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RELIABILITY ASSESSMENT ON THE PERFORMANCE EVALUATION FORMULA OF BUCKLING-RESTRAINED BRACE USING STEEL MORTAR PLANKS

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Buckling-restrained brace using steel mortar planks (BRBSM) designed to reduce plastic strain in the beam-to-column connections are being widely used in practical structures. Although a performance evaluation formula has been proposed to guide the design of BRBSM, the safety for this standard has not been assessed. In this paper, for reasonable design of BRBSM under reliability-based requirement, moment method is adopted to evaluate the relationship between the restraining index and its corresponding exceeding probability of cumulative plastic strain energy ratio based on a total of 101 previous experimental data. It can be observed that the exceeding probability of the cumulative plastic strain energy ratio changes with the increasing restraining index. To clearly indicate the reliability level of the performance of BRBSM, two boundaries with the nearly same exceeding probability of 5% and 10% respectively are then presented.

Keywords: Buckling-restrained brace, Performance evaluation formula, Moment method, Exceeding probability, Fourth-moment normal transformation

1 Introduction

When a strong earthquake occurs, the building may be severely damaged and not be able for continuous usage due to the large plastic strain yielded in the beam-column connections, to solve which, a seismic-response-controlled member called buckling-restrained brace was generally used in the practical engineering.

Currently, a variety of systems have been proposed for the buckling-restrained brace, among which a typic buckling-restrained brace using steel mortar planks (BRBSM), which enables increased design freedom at both ends of the core plate and strict quality control while providing stable hysteresis characteristics even under high strains, is developed by Iwata.[1] The details of BRBSM is shown in Fig.1. Lots of experimental tests have been carefully designed to investigate the performance of BRBSM, based on which a formula to evaluate the minimum performance of BRBSM is proposed as following equations. [1]

$$\omega = \begin{cases} 150R, R \le 6 \\ 900, \quad R > 6 \end{cases} \tag{1}$$

$$\omega = \frac{E_t}{W_y} = \frac{E_t}{P_y \delta_y}, \quad R = \frac{P_E}{P_y} \tag{2}$$

where ω is cumulative plastic strain energy ratio and R is restraining index; E_t is the cumulative plastic strain energy; P_y and δ_y is the yield load and the elastic-limit deformation of the brace, respectively; P_E is the buckling load of the restraining part.

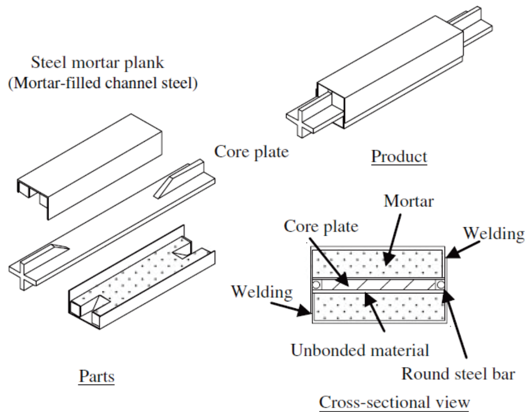

Fig. 1. Details of buckling-restrained brace using steel mortar planks (BRBSM)

Although this evaluation formula has been widely applied in the past practical applications to design the detailed dimensions for BRBSM, the safety for this evaluation formula has not been assessed. Furthermore, for accurate design of BRBSM under reliability-based requirement, the performance evaluation formula under specified exceeding probability should be investigated. In this paper, the exceeding probability with the performance evaluation formula is firstly evaluated by using moment method based on a total of 101 data from the past experiments and two boundaries for the relationship between restraining index and cumulative plastic strain energy ratio for specified reliability requirement are presented.

2 Moment method for calculating the exceeding probability

The LRFD format is expressed as follows

The evaluation formula of BRBSM's reliability problem is formulated in terms of a vector of basic random variables $\mathbf{X}=[x_1,x_2,\ldots,x_n]^T$, which represent uncertain quantities of restraining index, R. The performance function $Z=G(X)$ describing the data of cumulative plastic strain energy ratio ω is obtained from the previous experiments in terms of $\mathbf{X}$. The exceeding probability, i.e., the probability of ω that is smaller than the performance evaluation formula's mean value, A, is given by:

$$P_e = \text{Prob}[Z = G(\mathbf{X}) \leq A] = \int_{G(\mathbf{X})\leq A} f(\mathbf{X}) d\,\mathrm{X} \tag{3}$$

where $f(\mathbf{X})$ is the joint probability density function (PDF) of $\mathbf{X}$.

The standardized variable Z_s can be expressed as a function of S(u). Several forms of S(u) has been proposed, among which the fourth-moment normal transformation (FMNT) proposed by Zhao [2] is widely applied in engineering practice with sufficient accuracy, which is expressed as

$$Z_s = S(u) = a_4 u^3 + a_3 u^2 + a_2 u + a_1 \tag{4}$$

where the coefficients a_1, a_2, a_3 and a_4 are obtained by solving the following equations

$$\mu_G = a_3 + a_1 \tag{5a}$$

$$\sigma_G = a_2^2 + 2a_3^2 + 6a_2 a_4 + 15a_4^2 \tag{5b}$$

$$\alpha_{3G} = 6a_2^2 a_3 + 8a_3^3 + 72a_2 a_3 a_4 + 270 a_3 a_4^2 \tag{5c}$$

$$\alpha_{4G} = 3\left(a_2^4 + 20a_2^3 a_4 + 210a_2^2 a_4^2 + 3465a_4^4\right) + 12a_3^2\left(5a_2^2 + 5a_3^2 + 78a_2 a_4 + 375a_4^2\right) \tag{5d}$$

The inverse transformation of FMNT defined in Eq. (6) is listed in Table 1[3].

Table 1. Complete expression of the FMNT*

Parameter			Range of x	Normal transformation u	Type
$a_4<0$			$J_2^*<x<J_1^*$	$-2r\cos[(\theta+\pi)/3]-a/3$	I
$a_4>0$	$p<0$	$\alpha_{3x}\geq 0$	$J_1^*<x<J_2^*$	$2r\cos(\theta/3)-a/3$	II
			$x\geq J_2^*$	$\sqrt[3]{A}+\sqrt[3]{B}-a/3$	
		$\alpha_{3x}<0$	$J_1^*<x<J_2^*$	$-2r\cos[(\theta-\pi)/3]-a/3$	III
			$x\leq J_1^*$	$\sqrt[3]{A}+\sqrt[3]{B}-a/3$	
	$p\geq 0$		$(-\infty, +\infty)$	$\sqrt[3]{A}+\sqrt[3]{B}-a/3$	IV
$a_4=0$		$a_{3x}\neq 0$	$a_2^2+4a_3(a_3+x_s)\geq 0$	$[-a_2+\sqrt{a_2^2+4a_3(a_3+x_s)}]/2a_3$	V
		$a_{3x}=0$	$(-\infty, +\infty)$	x_s	VI

*Note: p; a; A; B; q; r; J_1^*, J_2^* *are expressed as Eq.* (7*a*) ~(7*e*), as follows:

$$p = \frac{3a_2 a_4 - a_3^2}{3a_4^2} \tag{7a}$$

$$\Delta = \left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2, q = \frac{2}{27}a^3 - \frac{ac}{3} - a - \frac{Z_s}{a_4}, a = \frac{a_3}{a_4}, c = \frac{a_2}{a_4} \tag{7b}$$

$$A = -\frac{q}{2} + \sqrt{\Delta}, B = -\frac{q}{2} - \sqrt{\Delta} \tag{7c}$$

$$\theta = \arccos\left(\frac{-q}{2r^3}\right), r = \sqrt{-\frac{p}{3}} \tag{7d}$$

$$J_1^* = \sigma_G a_4\left(-2r^3 + \frac{2}{27}a^3 - \frac{ac}{3} - a\right) + \mu_G,\ J_2^* = \sigma_G a_4\left(2r^3 + \frac{2}{27}a^3 - \frac{ac}{3} - a\right) + \mu_G \tag{7e}$$

Since the distribution information of random variables for BRBSM, **X**, are unknown, $f(\mathbf{X})$ cannot be determined. The exceeding probability P_e defined in Eq. (8) is calculated by assuming Z as a random variable, and then P_e is expressed as:

$$P_e = \text{Prob}\left[Z = G(X) \leq A\right] = \text{Prob}\left[Z_s \leq \frac{A-\mu_G}{\sigma_G}\right] = \text{Prob}\left[Z_s \leq \frac{A}{\sigma_G} - \beta_{2M}\right] = \Phi\left[S^{-1}(u)\right] \tag{8}$$

where $Z_s = (Z-\mu_G)/\sigma_G$ is the standardization of Z; μ_G and σ_G are the mean value and standard deviation of Z, respectively; $\beta_{2M} = \mu_G/\sigma_G$ is the second-moment reliability index; $\Phi(.)$ is the cumulative distribution function of standard normal random variable u; u is a standard normal random variable with the limitation that $\Phi(u) = F(A/\sigma_G-\beta_{2M})$; $F(.)$ is the CDF of Z. With the exceeding probability P_e obtained, the corresponding reliability index is expressed as :

$$\beta = -\Phi^{-1}(P_e) = -S^{-1}\left(\frac{A}{\sigma_G} - \beta_{2M}\right) \tag{9}$$

Table 2 List of test specimens

Number	Specimen	Core Plate Dimensions B×H (mm)	Core Plate Length (mm)	Core Plate Yield load Py (KN)	Restraining Part Dimensions b×h×t (mm)	BRBSM R	BRBSM ω	Number	Specimen	Core Plate Dimensions B×H(mm)	Core Plate Length (mm)	Core Plate Yield load P_y (KN)	Restraining Part Dimensions b×h×t (mm)	BRBSM R	BRBSM ω
1	B-1	176×16	2351	739	200×50×3.2	3.8	2005	51	5	176×16	1644	792	203.2×50×3.2	6.0	1082
2	B-2	176×16	2351	739	200×50×3.2	3.8	933	52	6	176×16	2351	763	203.2×70×3.2	6.2	906
3	C-1	176×16	2351	739	200×50×3.2	3.1	790	53	7	132×12	2351	479	159.2×31×3.2	1.4	275
4	C-2	176×16	2351	739	200×50×3.2	3.1	745	54	8	132×12	2351	479	159.2×53×3.2	4.1	575
5	D-1	176×16	2351	739	200×50×3.2	1.6	577	55	9	132×12	2351	479	159.2×37.5×3.2	2.0	465
6	S0	176×16	2351	739	200×50×3.2	3.2	642	56	10	132×12	2351	479	159.2×37.5×3.2	2.0	547
7	S1	176×16	2351	739	203.2×50×3.2	3.1	657	57	11	132×12	2351	479	159.2×31×3.2	1.4	519
8	S2	176×16	2351	739	203.2×35×3.2	1.6	563	58	12	132×12	2351	479	159.2×53×3.2	4.1	699
9	S3	176×16	2351	739	203.2×25×3.2	0.9	191	59	L450G05S	132×12	2351	443	159.2×45×3.2	3.1	1616
10	G1	176×16	2351	739	203.2×50×3.2	3.1	470	60	L450G05	132×12	2351	443	159.2×45×3.2	3.1	1867
11	BP5S11-2	132×12	2351	467	203.2×40×3.4	2.3	446	61	L650G08S	132×12	3058	443	159.2×57×3.2	3.1	1707
12	BP5S8-2	104×12	2351	368	131.2×40×3.2	2.5	630	62	L650G08	132×12	3058	443	159.2×57×3.2	3.1	1892
13	BP5M11-2	176×16	2351	814	203.2×45×3.2	2.3	487	63	L850G11S	132×12	3765	443	159.2×69×3.2	3.1	1624
14	BP10M8-2	138×16	2351	638	165.2×60×3.2	4.5	989	64	L850G11	132×12	3765	443	159.2×69×3.2	3.1	1436
15	BP5M8-2	138×16	2351	638	165.2.2×45×3.2	2.4	468	65	L520	132×12	2634	495	159.2×50×3.2	2.8	1114
16	BP10M6-2	104×16	2351	481	131.1×58×3.2	4.6	1680	66	L560S	132×12	2775	495	159.2×52.5×3.2	2.8	809
17	BP5M6-2	104×16	2351	481	131.1×42×3.2	2.3	690	67	L560	132×12	2775	495	159.2×52.5×3.2	2.8	1033
18	BP5L5-2	104×22	2351	636	131.1×45×3.2	2.3	950	68	L600	132×12	2917	495	159.2×55×3.2	2.8	1322
19	BP10L4-2	88×22	2351	538	115.2×60×3.2	4.4	824	69	L750S	132×12	3482	495	159.2×64.5×3.2	2.8	932
20	BP5L4-2	88×22	2351	538	115.2×45×3.3	2.4	725	70	L750	132×12	3482	495	159.2×64.5×3.2	2.8	918
21	K1	176×16	2351	780	203.2×50×3.2	3.0	603	71	L450	132×12	2351	495	159.2×45×3.2	2.8	1147
22	K2	176×16	2351	780	203.2×50×3.2	3.1	506	72	L450F65	132×12	2351	484	159.2×45×3.2	2.9	1040
23	K3	176×16	2351	780	203.2×35×3.2	1.5	290	73	L450F80S	132×12	2351	484	159.2×45×3.2	2.9	519
24	K4	176×16	2351	780	203.2×65×3.2	5.2	602	74	L800F50S	132×12	3765	484	159.2×69×3.2	2.9	496
25	M0	50×4.5	1011	55	56×20.6×2	3.3	1182	75	L800F70	132×12	3765	484	159.2×69×3.2	2.9	746
26	U0	50×4.5	1011	55	56×20.6×2	3.0	668	76	L800F90S	132×12	3765	484	159.2×69×3.2	2.9	412
27	M1	50×3.2	1011	33	56×19.3×2	5.0	1055	77	Type A	176×16	2351	766	203.2×61×3.2	4.6	2124
28	U1	50×3.2	1011	33	56×19.3×2	4.7	833	78	Type B	105×16	2351	457	203.2×61×3.2	7.7	1760
29	M2	50×6	1011	74	56×22.1×2	2.6	1518	79	Type C	105×16	2351	457	203.2×61×3.2	7.7	2541
30	U2	50×6	1011	74	56×22.1×2	2.4	541	80	1	84×12	1785	308	159.2×82×3.2	6.4	986
31	U3	50×4.5	1011	55	56×22.6×2	3.8	657	81	2	84×12	1785	308	159.2×106×3.2	11.0	1860
32	U4	50×4.5	1011	55	56×20.6×2	3.0	660	82	3	84×12	1785	298	159.2×82×3.2	6.6	1341
33	P30-05	176×16	2351	842	203.2×52×3.2	3.0	720	83	4	84×12	1785	298	159.2×95×3.2	9.0	2162
34	P30-15	176×16	2351	842	207.5×51.7×3.2	3.0	1655	84	5	84×12	1785	298	159.2×95×3.2	9.0	1264
35	P30-25	176×16	2351	842	207.5×51.7×3.2	3.0	1176	85	6	84×12	1785	298	159.2×95×3.2	9.0	1159
36	P09-05	176×16	2351	842	207.5×26.7×3.2	0.8	475	86	B2	114×19	2351	663		1.5	474
37	P09-10	176×16	2351	842	207.5×26.7×3.2	0.8	1137	87	B4	96×16	2351	568		1.5	492
38	P09-15	176×16	2351	842	207.5×26.7×3.2	0.8	496	88	B4n					1.5	405
39	P62M11	176×16	2351	763	206.4×70×3.2	6.3	918	89	H1	133×19	2351	773		6.5	917
40	P14M4	64×16	2351	310	91.2×30×3.2	1.4	1258	90	H1n					6.5	1248
41	P21L6	132×22	2351	815	159.2×45×3.2	2.1	682	91	H4	112×16	2351	423		6.5	1290
42	P22L6-C2	132×22	2351	815	159.2×45×3.2	2.2	666	92	KB11R1.2G	132×12	2351	611		1.2	1037
43	S1	176×16	2351	786	203.2×50×3.2	2.9	717	93	KB11R1.5G	132×12	2351	298		1.5	966
44	S2	176×16	2351	786	203.2×70×3.2	6.1	1353	94	KH6R6G	114×19	2351	298		5.9	1778
45	S4	176×16	2351	786	203.2×50×3.2	2.8	828	95	6					1.5	764
46	S5	176×16	2351	786	203.2×70×3.2	6.1	819	96	7					5.7	1576
47	1	176×16	2351	792	203.2×50×3.2	2.9	831	97	No.1	132×12	2351	604		1.2	1257
48	2	176×16	1804	852	203.2×40×3.2	2.9	454	98	No.2	133×19	2351	771		1.1	1835
49	3	132	2351	458	159.2×37.5×3.2	2.1	439	99	No.3	132×12	2351	466		1.2	1012
50	4	132	3765	466	159.2×58×3.2	2.0	735	100	No.4	132×12	2351	610		1.2	1060
								101	No.5					2.6	881

3 Results of the calculated exceeding probability under the performance evaluation formula

To calculate the exceeding probability under the evaluation formula of Eq. (1), a total of 101 experimental data[4] with different restraining index and other parameters are collected from the past experiments listed in Table 2. As the data for reliability analysis is not much, we can only divide R into four groups with the range of 0~2, 2~4, 4~6 and 6~ (the last group's range becomes larger because of the same value from the performance evaluation formula.), to confirm each group will contain as much data as possible in the smallest acceptable range for discussion. Each group's sample size and the first four moments are calculated and shown in Table. 3.

The results of each group's fourth-moment reliability index and exceeding probability under the performance evaluation formula calculated by the method of FMNT are shown in Table. 4. The exceeding probability is found to change with the increasing restraining index. Therefore, using this performance evaluation formula as a standard for BRBSM's performance design in applications is not accurate based on reliability requirement. Thus, new boundaries which can confirm specified exceeding probability to evaluate the performance of BRBSM is necessary.

Table 3. The sample size and the first four moment of ω in each group

R	Amount	μ_G	σ_G	α_{3G}	α_{4G}
0~2	22	731.5727	409.4874	1.0082	3.7922
2~4	52	911.2115	427.9156	1.0706	3.1809
4~6	12	1151.3667	512.2965	0.7037	2.2312
6~	15	1368.2267	503.0717	1.1493	3.7109

Table 4. The calculated 4M reliability index and exceeding probability in each group

R	Mean value of performance evaluation formula A	4M reliability index β_{4M}	Exceeding probability P_e
0~2	150	2.2448	0.0124
2~4	450	1.1770	0.1196
4~6	750	0.7290	0.2330
6~	900	0.9027	0.1833

4 Proposed boundaries in exceeding probability of 5% and 10%

Using the moment method of FMNT, the boundary value of cumulative plastic strain energy ratio in terms of restraining index under the exceeding probability of 5% and 10% has been calculated respectively and shown in Table 5.

Based on the value calculated, two boundaries which represent the minimum performance for BRBSM with a relatively stable exceeding probability of 5% and 10% are shown in Fig. 2 and Eq. (10), (11), respectively. From Fig. 2, it can be observed that, the proposed boundaries are with a same shape with the performance evaluation formula: as R's value smaller than 6, ω and R is in a proportional relationship, and as R's value bigger than 6, ω will not change with R's change.

Boundary in exceeding probability of 5%:

$$\omega = \begin{cases} 112R + 93, R \le 6 \\ 765, R > 6 \end{cases} \tag{10}$$

Boundary in exceeding probability of 10%:

$$\omega = \begin{cases} 109R + 157, R \le 6 \\ 811, R > 6 \end{cases} \tag{11}$$

Through the two boundaries, it can be observed that the value above the line can be considered safe with a relatively stable exceeding probability of 5% and 10%, respectively. For engineers in practical application, it will be a more accurate and convenient evaluation method for BRBSM's performance design from the viewpoint of reliability.

Table 5. Value of ω calculated with exceeding probability of 5% and 10%

R	Exceeding probability of 5%		Exceeding probability of 10%	
	ω	P_e	ω	P_e
0~2	207	0.0493	265	0.0997
2~4	378	0.0500	429	0.0992
4~6	426	0.0498	540	0.997
6~	765	0.0498	811	0.0991

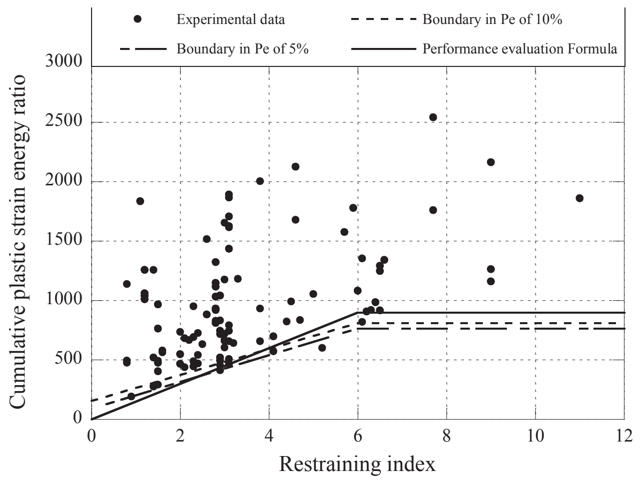

Fig.2 Boundary in specified exceeding probability

5 Conclusions

Based on the previous performance evaluation formula, the relationship between the BRBSM's restraining index and its exceeding probability of cumulative plastic strain energy ratio is investigated in this paper. The conclusions are summarized as follows:

(1) The exceeding probability based on the performance evaluation formula was generally found to change with BRBSM's restraining index, *R*, which is not convenient and accurate enough for the design in practical application.

(2) For the design of BRBSM under the specified exceeding probability of 5% and 10%, two boundaries are presented.

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