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RELIABILITY BASED CONTROL GAIN DESIGN OF SEISMIC STRUCTURES WITH MR DAMPERS

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Abstract: A physically based model of stochastic earthquake ground motion is first introduced, which takes into account the randomness inherent in the earthquake ground motion. In order to quantify the effect of the randomness achieving the optimal control of seismic structures, a probabilistic criterion involving function of structural reliability is employed. In conjunction with the bounded Hrovat algorithm, a reliability based control gain design is applied in the semi-active control of seismic structures with MR dampers, of which just the viscous damping coefficient and the maximum Coulomb force need to be defined. By tracing the optimal active control force and the relevant system state, the viscous damping coefficient and maximum Coulomb force are then defined. For illustrative purposes, a single-degree-of-freedom system of structure with MR damper subjected to random seismic ground motions is investigated. Numerical example shows that the safety of controlled structure gains a significant enhancement using the reliability based control gain design of MR damper.

Keywords: Stochastic seismic model, Reliability criteria, MR damper, bounded Hrovat algorithm.

1 Introduction

Magnetorheological (MR) damping control is regarded as the most promising manner for implementing semi-active and intelligent control modalities owing to the perfect dynamic damping behaviors of the MR damper (Casciati et al. 2006; Peng et al. 2016). The challenging issue, however, of the MR damping control is the design and optimization of control gain, which relies upon the structural state relevant to the external excitation and the demanded structural performance that the control policy is expected to achieve.

There is a common knowledge in earthquake engineering community that the earthquake ground motion arises to be a random process. However, this is not received sufficient attention in the design of structural control of seismic structures. Only a few of seismic acceleration records are utilized that ignores the randomness inherent in the seismic ground motion. In this study, a physically based model of stochastic earthquake ground motion is first introduced. In order to quantify the effect of randomness on the control gain of seismic structures with MR dampers, a probabilistic criterion involving function of structural reliability is employed. Since the bounded Hrovat algorithm, a well-recognized control gain format of semi-active control, includes the correlated parameters to be defined, such as viscous damping coefficient, maximum Coulomb force and optimal active control force, the reliability based probabilistic criterion is just posed on the stochastic optimization of active control force. By tracing the optimal control force and the relevant system state, the viscous damping coefficient and maximum Coulomb force are then defined. For illustrative purposes, a single-degree-of-freedom system of structure with MR damper subjected to random seismic ground motions is investigated.

2 Modeling of Stochastic Seismic Ground Motion

Considering the randomness inherent in the seismic sources, propagation paths, and site soil, the physically motivated stochastic ground motion model is re-visited in the present paper. For the sake of clarity, the local engineering site is modeled as a single-degree-of freedom (SDOF) system. The stochastic ground motion model in the frequency domain is thus given by (Li and Ai 2006)

$$\ddot{X}_g(\mathbf{\Theta},\omega) = H_g\left(\Theta_{\bar{\omega}_0},\ \Theta_{\bar{\varsigma}},\ \omega\right)\cdot \ddot{U}_b(\mathbf{\Theta}_b,\omega) \tag{1}$$

$$H_g\left(\Theta_{\bar{\omega}_0},\ \Theta_{\bar{\varsigma}},\ \omega\right) = \frac{\Theta_{\bar{\omega}_0}^2 + 2i\Theta_{\bar{\omega}_0}\Theta_{\bar{\varsigma}}\omega}{\Theta_{\bar{\omega}_0}^2 - \omega^2 + 2i\Theta_{\bar{\omega}_0}\Theta_{\bar{\varsigma}}\omega} \tag{2}$$

where $\ddot{X}_g(\mathbf{\Theta},\omega)$ and $\ddot{U}_b(\mathbf{\Theta}_b,\omega)$ denotes the frequency domain expressions of ground motions at the engineering site and bedrock, respectively. $\mathbf{\Theta} = \left\{\Theta_{\bar{\omega}_0},\ \Theta_{\bar{\varsigma}},\ \omega\right\}$ denotes the random vector characterizing the randomness involved in the ground motion at the surface of the engineering site; $\Theta_{\bar{\omega}_0}$, $\Theta_{\bar{\varsigma}}$ denote the parameters indicate the randomness of the predominant frequency of the engineering site $\overline{\omega_0}$ and the equivalent damping ratio $\bar{\varsigma}$, respectively. $\mathbf{\Theta}_b = \left\{\Theta_{b,j}\right\}_{j=1}^{s_b}$ denotes the random vector characterizing the randomness involved in the ground motion at the bedrock from the properties of the sources and the propagation path, with s_b being the number of the random variables involved at this stage. $H_g\left(\Theta_{\bar{\omega}_0},\ \Theta_{\bar{\varsigma}},\ \omega\right)$ denotes a frequency transfer function, in which ω denotes the circular frequency. And i denotes the unit of the imaginary number $\sqrt{-1}$.

The time history of the ground motions could then be obtained by the following inverse Fourier transformation:

$$\ddot{x}_g(\mathbf{\Theta},t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \ddot{X}(\mathbf{\Theta},\omega)e^{i\omega t}d\omega \tag{3}$$

where the probability distribution and statistical parameters of the two critical variables, $\overline{\omega_0}$ and $\bar{\varsigma}$, are identified according to the seismic acceleration records collected from a certain type of engineering site. In the present paper, $\overline{\omega_0}$ and $\bar{\varsigma}$ are assumed as being mutually independent random variables and both admit a log-normal distribution. The mean of $\overline{\omega_0}$ and $\bar{\varsigma}$ are 12 rad/s and 0.1 with the coefficients 0.42 and 0.35, respectively. Meanwhile, the initial phase angle of the inverse Fourier transformation of the ground motion is treated to admit normal distribution with π and 1.2 of mean and coefficient values, respectively.

Using the tangent spheres method (Chen and Li 2008), a collection of 221 representative points with corresponding assigned probabilities are selected. The sampling frequency is 50 Hz, and the time-domain duration of the ground motions is 20.48 s. To establish a non-stationary variation of the intensity of the seismic acceleration, a uniform modulation function is employed (Li and Chen 2009). The peak accelerations of the samples are 0.11g.

3 Control Gain of Stochastic Optimal Control

Stochastic optimal control involves the maximization or minimization of the specified cost function. The generalized form of the cost function is typically the quadratic combination of displacement, velocity, acceleration and control force. A standard quadratic cost function can be expressed as follows:

$$J_1\left(\mathbf{Z},\mathbf{U}_\alpha,\mathbf{\Theta}\right)=\frac{1}{2}\mathbf{Z}^{\mathrm{T}}\left(t_f\right)\mathbf{P}\left(t_f\right)\mathbf{Z}\left(t_f\right)+\frac{1}{2}\int_{t_0}^{t_f}\left[\mathbf{Z}^{\mathrm{T}}\left(t\right)\mathbf{QZ}\left(t\right)+\mathbf{U}_\alpha^{\mathrm{T}}\mathbf{RU}_\alpha\right]\mathrm{d}t \tag{4}$$

where $\mathbf{Z}=[\mathbf{X},\dot{\mathbf{X}}]^{\mathrm{T}}$ denotes the state vector of the structure system; $\mathbf{U}_a=\left\{U_{a,i}\left(\cdot\right)\right\}_{i=1}^{m}$ denotes the optimal control force; $\mathbf{Q}=\left\{Q_{a,i}\left(\cdot\right)\right\}_{2n_d\times 2n_d}$ denotes a positive definite matrix; t_0 and t_f denotes the starting and terminal time, respectively. The minimization of the random variable J_1 in Eq. (4) is the system state globally optimized in the case of given control parameters $\mathbf{Q}$ and $\mathbf{R}$, of which the solution can be expressed as follows:

$$\mathbf{U}_a\left(\Theta,t\right)=-\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{PZ}\left(\Theta,t\right)=-\mathbf{GZ}\left(\Theta,t\right) \tag{5}$$

where $\mathbf{B}=\left[B_{ij}\right]_{2n_d\times m}$ denotes the control influence matrix; $\mathbf{G}$ denotes the control gain matrix; and $\mathbf{P}=\left[P_{ij}\right]_{2n_d\times m}$ denotes the Riccati matrix, which can be solved from the following Riccati equation(Li et al. 2010):

$$\mathbf{PA}+\mathbf{A}^T\mathbf{P}-\mathbf{PBR}^{-1}\mathbf{B}^T\mathbf{P}+\mathbf{Q}=\mathbf{0} \tag{6}$$

As indicated previously, the control parameters $\mathbf{Q}$ and $\mathbf{R}$ need to be determined so as to implement the Riccati control. Utilizing the exceedance probability criterion with the constraint of acceleration, a better system performance through structural control can be achieved, which is expressed as follows (Li et al. 2011):

$$J_2=\frac{1}{2}\left[\mathrm{Pr}_{\overline{Z}}^{\mathrm{T}}\left(\overline{Z}-\overline{Z}_{\mathrm{thd}}>0\right)\mathbf{Q}_{\overline{Z}}\mathrm{Pr}_{\overline{Z}}\left(\overline{Z}-\overline{Z}_{\mathrm{thd}}>0\right)+\mathrm{Pr}_{\overline{U}}^{\mathrm{T}}\left(\overline{U}-\overline{U}_{\mathrm{thd}}>0\right)\mathbf{R}_{\overline{U}}\mathrm{Pr}_{\overline{U}}\left(\overline{U}-\overline{U}_{\mathrm{thd}}>0\right)\right]+H\left(\overline{A}_{\max}-\overline{A}_{\mathrm{con}}\right) \tag{7}$$

where $\overline{Z}$ and $\overline{U}$ denote the equivalent extreme-value of Z and U, respectively (Li et al. 2007). $\overline{Z}_{\mathrm{thd}}$ and $\overline{U}_{\mathrm{thd}}$ denote the thresholds of $\overline{Z}$ and $\overline{U}$, respectively.

4 Semi-Active Control of Seismic Structures with MR Dampers

For the randomly base-excited structures controlled semi-actively with MR dampers, the equation of motion can be expressed as follows:

$$\mathbf{M}\ddot{\mathbf{X}}\left(\mathbf{\Theta},t\right)+\mathbf{C}\ddot{\mathbf{X}}\left(\mathbf{\Theta},t\right)+\mathbf{KX}\left(\mathbf{\Theta},t\right)=\mathbf{B}_s\mathbf{U}_s\left(\mathbf{\Theta},t\right)+\mathbf{D}_s\mathbf{F}\left(\mathbf{\Theta},t\right) \tag{8}$$

where $\mathbf{X}$ denotes the inter-story drift vector which equals to the displacement of MR damper; $\mathbf{M},\mathbf{C},\mathbf{K}$ denote the mass, damping and stiffness matrix, respectively; $\mathbf{B}_s$ and $\mathbf{D}_s$ denote the location matrix of the dampers and the excitation, respectively; $\mathbf{U}_s$ denotes the damping force vector; $\mathbf{F}$ denotes the random excitation vector. For shear-valve MR damper, the damper force $\mathbf{U}_s$ in Eq. (8) can be expressed as:

$$\mathbf{U}_s\left(\mathbf{\Theta},t\right)=-\mathbf{C}_d\dot{\mathbf{X}}\left(\mathbf{\Theta},t\right)-\mathbf{U}_{dc}\left(\mathbf{\Theta},t\right) \tag{9}$$

where $\mathbf{C}_d$ and $\mathbf{U}_{dc}$ denote the viscous damping coefficient and the Coulomb force of MR damper, respectively. The damper force is determined mainly by the strength of the magnetic field and can be adjusted according to a control strategy. For the sake of fully usage of the MR damper's dynamic behaviors, a bounded Hrovat algorithm is applied as follows (Peng and Li 2010):

$$U_{\mathrm{s}}(\mathbf{\Theta},t)=\begin{cases}C_{\mathrm{d}}\dot{X}(\mathbf{\Theta},t)+U_{\mathrm{dc,max}}\,\mathrm{sgn}(\dot{X}(\mathbf{\Theta},t)), & \text{Case A: } U_{\mathrm{a}}\dot{X}<0 \text{ and } \left|U_{\mathrm{a}}\right|>U_{\mathrm{d,max}}\\ \left|U_{\mathrm{a}}\right|\mathrm{sgn}(\dot{X}(\mathbf{\Theta},t)), & \text{Case B: } U_{\mathrm{a}}\dot{X}<0 \text{ and } \left|U_{\mathrm{a}}\right|<U_{\mathrm{d,max}}\\ C_{\mathrm{d}}\dot{X}(\mathbf{\Theta},t)+U_{\mathrm{dc,min}}\,\mathrm{sgn}(\dot{X}(\mathbf{\Theta},t)), & \text{Case C: } U_{\mathrm{a}}\dot{X}>0\end{cases} \tag{10}$$

where $U_s(\boldsymbol{\Theta},t)$ denotes the damping force of MR damper; $U_a(\boldsymbol{\Theta},t)$ denotes the reference optimal control force; $U_{d,\max}(\boldsymbol{\Theta},t)=C_d\left|\dot{X}(\boldsymbol{\Theta},t)\right|+U_{dc,\max}$ denotes the maximum damping force of MR damper at a certain time; $\dot{X}(\boldsymbol{\Theta},t)$ denotes the damper velocity; $U_{dc,\max}$ and $U_{dc,\min}$ denote the maximum and minimum Coulomb force of MR damper, respectively.

With the assumption that the semi-active control force required in the Eq. (10) could be fully obtained by the MR damper, the maximum control force in the semi-active control equals to the maximum control force in the active optimal control, $U_{s,\max}=U_{a,\max}$, so as to make the former achieves the similar effectiveness with the latter. Meanwhile, with the condition that the inter-story velocity in case of the maximum semi-active control force reaches to that in case of the maximum active optical control, the following equations could be derived:

$$U_{s,\max}(\boldsymbol{\Theta})=C_d\left|\dot{X}_{s|U_{s,\max}(\boldsymbol{\Theta})|}\right|+\left|U_{dc,\max}\right|=C_d\left|\dot{X}_{a|U_{a,\max}(\boldsymbol{\Theta})|}\right|+\left|U_{dc,\max}\right|=U_{a,\max}(\boldsymbol{\Theta}) \tag{11}$$

Since the Coulomb force of the MR damper can be continuously adjusted by the current, there is:

$$U_{s,\max}(\boldsymbol{\Theta})=C_d\left|\dot{X}_{s|U_{s,\max}(\boldsymbol{\Theta})|}\right|+\left|U_{dc,\max}\right|=s\left(C_d\left|\dot{X}_{s|U_{s,\max}(\boldsymbol{\Theta})|}\right|+\left|U_{dc,\min}\right|\right) \tag{12}$$

In this paper, the minimum Coulomb force of the MR damper is assumed to be $U_{dc,\min}=0$. Associating Eq. (11) with Eq. (12), we have

$$U_{s,\max}(\boldsymbol{\Theta})=sC_d\left|\dot{X}_{s|U_{s,\max}(\boldsymbol{\Theta})|}\right|=sC_d\left|\dot{X}_{a|U_{a,\max}(\boldsymbol{\Theta})|}\right|=U_{a,\max}(\boldsymbol{\Theta}) \tag{13}$$

Then, the damping coefficient C_d and the maximum Coulomb force $U_{dc,\max}$ of the MR damper could be obtained:

$$\begin{cases} C_d=\dfrac{U_{a,\max}(\boldsymbol{\Theta})}{s\left|\dot{X}_{a|U_{a,\max}(\boldsymbol{\Theta})|}\right|} \\ U_{dc,\max}=(s-1)C_d\left|\dot{X}_{a|U_{a,\max}(\boldsymbol{\Theta})|}\right| \end{cases} \tag{14}$$

Therefore, the semi-active stochastic optimal control force and the parameters of the MR damper are determined. Then the state vector $\mathbf{X}$ and the control force vector $\mathbf{U}_s$ could be obtained, which both are the functions of $\boldsymbol{\Theta}$, indicating that both $\mathbf{X}$ and $\mathbf{U}_s$ are satisfied with the generalized probability density evolution equations (Li et al 2010):

$$\begin{cases} \dfrac{\partial p_{X\boldsymbol{\Theta}}(x,\boldsymbol{\theta},t)}{\partial t}+\dot{X}(\boldsymbol{\theta},t)\dfrac{\partial p_{X\boldsymbol{\Theta}}(x,\boldsymbol{\theta},t)}{\partial x}=0 \\ \dfrac{\partial p_{U_s\boldsymbol{\Theta}}(u,\boldsymbol{\theta},t)}{\partial t}+\dot{U}_s(\boldsymbol{\theta},t)\dfrac{\partial p_{U_s\boldsymbol{\Theta}}(u,\boldsymbol{\theta},t)}{\partial u}=0 \end{cases} \tag{15}$$

where $p_{X\boldsymbol{\Theta}}(x,\boldsymbol{\theta},t)$ and $p_{U_s\boldsymbol{\Theta}}(u,\boldsymbol{\theta},t)$ are the joint PDFs of the $(X(t),\boldsymbol{\Theta})$ and $(U_s(t),\boldsymbol{\Theta})$ respectively; $\dot{X}(\boldsymbol{\theta},t)$ and $\dot{U}_s(\boldsymbol{\theta},t)$ are the velocity arguments of $X(t)$ and $U_s(t)$ with the condition of $\boldsymbol{\Theta}=\boldsymbol{\theta}$. The initial conditions of Eq. (15) are as followed:

$$\begin{cases} p_{X\boldsymbol{\Theta}}(x,\boldsymbol{\theta},t)\big|_{t=0}=\delta(x-x_0)\,p_{\boldsymbol{\Theta}}(\boldsymbol{\theta}) \\ p_{U_s\boldsymbol{\Theta}}(u,\boldsymbol{\theta},t)\big|_{t=0}=\delta(u-u_0)\,p_{\boldsymbol{\Theta}}(\boldsymbol{\theta}) \end{cases} \tag{16}$$

where x_0,u_0 are the initial values of $X(t),U_s(t)$ respectively; $\delta(\cdot)$ is the Dirac's delta function. Then the instantaneous PDFs of $X(t)$ and $U_s(t)$ at any time could be obtained:

$$\begin{cases} p_X(x,t) = \int_{\Omega_\Theta} p_{X\Theta}(x,\boldsymbol{\theta},t)\,\mathrm{d}\boldsymbol{\theta} \\ p_{U_s}(u,t) = \int_{\Omega_\Theta} p_{U_s\Theta}(u,\boldsymbol{\theta},t)\,\mathrm{d}\boldsymbol{\theta} \end{cases} \tag{17}$$

where Ω_θ is the distribution domain of $\boldsymbol{\Theta}$. Generally, it is difficult to obtain the analytical solution of PDFs , and a numerical scheme is usually a preferable tool (Li and Chen 2009).

5 Case Study

In order to validate the effectiveness of the reliability based control gain design in semi-active control strategy, a SDOF structural system subjected to random seismic ground motion and controlled by a MR damper is investigated. The parameters of the structural system are as follows: structural mass $m = 1\times10^5\,\mathrm{kg}$; natural circular frequency $\omega_0 = 11.22\,\mathrm{rad/s}$; damping ratio $\xi = 0.05$; tunable times of MR damping force $s = 8$. Seismic accelerations are simulated by the stochastic ground motion model. Through the parameter optimization with respect to Eq. (7), the optimized weighting matrices are $\mathbf{Q} = \mathrm{diag}[1073.6, 505]$ and $\mathbf{R} = 10^{-10}$, and the control gain matrix is derived as $\mathbf{G} = [4.18, 21.56]\times10^5$. Using the functional relationship shown in Eq. (14), the damping coefficient and the maximum Coulomb force are designed to be $C_d = 0.269\,\mathrm{kN\cdot s/mm}$, $U_{dc,\max} = 90.33\,\mathrm{kN}$, respectively.

The root-mean-square displacements of the structural system with active/semi-active control and without control are shown in Fig. 1. It is seen that the structural performance gains a significant improvement both using the stochastic optimal controls. The semi-active stochastic optimal control can achieve almost the same effect with the active stochastic optimal control. Only in the time domain where a larger response of uncontrolled structure locates, the semi-active optimal control has a weaker effectiveness with the active optimal control. It is explained that in that time domain, the expected active control force surpasses the maximum output that the MR damper could yield in practices, i.e. $|U_a| > U_{d,\max}$.

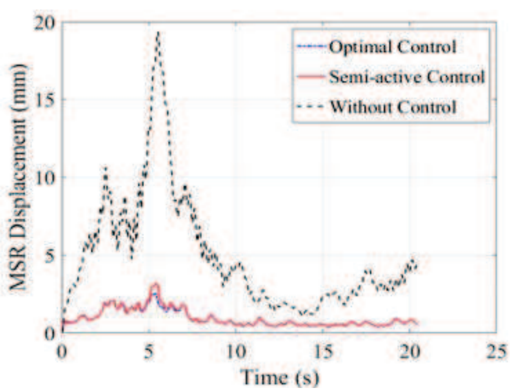


Figure 1. Root-mean-square displacement of structural system with and without control

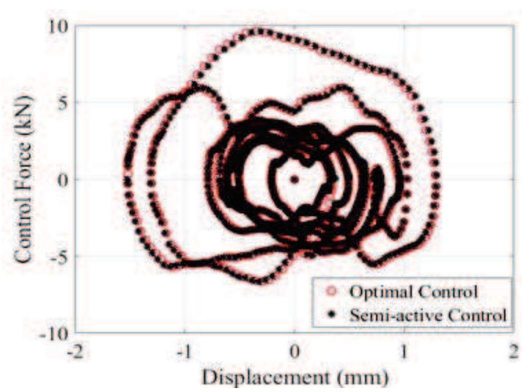
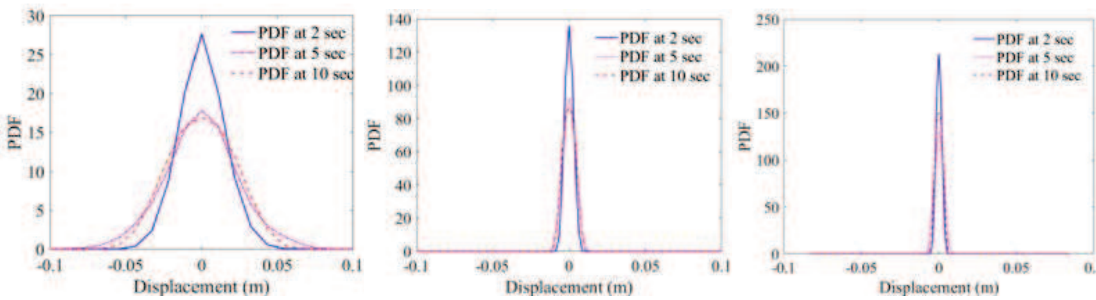


Figure 2. MR damping force traces active optimal control force in case of a typical sample seismic excitation

The details on the situation of MR damping force tracing active optimal control force in case of a typical sample seismic excitation are shown in Fig. 2. One could recognize that the semi-active control gains a good control-force tracing with the active optimal control. More accurate probabilistic representation is the probability density function. Fig. 3 shows the probability density functions of structural displacement at typical instants with and without control. It is seen that both the semi-active stochastic optimal control and the active stochastic optimal control reduce the variation of structural displacement significantly, where the distribution range of PDFs becomes narrower, indicating that the structural safety is enhanced remarkably, and meanwhile, the former can achieve almost the same effect with the latter.



(a) (b) (c)

Figure 3. Probability density functions of structural displacement at typical instants with and without control: (a) without control; (b) MR damping control in semi-active modality; (c) active stochastic optimal control.

6 Conclusion

The gain parameters of the active stochastic optimal control are optimized based on the reliability based probabilistic criterion, and the control efficiency of the semi-active stochastic optimal control tracing the stochastic optimal control force is investigated in the present paper. Numerical results reveal that the semi-active control strategy utilizing MR dampers can reduce both the amplitude and the variation of seismic response significantly, and strengthen the structural safety.

7 Reference

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