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ASSESSING SITE INVESTIGATION PROGRAM FOR SERVICEABILITY DESIGN OF SHALLOW FOUNDATIONS ON SPATIALLY VARIABLE SOIL

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The reliability of a shallow foundation may be tremendously dependent on the spatial variability of the underlying soil. The spatial variability can be modelled as random field. Site investigation can be done to obtain information to calibrate the random field model. The final influence of site investigation on shallow foundation design is seldom studied, which is focused in this paper. A reliability-based design method considering spatial variability is adopted. The effectiveness of a site investigation program is assessed through a series of Monte Carlo simulation. The effects of uncertainty of site investigation and random field parameters are studied. For the examples studied in this paper, there exists an optimal sampling spacing when the number of samples is constant. It is found that, for different numbers of samples, the optimal sampling spacing is of little difference. The optimal sampling spacing is slightly smaller than half of the mean of distribution of scale of fluctuation.

Keywords: Site investigation, random field, shallow foundations, spatial variability.

1 Introduction

Excessive or differential settlement may cause deformation and cracks of the super structures. When evaluating the serviceability of shallow foundations, settlement is often calculated as the summation of the compression of each stratum. Due to the randomness in the natural deposit histories, soil properties are inherently spatially variable even within the same stratum (e.g., Phoon and Kulhawy 1999). To characterize the spatial variability of soil properties, the random field model has been used by a number of researchers (e.g., Huang et al. 2010; Li et al. 2015; Hu et al. 2017). The performance of shallow foundations may be tremendously affected by the spatial variability of the underlying soil (e.g., Kasama et al. 2012). In the reliability-based design of shallow foundation, the spatial variability may be taken into consideration (e.g., Cherubini 2000).

In geotechnical engineering, the random field model should be calibrated. The fundamental parameters of a stationary random field model are mean, standard deviation and scale of fluctuation (SOF) (Vanmarcke 1983). The calibration of random field model calls for statistical inference based the observed data from site investigation. The site investigation program may affect the accuracy of estimating random field calibration and hence the shallow foundation design. It is essential to find an appropriate program that improve the shallow foundation design as much as possible.

Studies of assessing site investigation program for statistical characterization of geotechnical uncertain property have been done by several researchers for waste-management facilities (e.g., Freeze et al. 1992), estimating tunneling-induced ground settlement (e.g., Gong et al. 2014) and so forth. However, the influence of site investigation program on the serviceability design of shallow foundations has not been studied. The goal of this paper is to assess the site investigation program that provides data for design of shallow foundations on spatially variable soil. This paper is organized as follows. First, an example is imported as a model of design of shallow foundations on spatially variable soil. The random field model is used to describe the spatial variability. Then, the effectiveness of site investigation programs is evaluated based on the result from the Monte Carlo simulation. Finally, the optimal site investigation is discussed.

2 Shallow Foundation on Spatially Variable Soil

Shown in Fig. 1(a), consider the shallow foundation model referring to Fenton et al. (1996). The foundation is founded on a soil layer that is ten meters thick. It will support a load P = 1000kN/m from the super structure. The underlying soil is assumed to be elastic. The Poisson ratio v is 0.25. The reliability index target is β = 2.0. The allowable settlement s_{lim} is 0.08 m. The width of the shallow foundation, denoted as B, is evaluated based on reliability design.

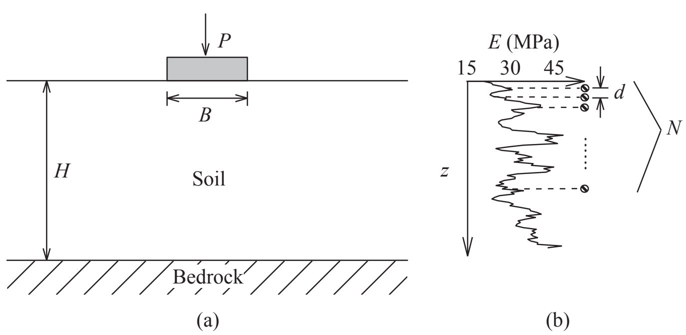

Figure 1. Shallow foundation on spatially variable soil

2.1 *Modelling of spatial variability*

A stationary random field is adopted to model the spatial variability of the elastic modulus. The elastic modulus E is modelled as lognormal random field. The mean and standard deviation of elastic modulus E are denoted as μ_E and σ_E, respectively. The correlation structure of a stationary random field is often described autocorrelation coefficient function (ACF). The isotropic exponential ACF is adopted (e.g., Fenton et al. 1996; Gong et al. 2014):

$$\rho(\tau) = \exp(-2|\tau| / \delta) \tag{1}$$

where τ is the distance between the two points, δ is the SOF, and ρ is the correlation coefficient between the soil properties of two points, also named autocorrelation coefficient. The correlation coefficient between ln$E$ is assumed to follow the ACF as Eq.(1). As an example, Fig. 1(b) illustrates the spatial variability of elastic modulus E along the vertical direction from a lognormal random field where μ_E = 35.0MPa, σ_E = 10.0MPa and δ = 2.0m.

2.2 *Modelling of site investigation*

For simplicity, the samples are assumed to be uniformly taken at a nearby location and the site investigation program can be characterized by the number of samples N and sampling spacing d, as shown in Fig. 1(b). Based on the observed data from site investigation, the maximum likelihood method can be used to obtain the predicted parameters: mean μ_{Ep}, standard deviation σ_{Ep} and SOF δ_p of elastic modulus and calibrate the random field model (Fenton 1999; Juang et al. 2015).

2.3 *Serviceability design*

For the shallow foundation in Fig.1, the empirical method proposed by Fenton et al. (1996) can be adopted as the design equations. This method is based on random elastic finite element analysis. In this method, the settlement are assumed to be lognormally distributed. The statistical property of settlement can be estimated by the empirical formula:

$$\mu_{\ln s} = \ln\left(s_{\det}\right) + \alpha_2 \sigma_{\ln s}^2 \tag{2}$$

where s is the settlement of shallow foundation; $s_{\det}$ is the settlement computed from deterministic analysis where the soil is homogenous with $E \equiv \mu_E$, which can be approximated via an empirical formula; $\mu_{\ln s}$ and $\sigma_{\ln s}$ are the mean and standard deviation of the logarithm of s, respectively; α_2 is an empirical coefficient. $\sigma_{\ln s}$ can be computed by:

$$\sigma_{\ln s} = \sqrt{\gamma\left(B, H; \delta\right)}\sigma_{\ln E} \tag{3}$$

where $\gamma(\cdot)$ is the variance reduction function; B and H are the width of the foundation and thickness of the soil layer, respectively; δ is the aforementioned SOF in Eq. (1). The computation of $s_{\det}$, α_2 and $\gamma(\cdot)$ refers to Eq.(7), Eq.(8), Eq.(10) and Eq.(11) in Fenton et al. (1996). Given a shallow foundation width B, the distribution of settlement can be estimated based on the above empirical method. Then, the reliability index can be computed. Otherwise, the shallow foundation width B can be obtained with a given reliability index or probability that the settlement exceeds the allowable settlement. Hence, the reliability-based design can be carried out.

3 Measurement of Effectiveness of Site Investigation

The site information can be characterized with μ_E, σ_E and δ. For given μ_E, σ_E and δ, an actual demand value of shallow foundation width B_d can be computed via the above method. Otherwise, the designed shallow foundation width B_p can be computed based on μ_{Ep}, σ_{Ep} and δ_p. The difference between the design and the actual demand may be used to assess the effectiveness of site investigation:

$$\Delta B = B_p - B_d \tag{4}$$

It can be seen that $\Delta B > 0$ indicates overdesign while $\Delta B < 0$ indicates unsatisfied reliability index.

For ease of illustration, an example is presented here. Assume the random field parameters are μ_E =35.0MPa, σ_E = 15.0MPa and δ = 2.0m, respectively, the actual demand value of shallow foundation width is calculated B_d = 1.20m. Then, with a given program that N = 30 and d = 0.2m, the observed data can be generated artificially from the random field to simulate the site investigation, as shown in Fig. 1(b). Afterwards, using data obtained from site investigation μ_{Ep}, σ_{Ep} and δ_p can be estimated: μ_{Ep} =34.8MPa, σ_E = 12.8MPa and δ = 1.8m. The designed shallow foundation width B_p = 1.18m can be computed with μ_{Ep}, σ_{Ep} and δ_p. Finally, we can get ΔB = -0.02m.

For the observed data from site investigation is uncertain, μ_{Ep}, σ_{Ep}, and δ_p are all random variables. Hence, the design width is also uncertain. Through Monte Carlo simulation, the samples of B_p and ΔB can be obtained and the distribution of ΔB can be estimated. Fig. 2 shows the histogram of ΔB when N = 30 and d = 0.2m. In Fig. 2(a), the random field parameters are known: μ_E = 35.0MPa, σ_E = 15.0MPa and δ = 2.0m.

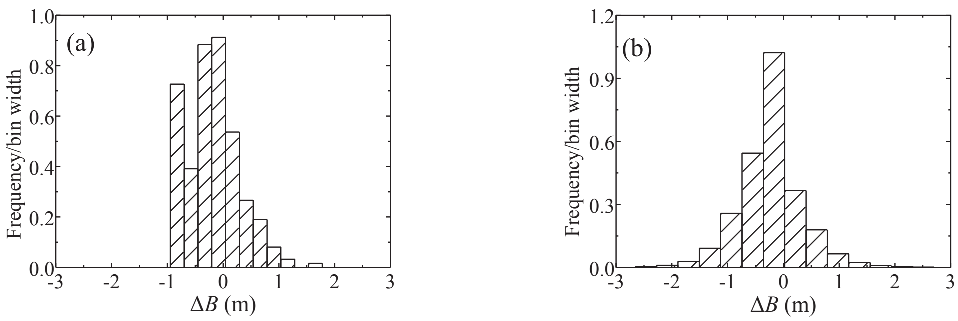

Figure 2. Histogram of ΔB: (a) the random field parameters is known; (b) the random field parameters is unknown

In the above case it is assumed that the random field parameters μ_E, σ_E and δ is known. However, they are usually unknown. To consider the uncertainty of μ_E, σ_E and δ, they can be modelled as random. The distribution of these random variables represents the experience from similar engineering project or database. In this paper, μ_E, σ_E and δ are assumed to be uniformly distributed. They are statistically independent, which is modelled as $\mu_E \sim U(30.0,40.0)$MPa, $\sigma_E \sim U(10.0,20.0)$MPa and $\delta \sim U(1.0,3.0)$m, respectively.

The actual demand value of shallow foundation width B_d also becomes a random variable. By generating μ_E, σ_E and δ sampled from these distributions and repeating the calculation mentioned above, the distribution of B_d and conditional distribution of B_p can be estimated. After that, the marginal distribution of ΔB can be obtained based on total probability theory. Fig. 2(b) shows the histogram of ΔB when the random field parameters are unknown. It can be seen that the distribution is single-peak and more symmetric, which is quite different from Fig. 2(a). As it is preferred that ΔB is closer to zero, the root-mean-square error (RMSE) of ΔB may be employed as a criterion to assess site investigation program.

$$\text{RMSE} = \sqrt{E\left(\left(\Delta B_i - 0\right)^2\right)} \tag{5}$$

4 Results and Discussions

4.1 *Effects of uncertainty of site investigation*

First, the random field parameters are assumed to be known: μ_E = 35.0MPa, σ_E = 15.0MPa and δ = 2.0m, i.e. the uncertainty of random field parameters is eliminated and the uncertainty of site investigation is mainly focused.

The RMSE as a function of sampling spacing is illustrated in Fig. 3(a). As expected, when N is larger, the RMSE is generally smaller. When $N = 10$, the RMSE value is minimized when $d = 0.75$m. When N is larger, the optimal point is similar. If N is the same, too large or too small sampling spacing is not the best choice. According to Eq. (1), if the sampling spacing is too large, the correlation may be too weak to extract appropriate information to calibrate the random field. Otherwise, too small sampling spacing may lead to insufficient domain of site investigation. The optimal sampling spacing seems not sensitive to the number of samples.

The RMSE of different pre-known SOF δ values is shown in Fig. 3(b). The marker of each optimal point is filled with solid color. It can be seen that the optimal sampling spacing varies with different pre-known SOF. The effectiveness of site investigation may be highly dependent on the SOF. For instance, if a program with $N = 50$ and $d = 0.4$m is decided, this program is the best one when $\delta = 1$m, where RMSE = 0.36 m. However it will lead to RMSE = 0.59 m when $\delta = 3$m. Hence, the uncertainty of random field parameters should not be neglected.

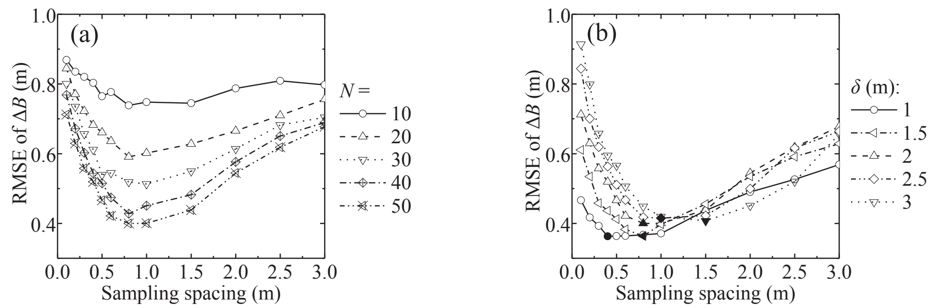

Figure 3. RMSE of ΔB: (a) $\delta = 2$m; (b) $N = 50$

4.2 *Effects of uncertainty of random field parameters*

Then the joint distribution of μ_E, σ_E and δ is imported to model the uncertainty of random field parameters. The RMSE as a function of sampling spacing is illustrated in Fig. 4(a). From $N = 10$ to 50, the shape of each curve is similar. The RMSE is generally larger than the result in 4.1. There is one optimal point that minimize RMSE between $d = 0.75$ and $d = 1$m, similar to the result in 4.1.

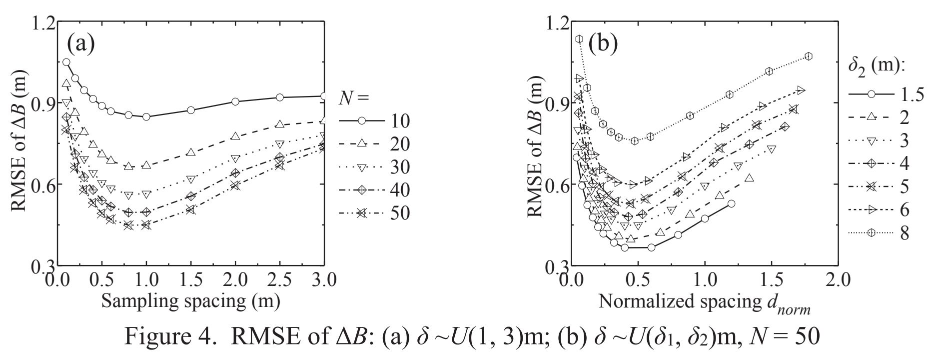

Figure 4. RMSE of ΔB: (a) $\delta \sim U(1, 3)$m; (b) $\delta \sim U(\delta_1, \delta_2)$m, $N = 50$

As is discussed above, the optimal point is related to the value of SOF. Here the distribution of SOF is assumed to be $\delta \sim U(\delta_1 = 1\text{m}, \delta_2)$. As is shown in Fig. 4(b), N is 50 constantly and the horizontal axis is scaled with normalized spacing $d_{norm}=d/\delta_m$ (where $\delta_m = (\delta_1+\delta_2)/2$ is the mean of prior SOF distribution). It can be found that the optimal points of different δ_2 stand quite close. The minimum value of RMSE appears while the normalized spacing is equal to or slightly smaller than 0.5, i.e. the half of the mean of distribution of δ.

5 Concluding Remarks

This paper assesses the site investigation program for shallow foundations on spatially variable soil via Monte Carlo simulation. For the examples studied in this paper, there exists an optimal sampling spacing when the number of samples is constant. The optimal sampling spacing is not sensitive to the number of samples. It is generally slightly smaller than half of the mean value of the distribution that models the uncertainty of scale of fluctuation.

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