Proc. of the 6th Intl. Symposium on Reliability Engineering and Risk Management (6ISRERM)
31 May – 1 June 2018, Singapore
Editor(s) Xudong Qian, Sze Dai Pang, Ghim Ping Raymond Ong, Kok-Kwang Phoon

Geotechnical Full Probabilistic Design Using Monte Carlo Simulation Methods in EXCEL Spreadsheet

Guo-Hui Gao[1], Zi-Jun Cao[1], Dian-Qing Li[1], and Yu Wang[2]

[1] *State Key Laboratory of Water Resources and Hydropower Engineering Science, Institute of Engineering Risk and Disaster Prevention, Wuhan University, 8 Donghu South Road, Wuhan 430072, P. R. China. E-mail: zijuncao@whu.edu.cn (corresponding author)*

[2] *Department of Architecture and Civil Engineering, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong*

Although several semi-probabilistic design approaches (e.g., load and resistance factor design methods and partial factor design methods) for geotechnical structures have been developed and implemented around the world, their development and implementation for general purposes of geotechnical designs (e.g., retaining structures) remain a challenging task. This is, at least partially, due to difficulties in dealing with multiple failure modes, spatial variability, and the correlation between load and resistance in geotechnical design practice. Such difficulties can be tackled with relative ease using full probabilistic design approaches, e.g., Monte Carlo simulation (MCS)-based design, which is, however, often criticized because of the hurdle of reliability algorithms involved in uncertainty modeling and propagation. This paper implements MCS-based full probabilistic design methods in an EXCEL spreadsheet to remove the hurdle of reliability algorithms by developing an EXCEL add-in, i.e., Geotechnical Reliability-based Design with Monte Carlo Simulation (GeoRBD/S). To improve the practicality, the implementation of MCS-based full probabilistic design with GeoRBD/S is deliberately divided into three parts, including uncertainty modeling, deterministic modeling, and uncertainty propagation. For illustration, the GeoRBD/S add-in is applied to designing a sheet pile wall example.

Keywords: Full probabilistic design, MCS, GeoRBD/S, Sheet pile wall, Spreadsheet.

1 Introduction

During the past two decades, several semi-probabilistic design approaches (e.g., load and resistance factor design (LRFD), partial factor design, Quantile Value Method (QVM) (Ching and Phoon, 2011), and Robust LRFD (Gong et al., 2016)) for geotechnical structures have been developed around the world. Although semi-probabilistic design approaches are a preferred method in prevailing geotechnical design codes (e.g., CAN/CSAS614, 2014; Eurocode 7, 2004), development and implementation of semi-probabilistic design approaches for general purposes of geotechnical designs (e.g., retaining structures) remain a challenging task. This is, at least partially, attributed to difficulties in dealing with multiple failure modes, spatial variability, and the correlation between load and resistance in geotechnical engineering. These difficulties can be tackled with relative ease under a full probabilistic design framework, such as Expanded reliability-based design (RBD) using direct Monte Carlo simulation (MCS) (Wang et al., 2011a; Wang, 2011, 2013) or Subset Simulation (SS) (Wang et al., 2011b; Wang and Cao, 2013; Li et al., 2016). However, the full probabilistic design approach is often criticized because of the hurdle of reliability algorithms involved in uncertainty modeling and propagation.

This paper implements MCS-based full probabilistic design methods in an EXCEL spreadsheet to remove the hurdle of reliability algorithms by developing an EXCEL add-in, i.e., Geotechnical Reliability-based Design with Monte Carlo Simulation (GeoRBD/S). The GeoRBD/S add-in allows performing geotechnical designs in EXCEL spreadsheet using direct MCS, SS, and Generalized Subset Simulation (GSS). For illustration, the three full-probabilistic design approaches are applied to designing a sheet pile wall example and their design results are compared for cross-validation.

2 MCS-based Full Probabilistic Design in Excel Spreadsheet

In general, geotechnical full probabilistic design consists of three steps: (i) determine a calculation model for deterministic analysis, random variables or systems parameters, and possible designs based on the engineering experience and judgments; (ii) calculate the failure probability of each possible design using a reliability method (e.g., direct MCS, SS, or GSS); and (iii) determine the optimal design based on the target failure probability p_T and economically-optimized limit state (Wang et al., 2011a). In this paper, three MCS-based full probabilistic design methods (i.e. Expanded RBD based on direct MCS, Expanded RBD based on SS, and RBD based on GSS) are implemented in a spreadsheet environment by a package of worksheets and add-ins, i.e., GeoRBD/S. The implementation is divided into three parts: uncertainty modeling, deterministic modeling, and uncertainty propagation, which are deliberately decoupled from each other to remove the hurdle of reliability algorithms involved in full probabilistic geotechnical designs. The three parts of the spreadsheet implementation are linked together through some input-output (I-O) cells in EXCEL worksheet. By this means, the full probabilistic design can proceed as an extension of the deterministic design. The next three subsections briefly introduce uncertainty modeling, deterministic modeling, and uncertainty propagation using GeoRBD/S, respectively.

2.1 Uncertainty Modeling

An uncertainty model worksheet is developed to define and simulate uncertain system parameters that are treated as random variables in the full probabilistic design. Possible designs in design space are also generated in the worksheet by GeoRBD/S using Visual Basic Application (VBA) function. From the I-O respective, the uncertainty model worksheet takes no input but returns a set of random samples and possible designs as output to the deterministic model worksheet. Details of generating random samples in the uncertainty model worksheet are referred to Au et al. (2010) and Wang and Cao (2013).

2.2 Deterministic Modeling

Deterministic modeling is the process of calculating the system response (e.g., FS) concerned for a given set of system parameters. The calculation process of the deterministic model is implemented in a series of worksheets assisted by VBA functions (Wang and Cao, 2013). From the I-O perspective, the deterministic analysis worksheets take a given set values as input to calculate the system response as output for uncertainty propagation. No probability concept is involved in deterministic model worksheets, and it can be developed by practitioners without the background of probability theory and statistics.

When the deterministic model worksheet and the uncertainty model worksheet are accomplished, they are linked together through I-O cells to performing the probability analysis and design. The connection is carried out by simply setting the input cells in the deterministic model worksheet to be the output cells in the uncertainty model worksheet in Excel. After that, the values of uncertain system parameters shown in the deterministic worksheet are equal to those generated in the uncertainty model worksheet, and the values of the geotechnical system response calculated in the deterministic model worksheet are random.

2.3 Uncertainty Propagation by MCS Methods

When the deterministic model worksheet and uncertainty model worksheets are completed and linked together, the direct MCS, SS, and GSS are invoked for uncertainty propagation. The algorithms of the three methods are introduced below.

2.3.1 Expanded RBD Based on Direct MCS

Expanded RBD can be viewed as an augmented reliability analysis of a geotechnical system, in which a set of design parameters are artificially considered as uncertain with probability distributions specified by the user for design exploration purposes (Wang et al., 2011a; Wang, 2011). Consider, for example, the embedded sheet pile wall with an embedded depth of D. The design process aims to find the D value that satisfies design requirements (i.e., p_T). In the context of Expanded RBD, the design parameters D is considered as a discrete uniform random variable with a probability mass function $p(D)$. The design process of the sheet pile wall then is formulated as a process of calculating the failure probability of designs with different D values (i.e., the conditional probability $p(\text{Failure}|D)$) and comparing them with p_T. Using the Bayes' Theorem, the $p(\text{Failure}|D)$ is given by (Wang et al. 2011a; Wang, 2011, 2013):

$$p\left(\text{Failure} \mid D\right) = \frac{p\left(D \mid \text{Failure}\right)}{p\left(D\right)} p_f \tag{1}$$

in which $p(D|\text{Failure})$ is the conditional probability of D given failure. Since D is a discrete uniform random variable, $p(D)$ is taken as equal to $1/n_D$, where n_D is the number of possible values for D. The values of $p(D|\text{Failure})$ and p_f in Eq. (1) can be estimated using a single run of direct MCS. Details of Expanded RBD based on direct MCS are referred to Wang et al. (2011a).

2.3.2 Expanded RBD Based on SS

The $p(D|\text{Failure})$ and p_f on the right-hand side of Eq. (1) for expanded RBD can also be evaluated using SS, which provides an efficient simulation algorithm to explore target failure domains. SS converts a small failure probability into a product of a sequence of relativity large conditional probabilities by introducing intermediate events adaptively, and employs specially designed Markov chains to generate conditional samples of these intermediate events until the target failure domain is achieved (Au et al., 2010; Wang and Cao, 2013; Li et al., 2016). Compared with the direct MCS, SS efficiently generates a large number of failure samples to improve the accuracy of $p(D|\text{Failure})$ and p_f estimated from failure samples, which subsequently leading to significant improvement in accuracy and resolution of estimated $p(\text{Failure}|D)$ in Eq. (1). When the failure probabilities of all designs are calculated using the Expanded RBD based on SS, the feasible designs are again selected by comparing failure probabilities of different designs with the target failure probability. Details of Expanded RBD based on SS are referred to Wang and Cao (2013).

2.3.3 RBD Based on GSS

Generalized Subset Simulation, which is developed from the Subset Simulation (Li et al., 2015), allows, efficiently and simultaneously, estimating failure probabilities of multiple failure events in a single simulation run. Using GSS, each possible design D in design space can be viewed as an equivalent failure event. The failure probabilities of all possible designs in design space can be calculated using a single GSS run without the need of Bayesian analysis in Eq. (1). The GSS explores the design space progressively from designs with relatively large failure probabilities to those at small probability levels. Hence, the GSS run can be stopped as the failure probability level is less than p_T to avoid unnecessary exploration in the feasible design domain, which leads to considerable computational saving. Interested readers can refer to Li et al (2015, 2017) for detailed algorithms of GSS.

The Expanded RBD based on direct MCS, Expanded RBD based on SS and RBD based on GSS are implemented in GeoRBD/S add-in through three user-forms. For example, Figure 1 shows the GSS user-form. The upper four input fields of the user-form (e.g., the number of GSS runs, the number of samples per level N, conditional probability p_0 from one level to the next level, and the target reliability index β_T) control the total number of the samples generated by GSS. The middle four input fields of the user-form record the design parameters and possible designs, the random variables, their PDF values and the driving variables, respectively. The lower three input fields of the user-form record the variables V of interest for some design and its failure modes during the simulation. After setting up the user-form, GSS procedure can be performed by clicking the "Run" button.

3 Illustration Example

For illustration, this section redesigns an embedded sheet pile wall example using the GeoRBD/S. As shown in Figure 2, the embedded sheet pile wall is designed for a 3-m deep excavation and is installed in a sand layer. The aim of the sheet pile wall design is to find an embedded depth d that satisfies the moment equilibrium about point O and to determine an additional embedded depth Δd by solving the horizontal force equilibrium equation (Wang, 2013). For simplification, Δd is commonly taken as $0.2d$ (e.g., Craig, 2004; Wang, 2013; Li et al., 2016). Then, the required depth D of the sheet pile wall example is equal to $1.2d$, and here it ranges from 1 to 8m with an increment of 0.1m. For a given D value, d and Δd are calculated, and the net resistance moment M_R about point O provided by passive earth pressure is evaluated, as well as the net overturning moment M_O resulted from the active pressure acting. After that, the FS is obtained, details of which are referred to Craig (2004) and Wang (2013).

Figure 3 shows the failure probabilities of sheet pile wall design example estimated from direct MCS, SS, and GSS by the GeoRBD/S add-in through solid lines with squares, circles, and triangles, respectively. The number of samples per level N, conditional probability, and the target reliability index in GSS are taken as 2000, 0.1, and 3.8, respectively. In this example, only 9193 random samples are generated in GSS, but 55000 random samples are used in SS and 10,000,000 random samples are used in direct MCS. The three lines are almost overlapped with each other. The minimum feasible design of D estimated from the Expanded RBD based on direct MCS, Expanded RBD based on SS, and RBD based on GSS are 8.0, 8.0, and 7.9m, respectively. The design results obtained from the three approaches are in a good agreement.

Figure 4 shows the failure probabilities of sheet pile wall designs estimated from the GSS by a solid line with triangles, where $N = 2000$, $p_0 = 0.1$, and $\beta_T = 2.0$. When D values range from 1.0 to 5.3m, the failure probabilities of designs are greater than the p_T and the failure probabilities estimated from GSS are consistent with those estimated by Expanded RBD based on direct MCS with 10,000,000 random samples (see the solid line with squares). When D is greater than 5.3 m, the failure probability level of designs is less than the target failure probability, which means the feasible design domain has been arrived in GSS. The RBD based on GSS does not provide estimates of failure probabilities in the feasible failure domain in this example because the GSS run is terminated based on the prescribed target reliability level (e.g., $\beta_T = 2.0$) to avoid unnecessary explorations in the feasible design domain, which provides considerable

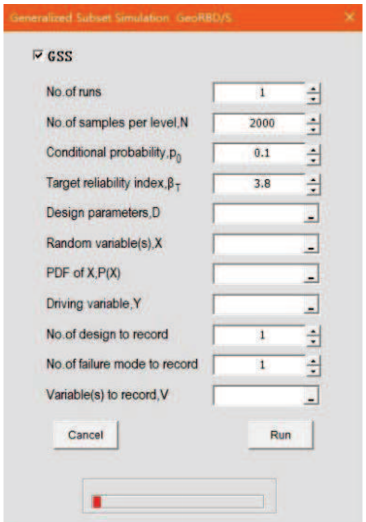

Figure. 1 Generalized Subset Simulation user-form

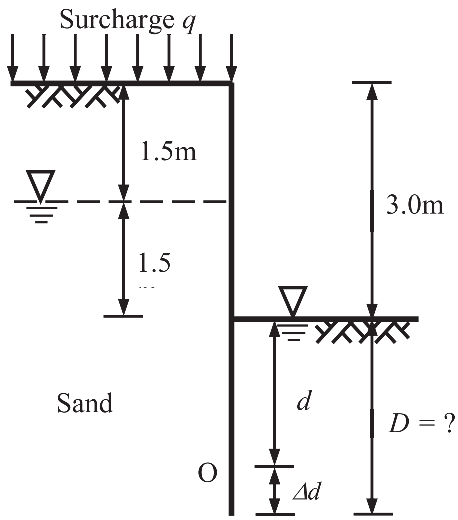

Figure. 2 Embedded sheet pile wall design example (after Craig, 2004; Wang, 2013; Li et al., 2016)

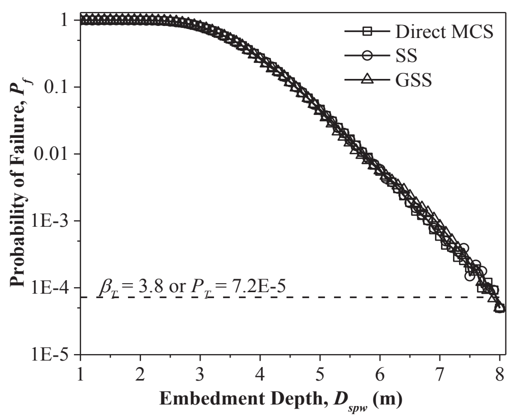

Figure. 3 Failure probabilities of the sheet pile wall design example estimated from different design approaches for $\beta_T = 3.8$

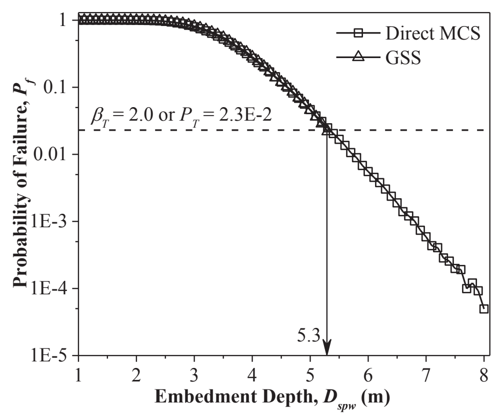

Figure. 4 Failure probabilities of sheet pile wall design example estimated from GSS for $\beta_T = 2.0$ and direct MCS

computational saving because the designs in feasible design domain usually have relatively small failure probabilities. Among the three RBD approaches implemented in GeoRBD/S add-in, the RBD approach based on GSS is the most efficient one, but Expanded RBD based on direct MCS is the most simple and robust approach.

4 Summary and Conclusions

This paper presented an EXCEL spreadsheet-based approach for full probabilistic geotechnical design. An RBD add-in, named Geotechnical Reliability-based Design with Monte Carlo Simulation (GeoRBD/S), was developed to implement Expanded RBD based on direct MCS, Expanded RBD based on SS, and RBD based on GSS. With the GeoRBD/S add-in, the major procedures of full probabilistic geotechnical design using MCS are deliberated decoupled into three parts (including uncertainty modeling, deterministic modeling, and uncertainty propagation) so that they can be implemented by personals with different expertise. This removes the hurdle of reliability algorithms involved in uncertainty modeling and propagation. For illustration, the GeoRBD/S add-in is applied to a sheet pile wall design example. Results showed that the three approaches provide consistent design results. It was shown that the RBD approach based on GSS is the most efficient one, but Expanded RBD based on direct MCS is the most simple and robust approach. More importantly, with the GeoRBD/S add-in, the user can implement the MCS-based full probabilistic design approaches with relative ease. By this means, the practicality of full probabilistic geotechnical design approaches is significantly improved.

Acknowledgments

This work was supported by the National Key R&D Program of China (Project No. SQ2016YFSF060351), the National Natural Science Foundation of China (Project Nos. 51679174, 51579190, 51528901). The financial support is gratefully acknowledged.

References

Au, S. K., Cao, Z. J. and Wang, Y., Implementing Advanced Monte Carlo Simulation under Spreadsheet Environment, *Structural Safety*, 32(5), 281-292, Sep, 2010.

CAN/CSAS614:2014, Canadian Highway Bridge Design Code, Mississauga, ON, Canadian Standards Organization.

Ching, J. and Phoon, K. K., A Quantile-based Approach for Calibrating Reliability-based Partial Factors, *Structural Safety*, 33, 275–285, Jul, 2011.

Craig, R. F., *Craig's Soil Mechanics*, Taylor & Francis, London, 2004.

EN 1997–1:2004, Eurocode 7: Geotechnical Design – Part 1: General Rules. Brussels, European Committee for Standardization (CEN).

Gong, W., Khoshnevisan, S., Juang, C. H. and Phoon, K. K., R-LRFD: Load and Resistance Factor Design Considering Design Robustness, *Computers and Geotechnics*, 74, 74–87, Apr, 2016.

Li, D. Q., Yang, Z. Y., Cao, Z. J., Au, S. K., and Phoon, K. K., System Reliability Analysis of Slope Stability Using Generalized Subset Simulation, *Applied Mathematical Modelling*, 46, 650-664, Jun, 2017.

Li, D. Q., Shao K. B., Cao Z. J., Tang, X. S. and Phoon, K. K., A Generalized Surrogate Response Aided-Subset Simulation Approach for Efficient Geotechnical Reliability-based Design, *Computers and Geotechnics*, 74, 88-101, Apr, 2016.

Li, H. S., Ma, Y. Z., and Cao, Z. J., A Generalized Subset Simulation Approach for Estimating Small Failure Probabilities of Multiple Stochastic Responses, *Computers & Structures*, 153, 239-251, Jun, 2015.

Wang, Y. and Cao, Z. J., Expanded Reliability-based Design of Piles in Spatially Variable Soil Using Efficient Monte Carlo Simulations, *Solis and Foundations*, 53(6), 820-834, Dec, 2013.

Wang, Y., MCS-based Probabilistic Design of Embedded Sheet Pile Walls, *Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards*, 7(3), 151-162, Mar, 2013.

Wang, Y., Au, S. K. and Kulhawy, F. H. Expanded Reliability-based Design Approach for Drilled Shafts, *ASCE Journal of Geotechnical and Geoenvironmental Engineering*, 137 (2), 140–149, Feb, 2011a.

Wang, Y., Cao, Z. J. and Au, S. K., Practical Reliability Analysis of Slope Stability by Advanced Monte Carlo Simulations in Spreadsheet, *Canada Geotechnical Journal*, 48(1), 162-172, Dec, 2011b.

Wang, Y., Reliability-based Design of Spread Foundations by Monte Carlo Simulations, *Géotechnique*, 61(8), 677-685, Aug, 2011.