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CPTU-BASED EVALUATION OF CONSOLIDATION PARAMETERS OF JIANGSU SOFT CLAYS USING MULTIVARIATE DISTRIBUTION MODEL

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Evaluation of consolidation characteristics of cohesive soils has been a critical issue for settlement-related issues in geotechnical engineering. This study presents an investigation on the multivariate correlations among the consolidation and CPTU parameters for the Jiangsu soft clays using the method of multivariate distribution model. A database containing two consolidation parameters (c_h and k_h) and four CPTU indices (I_c, B_q, $\Delta u_2/\sigma'_{v0}$, t_{50}) was compiled and portioned into a calibration dataset and a validation dataset. The analyses of the calibration data showed that all these four CPTU indices could be useful to predict c_h and k_h. A comparison between the correlations developed for the Jiangsu soft clays and the validation data, data points and existing correlations collected from the literature demonstrated the rationality of the multivariate distribution model. Moreover, the accuracy of the multivariate correlations can be significantly improved by incorporating the four complete CPTU indices with increasing the unbiasedness and precision of the predicted c_h and k_h values.

Keywords: Multivariate distribution model, consolidation properties, piezocone, soft clay.

1 Introduction

The consolidation characteristics are main considerations in geotechnical settlement-related designs in clayey soils. The piezocone penetration test (CPTU) has been shown a powerful tool to access these properties and its accuracy has been acknowledged in the literature (Teh and Houlsby 1991, Robertson et al. 1992, Lunne et al. 1997). In most studies, only one CPTU parameter such as the time for 50% dissipation (t_{50}) is utilized to predict the consolidation parameters. However, it has been demonstrated that more than one CPTU index can be related to this behavior. For example, the CPTU soil behavior type index, I_c, is shown to be an effective indictor of the soil type and fines content, and it hence has been correlated to the hydraulic conductivity (Robertson 2010). The CPTU pore pressure parameter ratio (B_q) and normalized excess pore water pressure ($\Delta u_2/\sigma'_{v0}$) are also related to the consolidation properties based on theoretical solutions and experience (Schneider et al. 2008, Chai et al. 2011). Notwithstanding the presence of the multi-dependency of these soil parameters, the prediction of the consolidation properties using multiple CPTU indices has been insufficiently studied.

The multivariate distribution model has been shown an effective tool to capture the multivariate dependency among different soil parameters (Liu et al. 2016; Ching et al. 2017). In this study, this method is used to develop the multivariate correlations among the consolidation and CPTU parameters for the Jiangsu soft clay. The involved consolidation parameters include the horizontal coefficient of consolidation (c_h) and horizontal hydraulic conductivity (k_h). The CPTU parameters include I_c, B_q, $\Delta u_2/\sigma'_{v0}$, and t_{50}. To achieve this objective, a database containing these six soil parameters is presented. Based on this database, bivariate and multivariate correlations for c_h and k_h are derived and validated. The model errors, defined as the ratios of measurements over predictions, of the multivariate correlations for the Jiangsu soft clays are also discussed.

2 Consolidation and CPTU Data of Jiangsu Soft Clay

More than 200 CPTU soundings have been performed in the Jiangsu soft clays. The detailed schematic of the equipment and representative CPTU profiles are available in the literature (e.g., Cai et al. 2010). Only the horizontal consolidation characteristics are considered as they primarily dominate the behavior of the CPTU indices. Three principal methods for obtaining c_h and k_h are used, including: (a) back analyses of field settlement data; (b) laboratory horizontal permeability tests; and (c) laboratory oedometer tests. In the first method, only the settlement observations of soils with vertical wick drains are analyzed to ensure that the drainage paths mainly occur in the horizontal direction. The second method only provides k_h data. In this case, c_h is estimated using $c_h = k_h E_s/\gamma_w$, where E_s and γ_w are the constrained modulus and unit weight of water, respectively. The E_s is firstly determined from the laboratory tests. When laboratory data are not available, it is then evaluated using $E_s = 3.53q_t$, as suggested by Cai et al. (2010) for Jiangsu clays. The third method provides the vertical coefficient of consolidation (c_v) and vertical hydraulic conductivity (k_v). Then $c_h = c_v \cdot k_h/k_v$ and k_h are estimated with a given ratio of k_h/k_v. This ratio is also firstly determined from laboratory tests, and when laboratory data are absent it is chosen to be $k_h/k_v = 2$, which is approximately the mean k_h/k_v value for the Jiangsu clays. This ratio of $k_h/k_v = 2$ corresponds to no evidence of layering to slightly layering for clays according to Robertson et al. (1992).

Using the above methods, 156 sets of $\{I_c, B_q, \Delta u_2/\sigma'_{v0}, t_{50}, c_h, k_h\}$ data are compiled. These data are portioned into two datasets, one calibration dataset for constructing the multivariate distribution model and developing the correlations for c_h and k_h and one validation dataset for validating the model and correlations. The validation dataset are formed by extracting one to four sets of $\{I_c, B_q, \Delta u_2/\sigma'_{v0}, t_{50}, c_h, k_h\}$ data from each site. Some extreme values which are considered nonrepresentative of the soft clays have been discarded. The sample sizes of the calibration and validation datasets are 124 and 32, respectively. Fig. 1 illustrates the locations of the two datasets on the Robertson (1990) and Schneider et al. (2008) soil classification charts. All the data are identified to be clayey soils according to these two charts. Besides, the calibration and validation data show similar range and trend. Therefore, they are considered representative for the Jiangsu soft clays.

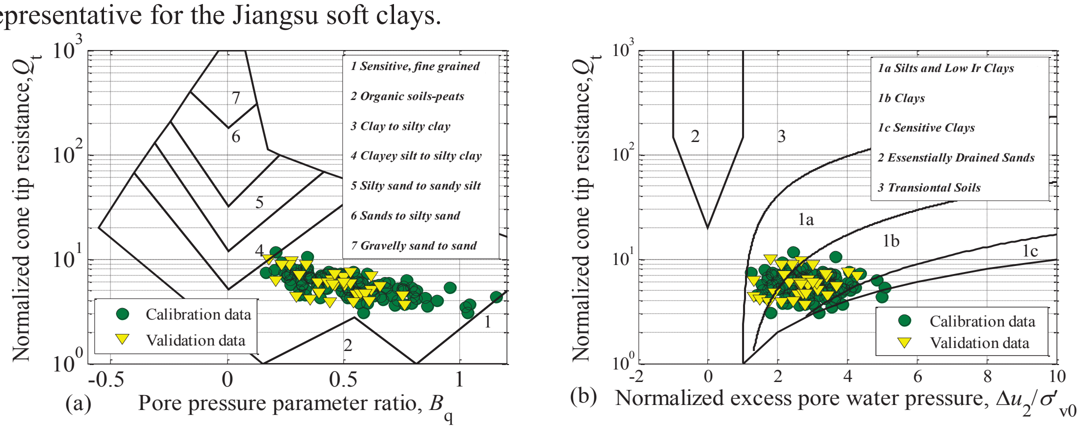

Figure 1. Compiled datasets on the Robertson (1990) and Schneider et al. (2008) charts

3 Construction of Multivariate Distribution Model for Jiangsu Soft Clay

The construction of the multivariate distribution model involves a data transformation to individually convert the non-Gaussian soil parameters to Gaussian variables and a calculation of the Pearson correlation coefficients among the transformed variables. In this study, the method of Box-Cox transformation is used due to its simplicity (Liu et al. 2016, Zou et al. 2017). In this method, a soil parameter (Y_i) is raised to a power (λ) to approximate a Gaussian variable and then it is standardized with a shifting parameter (a) and a scaling parameter (b) to produce a standard normal variable (X_i). More details are available in the literature (Liu et al. 2016, Zou et al. 2017), and therefore they are not presented here. Table 1 lists the Box-Cox transformation parameters for the Y_i variables. In this study, $Y_1 = I_c$, $Y_2 = B_q$ and $Y_3 = \Delta u_2/\sigma'_{v0}$ are the direct physical CPTU parameters, whereas $Y_4 = \lg t_{50}$ (t_{50} in s), $Y_5 = -\lg c_h$ (c_h in m^2/s), and $Y_6 = -\lg k_h$ (k_h in m/s) are the logarithmic soil parameters to reduce the data scatter. Table 1 also shows the results (P values) of the Kolmogorov-Smirnov (KS) test on the transformed variables. All the KS P values are larger than 0.05. Therefore, there is no strong evidence to reject the hypothesis that each transformed variable individually follows a standard normal distribution.

Table 1. Box-Cox transformation parameters for Y variables

Parameters	λ	a	b	KS P value
$Y_1 = I_c$	-2.455	0.382	0.004	0.428
$Y_2 = B_q$	0.238	-0.641	0.326	0.875
$Y_3 = \Delta u_2/\sigma'_{v0}$	0.515	1.371	0.476	0.749
$Y_4 = \lg t_{50}$ s	2.163	4.456	1.384	0.950
$Y_5 = -\lg c_h$ m^2/s	2.916	74.813	14.207	0.924
$Y_6 = -\lg k_h$ m/s	1.706	22.196	2.525	0.794

The Pearson correlation coefficients among the transformed X_i variables are presented in Table 2. From Table 2, the following findings are summarized:

- t_{50} is the best index to predict c_h and k_h, as their correlation coefficients are all larger than 0.75. This is reasonable since t_{50} directly evaluates the rate of soil consolidation.
- I_c is deemed effective to predict k_h as their correlation coefficient is larger than 0.70. This is perhaps due to the fact that I_c implies the change of fines content, which impacts k_h.
- B_q and $\Delta u_2/\sigma'_{v0}$ also provide useful information on c_h and k_h, as their correlation coefficients are around 0.60. It is justified that the generation of the excess pore water pressure depends on the drainage conditions of the surrounding soils, and hence varies with c_h and k_h.

Table 2. Pearson correlation coefficients among X variables

Variables	X_1	X_2	X_3	X_4	X_5	X_6
X_1	1.00	0.50	0.23	0.46	0.65	0.74
X_2	0.50	1.00	0.77	0.49	0.60	0.66
X_3	0.23	0.77	1.00	0.43	0.58	0.61
X_4	0.46	0.49	0.43	1.00	0.82	0.75
X_5	0.65	0.60	0.58	0.82	1.00	0.77
X_6	0.74	0.66	0.61	0.75	0.77	1.00

4 Bivariate Correlation Analyses for Jiangsu Soft Clay

Using a Bayesian updating and a back transformation, the correlations among {c_h, k_h} and {I_c, B_q, $\Delta u_2/\sigma'_{v0}$, t_{50}} can be derived with the hypothesis that the transformed X variables jointly follow a multivariate normal distribution (Liu et al. 2016). Fig. 2 illustrates the bivariate c_h-t_{50}, k_h-t_{50}, k_h-I_c, and k_h-$\Delta u_2/\sigma'_{v0}$ correlations in terms of median and 95% confidence interval (CI). The compiled calibration data categorized by the sources of c_h and k_h data are also shown in Fig. 2. It is evident that the derived median correlations fit the data well and the 95% CIs also agree with the scatter of the data. The calculated coefficients of determination (R^2) are 0.65, 0.57, 0.54 and 0.37 for the median-based c_h-t_{50}, k_h-t_{50}, k_h-I_c, and k_h-$\Delta u_2/\sigma'_{v0}$ correlations, respectively. Among the three CPTU indices, t_{50} is the best one to predict c_h and k_h, whereas the performance of $\Delta u_2/\sigma'_{v0}$ is not satisfactory. This conclusion is consistent with the previous findings arrived in the analysis of the correlation coefficients among these parameters.

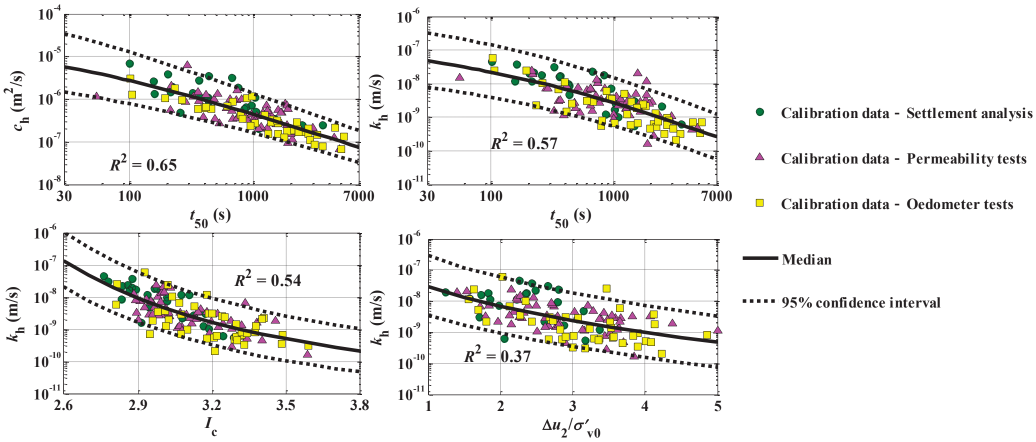

Figure 2. Bivariate c_h and k_h correlations for the calibration data of Jiangsu soft clays

Another conclusion drawn from Fig. 2 is that the sources of c_h and k_h data may impact the derived correlations. Most (approximately 80%) c_h and k_h data from the settlement analysis are above the median-based correlations. On the contrary, most (about 80%) data from the oedometer tests are below the median-based correlations. According to the complied data, the c_h and k_h results from the settlement analysis are approximately 2 to 3 times those from the oedometer tests. This is perhaps due to the impact of the small size of sample in the oedometer tests and the unknown influence of soil fabric such as layering in the in situ condition. The c_h and k_h values provided by the settlement observations shall provide a better evaluation of the in situ consolidation behavior of the soils.

Fig. 3 presents the comparison between the above four bivariate correlations and the corresponding validation data of Jiangsu soft clays. Data points provided by a comprehensive literature review in Robertson et al. (1992) and some existing correlations published in the literature are also illustrated in Fig. 3. Fig. 3 shows that the median-based correlations for the Jiangsu soft clay agree well with the validation data in both trend and range. These correlations are also consistent with the trends indicated by the existing correlations. However, the lower bounds of the 95% CIs for the Jiangsu clays are slightly beneath those given by the theoretical solutions. This is perhaps because large amounts of c_h and k_h data are obtained from the results of oedometer tests and they are considered less than the actual values. In fact, a comparison between Fig. 2 and Fig. 3 shows that these theoretical solutions agree well with the c_h and k_h data obtained from the settlement analysis as they are lying within comparable ranges, i.e., between the medians and upper bounds of the 95% CIs. Therefore, these bivariate correlations are deemed reasonable with respect to the existing experience in the geotechnical practice.

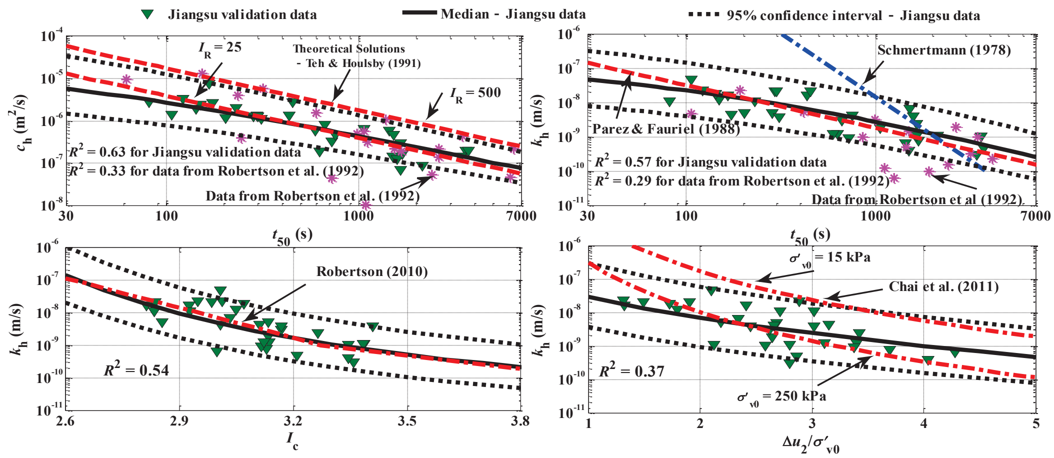

Figure 3. Validation of the bivariate c_h and k_h correlations for Jiangsu soft clays

5 Median-based Multivariate Correlations for Jiangsu Soft Clay

Based on the multivariate distribution model, multivariate correlations for c_h and k_h are derived:

$$\lg c_h = -\left[-2737.649 I_c^{-2.455} - 37.253 B_q^{0.238} + 84.087\left(\Delta u_2/\sigma'_{v0}\right)^{0.515} + 278.804\right]^{0.343} \quad (1)$$

$$\lg c_h = -\left[-1827.174 I_c^{-2.455} - 70.081 B_q^{0.238} + 59.076\left(\Delta u_2/\sigma'_{v0}\right)^{0.515} + 7.698\left(\lg t_{50}\right)^{2.163} + 210.343\right]^{0.343} \quad (2)$$

$$\lg k_h = -\left[-314.785 I_c^{-2.455} - 0.559 B_q^{0.238} + 8.202\left(\Delta u_2/\sigma'_{v0}\right)^{0.515} + 45.058\right]^{0.586} \quad (3)$$

$$\lg k_h = -\left[-248.337 I_c^{-2.455} - 2.955 B_q^{0.238} + 6.377\left(\Delta u_2/\sigma'_{v0}\right)^{0.515} + 0.561\left(\lg t_{50}\right)^{2.163} + 40.062\right]^{0.586} \quad (4)$$

Fig. 4 shows a comparison between the c_h and k_h measurements against their predicted medians using the above four formulas. The predicted medians are uniformly scattered around the reference line, indicating that the predictions are almost unbiased, regardless of the sources of c_h and k_h data. However, obtaining a continuous c_h profile using $\{I_c, B_q, \Delta u_2/\sigma'_{v0}\}$ may not be satisfactory as the scatter between the predictions and the actual measurements is significant ($R^2 < 0.70$). Nonetheless, it is still possible to achieve a relatively accurate k_h profile based on $\{I_c, B_q, \Delta u_2/\sigma'_{v0}\}$ since the R^2 value can reach 0.74. The best case is to perform a CPTU dissipation test and then use t_{50} in the predictions. When t_{50} is involved, the R^2 values can reach more than 0.80.

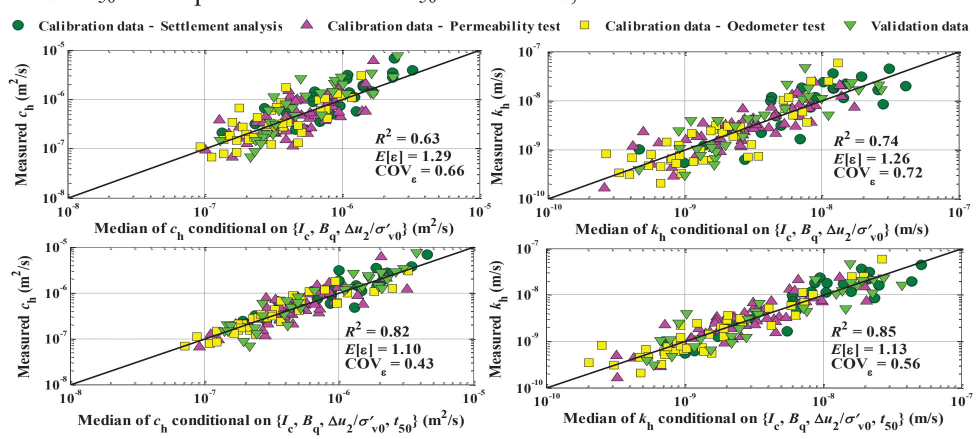

Figure 4. Multivariate c_h and k_h correlations for Jiangsu soft clays

A more rational evaluation of the performance of the correlations is to use a model error (ε), defined as the ratio of measurement over the predicted median. Fig. 4 also presents the mean values and coefficients of variation of ε (COV_ε) for the above four multivariate correlations. It is evident that the inclusion of t_{50} increases the unbiasedness of the multivariate correlations with $E[\varepsilon]$ closer to 1.0 and simultaneously improves the precision with the reduction of COV_ε. Therefore, the multivariate distribution model is an effective tool to capture the multi-dependency of the consolidation and CPTU parameters for the Jiangsu soft clays.

Conclusions

In this study, the multivariate correlations among the consolidation and CPTU parameters of Jiangsu soft clays were analyzed. A database containing two consolidation parameters (c_h and k_h) and four CPTU indices (I_c, B_q, $\Delta u_2/\sigma'_{v0}$, and t_{50}) was compiled. It was shown that all these four CPTU indices provide valuable information on c_h and k_h and rational bivariate correlations for these two parameters can be derived. Moreover, the uncertainties associated with c_h and k_h can be significantly reduced by incorporating the four complete CPTU indices with the increasing of R^2 value and decreasing of model scatter between the measurements and predictions.

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