \mathcal{M}_1, \mathcal{M}_2, \cdots, \mathcal{M}_n, \cdots \mathcal{M}_{N_m} \right\} \tag{2}$$

where the n-th modal component $\mathcal{M}_n$ contains modal parameters of this mode, i.e., $\mathcal{M}_n$ = { ω_n, ξ_n, $\boldsymbol{\varphi}_n$, $y_n(0)$, $\dot{y}_n(0)$ }, where ω_n is the modal frequency; ξ_n is the modal damping ratio; $\boldsymbol{\varphi}_n$ is the mode shape; $y_n(0)$ and $\dot{y}_n(0)$ are the modal initial displacement and modal initial velocity, respectively. Modal-component sampling is to sample from the PDF of the n-th modal component conditional on other modal components, i.e., $p\left(\mathcal{M}_n | \mathcal{M}_1, \mathcal{M}_2, \cdots, \mathcal{M}_{n-1}, \mathcal{M}_{n+1}, \cdots \mathcal{M}_{N_m}, \mathcal{D}\right)$. In this way, the parameter space is explored locally, instead of directly sampling in the whole vast parameter space. It turns out that this setting is quite efficient and guarantee the convergence of modal parameters in a matter of minutes for full-scale structures. The application procedures of the proposed method are summarized briefly in the following.

(1) Obtain the initial values of modal parameters from singular value spectra calculated by singular value decomposition of power spectral densities of measured accelerations.

(2) To generate one sample of $\boldsymbol{\theta}$, sampling is conducted for each modal component using the set of PDFs $\left\{ p\left(\mathcal{M}_n | \mathcal{M}_1, \mathcal{M}_2, \cdots, \mathcal{M}_{n-1}, \mathcal{M}_{n+1}, \cdots \mathcal{M}_{N_m}, \mathcal{D}\right) : n = 1, 2, \cdots N_m \right\}$, where N_m is the number of modes to be identified. Sampling is then conducted for modal components one at a time, while keeping other modal components unchanged at their current values. Continuing generate samples of $\boldsymbol{\theta}$ until convergence.

(3) The MPV and uncertainty of $\boldsymbol{\theta}$ is approximated using the samples.

3 Bayesian Operational Modal Analysis of an Academic Building

The proposed method was applied for modal analysis of a 22-story building (see Figure 1) to illustrate its efficiency. The main building measures 55.00 m length × 38.84 m width × 97.00 m height. Figure 2 shows the typical floor plan of the building. A considerable number of students have their lectures in this building and many students from other places of the campus have their meals in the canteen of this building. The loading introduced to this building is intense. In addition, this building has a special shape. It is therefore of great interest to obtain its dynamic properties.

The main building was focused in this test. The measurement was conducted in multiple setups. In order to avoid interfering the normal operation of the building, the measurement was mainly conducted within the 3 staircases (see Figure 2). The 3 staircases were measured separately. The 19-th floor was then measured. The mode shapes were assembled by using the references on the 19-th floor. The blue balls in Figure 2 show the sensor arrangement on the 19-th floor. The north direction of all sensors is shown in Figure 2. The sensors spread over the perimeter of the floor and therefore the structural information of the building can be well captured. The 3 locations at the 3 staircases were measured on the 19-th floor such that the mode shapes of the staircases and the floor slab could be assembled.

Figure 1. The Academic building.

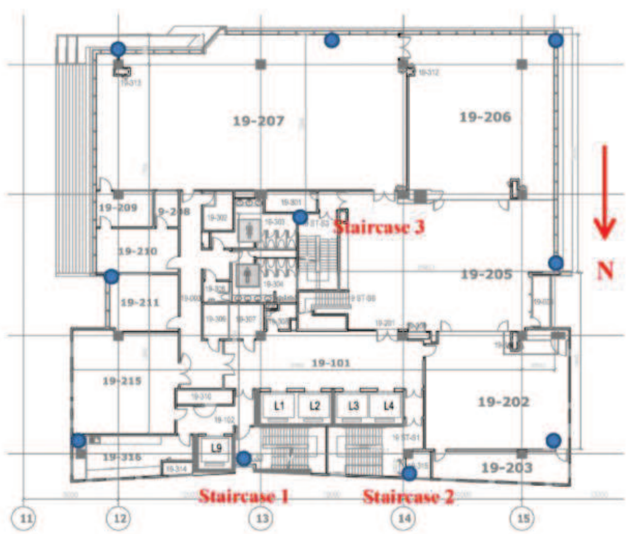
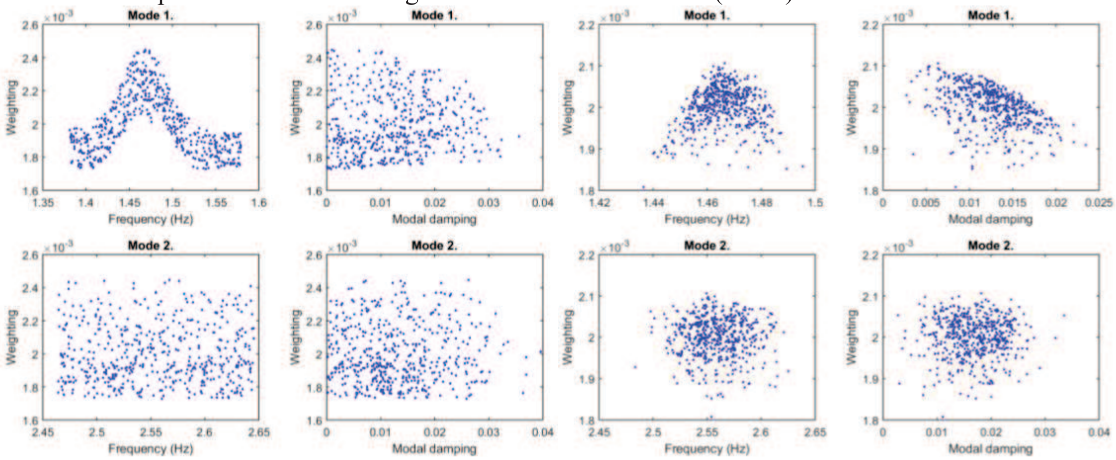


Figure 2. Typical floor plan.

Only two modes are presented due to the limited space. Figure 3 shows how the samples of the two modes based on the data of setup 1 converge. Only the samples of frequencies and modal damping are shown. The horizontal axis corresponds to frequency or modal damping and the vertical axis corresponds to the weighting of a sample. Figure 3(a) shows that the samples at the beginning of the sampling process distribute in relatively wide regions. Figure 3(b) shows that the samples at the end of the sampling process converge in certain regions. The computational time for one setup is only about 2 minutes. The first mode has a modal frequency 1.46 Hz and a modal damping ratio 1.43%. The global mode shape assembled from partial mode shapes of different setups is shown in Figure 4. It can be seen that this is a torsional mode and the rotational center is approximately at the middle of the building. The second mode has a modal frequency 2.55 Hz and a modal frequency 1.59 %. Figure 5 shows that this mode is a translational mode and bending effects can be observed from the staircases.

Apart from the MPVs of the modal parameters, the proposed method can also calculate their uncertainties. Table 1 presents the uncertainties of the modal parameters for the first three setups.

It is shown that the uncertainties of the modal damping ratios are relatively large. This fits the intuition because it is known that modal damping ratios are uncertain. Note that the uncertainties of mode shapes are calculated using modal assurance criterion (MAC).



(a) (b)

Figure 3. Sampling process: (a) at the beginning, (b) in the end.

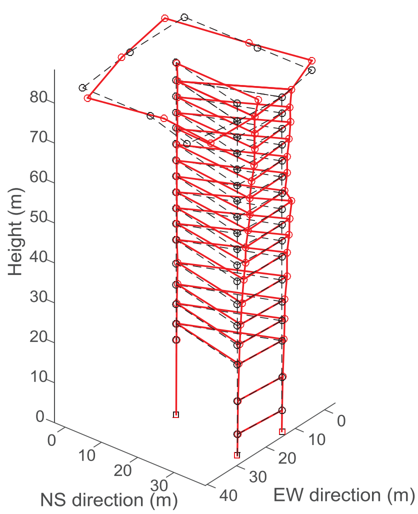


Figure 4. Torsional mode.

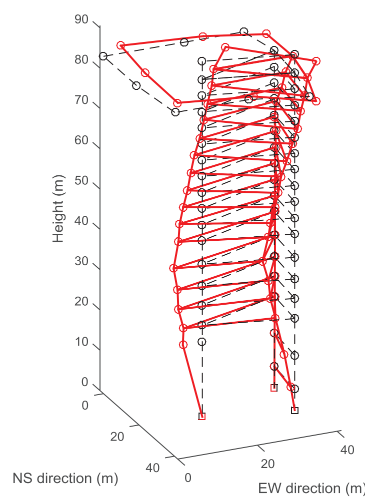


Figure 5. Translational mode.

Table 1. The uncertainties of the modal parameters.

	Modal frequency		Modal damping		Mode shape ($\times 10^{-3}$)	
	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
Setup 1	0.64%	0.93%	33.47%	28.78%	0.0011	0.0181
Setup 2	1.65%	0.80%	24.75%	29.32%	0.0135	0.0147
Setup 3	1.69%	0.93%	32.10%	27.80%	0.0151	0.0082

4 Conclusion

This paper proposes a Bayesian operational modal analysis method based on modal-component sampling. The modal-component sampling algorithm facilitate the efficient sampling from the posterior PDF for a high-dimension problem. The proposed method successfully identified the modal parameters and their uncertainties of an academic building. The identified modal parameters reveal interesting dynamic behaviors of this building.

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