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# STOCHASTIC RESPONSE OF LARGE WIND TURBINES UNDER TWO-DIMENSIONAL WIND FIELDS

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Wind loads are the critical loads for large wind turbines. Therefore, the simulation of wind speed random field is of great significance. At present, the spectral representation method (SRM) is widely used because of its simple algorithm and rigorous theory. However, the traditional procedure for wind field simulation based on SRM involves Cholesky decomposition at each discretized frequency point with respect to the cross-power spectrum density (PSD) matrix, which results in large computational efforts, especially in case a large-scale simulation of wind field is required. Recently, a wavenumber-frequency joint power spectrum based SRM for the wind field in one spatial dimension was proposed, bypassing the Cholesky decomposition. The method was extended by the authors to two spatial dimensions. In the present paper, the method is employed to generate the wind speed field in two spatial dimensions for wind turbines. Then the response of a large wind turbine is studied and compared with the results when the wind speed field is modeled by the physically based model, showing the consistency between the two approaches.

*Keywords:* offshore wind turbine, StoDRAOWT model, fluctuating wind speed field, spectral representation method, wavenumber-frequency joint power spectrum, uneven discretization

## 1 Introduction

Offshore wind energy has received a rapid development around the world in the last two decades due to its advantages over onshore wind energy harvesting (Burton et al. 2011). However, compared to the onshore wind turbines, for the offshore wind turbine the environmental conditions are more severe, and the support structures become larger, taller and more flexible with an increasing power capacity. On the other hand, the maintenances and repairs of offshore wind turbines are much more costly. Therefore, refined dynamic response analysis and reliability assessment of offshore wind turbines are attached increasing importance (Sichani and Nielsen 2013).

Wind speed field simulation is crucial in the analysis of offshore wind turbines because the wind load is dominating for power output and reliability of the system. Traditionally, the wind speed field is modeled as a multivariate stochastic vector process, which is continuous in time domain and discretized in space domain (Shinozuka 1971), and the cross-PSD matrix is used to describe its statistical characteristics and correlation characteristics (Di Paola 1998). The spectral representation method (SRM) is widely used for its simple algorithm and rigorous theory (Benowitz and Deodatis, 2015). However, because the Cholesky decomposition of the cross-PSD matrix at each discretized frequency is usually invoked, the simulation efficiency decreases noticeably and numerical instability even occurs as the number of simulation spatial points becomes large. Although some improved methods were investigated (Ding et al. 2011), the decomposition of the cross-PSD matrix is unavoidable and the efficiency cannot be enhanced in a large degree essentially.

In fact, the wind speeds in space is a random space-time field. Along this line, a wavenumber-frequency joint spectrum was proposed to simulate wind speed field in one spatial dimension (Benowitz and Deodatis 2015; Peng et al. 2017). In this method the Cholesky decomposition is not needed. The basic idea has recently been extended to wind fields in two dimensions (Chen et al. 2017b), which is needed in the analysis of wind turbines. In the present paper, this new method is firstly outlined, and then employed to generate the speed wind field in two spatial dimensions for wind turbines. Then the dynamic response analysis of a 5-MW offshore wind turbine is carried out, showing its efficiency and accuracy.

## 2 Fluctuating Wind Field Simulation via a Wavenumber-Frequency Joint Spectrum

### *2.1 Joint spectrum for fluctuating wind field in two spatial dimensions*

Fluctuating wind speed field simulation for rotating blades of a 5-MW wind turbine in the two dimensional z-y coordinates is studied in this paper. For clarity, the basic idea for the simulation of homogeneous fluctuating wind speed field in two spatial dimensions via the wavenumber-frequency joint spectrum is outlined here (Chen et al. 2017b).

According to the theories in wind engineering (Simiu and Scanlan 1996), the wind speed can be decomposed into average wind speed and fluctuating speed, while the fluctuating wind speed $X(z,y,t)$ at an arbitrary spatial point $P_0(z,y)$ is a zero-mean stochastic process. Considering the arbitrariness of $P_0$, $X(z,y,t)$ is essentially a zero-mean random space-time field. Further, $X(z,y,t)$ is a homogeneous random field if the correlation function $R_{X_1X_2}$ is not dependent on the coordinates (Vanmarcke 2010), i.e.,

$$R_{X_1X_2} = E[X(z_1,y_1,t_1)X(z_2,y_2,t_2)] = R(\xi_z,\xi_y,\tau) \tag{1}$$

where $X_1$ denotes the wind speed at $P_1(z_1,y_1)$, i.e., $X(z_1,y_1,t_1)$, and $X_2$ denotes the wind speed at $P_2(z_2,y_2)$, i.e., $X(z_2,y_2,t_2)$, and $\xi_z = z_1 - z_2, \xi_y = y_1 - y_2, \tau = t_1 - t_2$, are the space difference and time difference, respectively.

The Fourier transform of $R(\xi_z,\xi_y,\tau)$ with respect to $\tau$ yields the cross-PSD of the wind speeds at the two points $P_1$ and $P_2$

$$S_{X_1X_2}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R(\xi_z,\xi_y,\tau)e^{-i\omega\tau}\mathrm{d}\tau \tag{2}$$

In practice, the cross-PSD can be determined by (Simiu and Scanlan 1996)

$$S_{X_1X_2}(\omega) = \sqrt{S_{X_1}(\omega)S_{X_2}(\omega)} \cdot \rho_{X_1X_2}(\omega) \tag{3}$$

where $S_{X_1}(\omega)$, $S_{X_2}(\omega)$ are the auto-PSD of the fluctuating wind speed at $P_1$ and $P_2$, respectively, $\rho_{X_1X_2}(\omega)$ is the corresponding coherence function, and usually takes

$$\rho(\omega,\xi_z,\xi_y)=\exp\left(-\tfrac{1}{2\pi U_{10}}|\omega|\sqrt{C_{1z}^2{\xi_z}^2+C_{1y}^2{\xi_y}^2}\right) \tag{4}$$

where $U_{10}$ is the 10-min mean wind speed at 10m, and $C_{1z}$ and $C_{1y}$ are constants, taking $C_{1z}=C_{1y}=7$ for simplicity.

If $S_X(\omega)$ involved in Eq.(3) does not vary with the spatial position (the height), such as one of the most famous fluctuating wind spectra, the Davenport spectrum (Davenport 1961)

$$S_0(\omega)=2.0u_*^2\left(\tfrac{1200}{2\pi U_{10}}\omega\right)^2\Big/\left[|\omega|\left(1+\left(\tfrac{1200}{2\pi U_{10}}\omega\right)^2\right)^{4/3}\right] \tag{5}$$

Then Eq.(3) can be rewritten as

$$S(\omega,\xi_z,\xi_y)=S_0(\omega)\cdot\rho(\omega,\xi_z,\xi_y) \tag{6}$$

which represents a homogeneous random field.

As Eq.(2) takes the form of Eq.(6), then a two-fold Fourier transform of $S(\omega,\xi_z,\xi_y)$ in terms of $\xi_z$ and $\xi_y$, respectively, yields

$$S^{(\mathrm{WF})}(k_z,k_y,\omega)=S_0(\omega)\cdot\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left(-\tfrac{|\omega|\sqrt{C_{1z}^2{\xi_z}^2+C_{1y}^2{\xi_y}^2}}{2\pi U_{10}}\right)e^{-i(k_y\xi_y+k_z\xi_z)}d\xi_y\mathrm{d}\xi_z \tag{7}$$

which finally leads to (Chen et al. 2017b)

$$S^{(\mathrm{WF})}(k_y,k_z,\omega)=S_0(\omega)\cdot\frac{1}{2\pi C_{1z}C_{1y}}\frac{1}{\left(\tfrac{1}{2\pi U_{10}}|\omega|\right)^2}\frac{1}{\left(1+\left[\left(\tfrac{1}{C_{1y}}k_y\right)^2+\left(\tfrac{1}{C_{1z}}k_z\right)^2\right]\Big/\left(\tfrac{1}{2\pi U_{10}}|\omega|\right)^2\right)^{\frac{3}{2}}} \tag{8}$$

### *2.2 Discretized schemes for the joint spectrum based representation method*

Once the joint spectrum is obtained, the SRM method (Shinozuka and Deodatis 1996) can be adopted to generate fluctuating wind field samples. It is noted that the joint spectrum of Eq.(8) has a relatively long but non-negligible tail, and the spectral value is concentrated highly and varies rapidly in the range close to the origin. Thus, if the uniform discretization scheme is utilized as shown in Shinozuka and Deodatis (1996), then as many as $10^9$ points are needed to achieve a satisfactory simulation result, which is prohibitively large for practical applications.

To reduce the computational efforts, the acceptance-rejection based unevenly discretized scheme is proposed, for which the essence is to take denser discretized points where the value of the joint PSD is greater. To this end, the radial wavenumber-frequency joint spectrum corresponding to Eq.(8) (Mantoglou and Wilson 1982) is firstly invoked

$$S^{(\mathrm{WF})}(k_r,\omega)=S_0(\omega)\cdot\frac{1}{C_{1z}C_{1y}\left(\tfrac{1}{2\pi U_{10}}|\omega|\right)^2}\frac{k_r}{\left(1+k_r^2\Big/\left(\tfrac{1}{2\pi U_{10}}|\omega|\right)^2\right)^{3/2}} \tag{9}$$

where $k_r=\sqrt{\left(k_y/C_{1y}\right)^2+\left(k_z/C_{1z}\right)^2}$ is the radial wavenumber coordinate.

Then a number of $n_{pt}$ points $\left(k_j^{(r)},\omega_j\right)_{j=1}^{n_{\mathrm{pt}}}$ can be obtained with the acceptance-rejection sampling method. Further, the two-dimensional wavenumber vector $\left(k_{i,j}^{(y)},k_{i,j}^{(z)}\right)_{i=1}^{n_j}$ can be specified corresponding to $k_j^{(r)}$. Thus, the final discretized wavenumber-frequency point set is $\left(k_{i,j}^{(y)},k_{i,j}^{(z)},\omega_j\right)$, and its total number $m=\sum_{j=1}^{n_{\mathrm{pt}}}n_j$. About this sampling method, refer to Li and

Chen (2009). In this case, the SRM is expressed as

$$X(z,y,t) = \sum_{j=1}^{n_{\mathrm{pt}}} \sum_{i=1}^{n_j} \sqrt{4S^{(\mathrm{WF})}(k_{i,j}^{(z)}, k_{i,j}^{(y)}, \omega_j) V_{i,j}} \times \\ \left[\cos(k_{i,j}^{(z)} z + k_{i,j}^{(y)} y + \omega_j t + \phi_{i,j}^{(1)}) + \cos(k_{i,j}^{(z)} z + k_{i,j}^{(y)} y - \omega_j t + \phi_{i,j}^{(2)}) + \right. \\ \left. \cos(k_{i,j}^{(z)} z - k_{i,j}^{(y)} y + \omega_j t + \phi_{i,j}^{(3)}) + \cos(k_{i,j}^{(z)} z - k_{i,j}^{(y)} y - \omega_j t + \phi_{i,j}^{(4)})\right] \tag{10}$$

where $V_{i,j}$ is the representation volume of the point $(k_{i,j}^{(z)}, k_{i,j}^{(y)}, \omega_j)$, $\phi_{i,j}^{(1)}$, $\phi_{i,j}^{(2)}$, $\phi_{i,j}^{(3)}$ and $\phi_{i,j}^{(4)}$ are four different sets of independent random phase angles uniformly distributed in $[0,2\pi]$.

With this scheme, about $1.3 \times 10^5$ representative discretized points are obtained and applied in the simulation of the wind field in section 4. To verify the efficiency of the method, wind fields of different simulation points are simulated with the proposed method and the Cholesky decomposition based method respectively, and the consumed time is shown in Table 1.

**Table 1.** Consumed time with two methods for different number of simulation points

| Simulation Method | Consumed time (unit: minutes) | | |
|---|---|---|---|
| | N = 30 | N = 90 | N = 150 |
| Cholesky Decomposition | 2.15 | 17.84 | 49.8 |
| Joint Spectrum | 7.12 | 21.52 | 35.8 |

Table 1 indicates that the consumed time increases linearly with the number of simulation points for joint spectrum based method while it increases more rapidly for Cholesky decomposition based method. Therefore the joint spectrum based method is preferred for large wind field simulation such as in the case of offshore wind turbine. Besides, if FFT is incorporated the efficiency will be improved considerably, and will be discussed elsewhere.

## 3 The Integrated Analysis Model for a 5-MW Wind Turbine

The stochastic dynamic reliability analysis of offshore wind turbine (StoDRAOWT) model (Chen et al. 2017a) is employed in this paper. This model is based on the definition of a 5-MW offshore wind turbine by the NREL in the U.S. (Jonkman et al. 2009), and has been demonstrated to be accurate and efficient (Chen et al. 2015; Chen et al. 2017a). A brief introduction to the StoDRAOWT model is as follows.

To establish the structural dynamic model, the finite element method and the flexible multi-body dynamics method are combined, which can consider the rotating effects and the geometric nonlinearity of the blades at the same time. The Euler-Bernoulli beam element is employed, and a total of 391 degrees of freedom are used to describe the system. In addition, the aero-elastic, blade-pitch control and nonlinear generator torque control are also considered.

To consider the interaction of the pile with soil, the nonlinear soil spring model is adopted, and the force-displacement relation of the soil spring is determined by the $p$-$y$ curve suggested by API (2002). Geological conditions of the wind farm can be referred to Chen et al. (2017a).

The Pierson-Moscowitz spectrum (Pierson and Moscowitz 1964) and the SRM are adopted to simulate the stochastic waves in this model. Then, the linear wave theory and the Morison's equation are employed to calculate the wave loads (Zhu 1991).

As for the wind field simulation, both the proposed method and the the random Fourier function based physical model (Li et al. 2012) are employed. First, the Davenport spectrum is invoked to determine the Fourier amplitude spectrum. Then, the evolutionary phase angle method (Li et al. 2013) and the delay phase angle model (Yan et al. 2013) are adopted to determine the Fourier phase spectrum, by which the correlations of the wind field can be considered. Lastly, the wind speed time histories can be obtained by the inverse Fourier transform. The classical momentum-blade theory is employed to calculate the aerodynamic loads on the wind turbine (Hansen 2008).

## 4 Responses Comparison of the Wind Turbine under the Wind Fields Simulated by Two Different Methods

In this section, responses of the wind turbine under two different wind fields simulated by the physical model and by the proposed model, respectively, are studied and compared, as shown from Fig.1 to Fig.4. The 10-min mean wind speed at the hub is 20m/s for both the two cases.

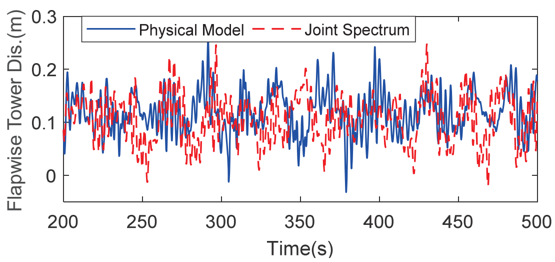


Fig.1 Time histories of tower top displacement

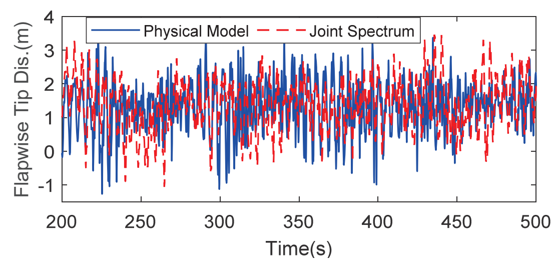


Fig.2 Time histories of blade tip displacement

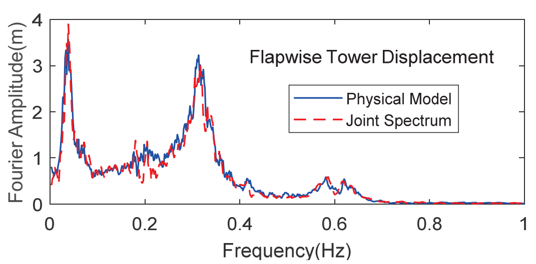


Fig.3 Fourier Amplitude of tower top displacement

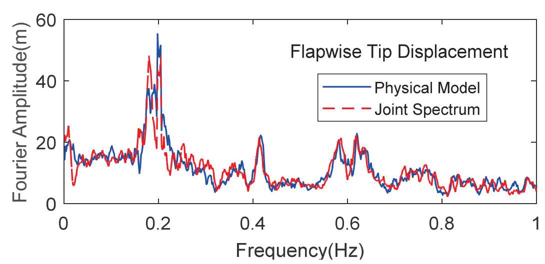


Fig.4 Fourier Amplitude of blade tip displacement

Fig.1 and Fig.2 show that the amplitudes of the displacement time histories of the blade tip and the tower top under two different wind fields are consistent, while Fig.3 and Fig.4 indicate that the properties in frequency domain of blade tip and tower top displacement are almost the same, which demonstrate the accuracy of wind simulation method proposed in this paper.

It is worth noting that the first peak in Fig.3 is around the peak frequency of wave spectrum, and the second peak is around the fundamental frequency of tower, while the peaks in Fig.4 are at the multiples of rotational sampling frequency, i.e. 0.2Hz. Therefore the responses of tower are mainly dominated by wave loads while the blade responses are dominated by wind loads.

## 5 Concluding remarks

A spectral representation method based on the wavenumber-frequency joint spectrum for wind field simulation has been extended to two spatial dimensions. The acceptance-rejection based unevenly discretized scheme has been proposed to reduce the computational costs. Based on the StoDRAOWT model, dynamic responses of wind turbine under wind fields simulated by the proposed method and by the physical model are compared, demonstrating the reliability of proposed method for wind field simulation.

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