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IMPROVED RANDOM FIELD SIMULATION OF SOIL DEPOSITS BY A POTENT AUTOCORRELATION MODEL

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The autocorrelation function of the soil profile in a particular site in China is studied using cone penetration test (CPT) data. This is done in context with a random field model of soil deposits. Pertinent autocorrelation function models are obtained by least squares approximation method fitting for the four different types of soils in the site. Based on the features of the autocorrelation function data, a potent autocorrelation model is selected involving a linear, an exponential and cosine terms. Further, the convergence of the selected model is compared with the normally used models in connection with the classical Karhunen- Loéve expansion of stochastic process. It is found that the selected model can converge quickly as the number of terms increases. Furthermore, the random field of the soil profile is modeled using a two-dimensional Karhunen-Loéve expansion for the model, separable in two dimensions. As a case study, the settlement of one site is calculated using the random field model, and is juxtaposed with the results of uniform value field. It is found that the random field results are drastically more physically sound than those pertaining to the uniform random field counterpart.

Keywords: CPT data, new autocorrelation model (LNCS), Karhunen-Loéve expansion, random field simulation, foundation settlement

1 Introduction

Uncertainty quantification (UQ) of soil related problems is important as simulation, optimization, and decision making analyses involve capturing the stochastic nature of the soil (Ang, 1984; Goovaerts, 1997). Due to the fact that soils are spatially variable, the mean, variance, and covariance structure of a specific soil site are needed for any realistic stochastic modeling (Baecher, 2003; Gao, 1996). In this context, and in attempting to establish the correlation structure of various soil profiles, the standard approximation procedure is to analyze the pertinent data and to establish the corresponding autocorrelation function (ACF) to the prescribed kind of soil. Based on the ACF, the random field simulation has been widely adopted to describe the uncertainty of the soil properties (Vanmarcke, 2010).

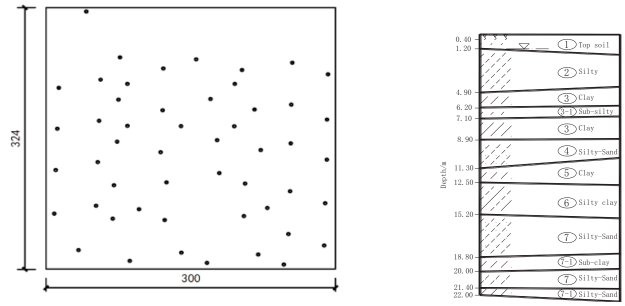
This paper addresses the correlation structure of the soil profiles based on cone penetration test (CPT) data, and establishes the autocorrelation function by a least squares approximation method fitting for the four different types of soils in the site. In this context, a potent autocorrelation model (Yue et al, 2018) is adopted involving a linear, an exponential, and a cosine terms to ensure differentiability at the origin of the spatial axis, and to accommodate negative value of the correlation function. Further, the convergence of the new model is studied. Furthermore, the random field of the soil profile is modelled using a two-dimensional Karhunen-Loéve expansion for the model. Finally, the settlement of the site soil is calculated using the newly developed random field model.

2 The correlation structures of soil properties

2.1 *CPT site data analysis*

CPT data have been used by many researches as a powerful tool to analyze the spatial correlation structure of soil sites (Liu etc. 2010; Uzielli etc., 2005). A CPT data provides cone tip resistance and sleeve friction information with an equal sampling interval distance. In this paper, the CPT data were gathered in the Shandong province, China, and comprise measurements of cone tip resistance q_c (MPa), and sleeve friction f_s

(KPa). The measurements were recorded at vertical intervals of 0.01m. One site including 51 soundings data is considered (Figure 1).



(a) Site and locations of CPT soundings /m (b)Soil profile

Figure 1. CPT layout and soil profile

The soil data was standardized by the equation

$$x(z) = \frac{[x_i(z) - \overline{x}]}{\tilde{x}} \quad , \tag{1}$$

where $\overline{x}$ is the mean value of the layer soil, and $\tilde{x}$ is the standard deviation.

2.2 *Spatial correlation function simulation*

2.2.1 *The autocorrelation function (ACF)*

Dealing with the CPT data of soil parameters, assume that $x_i(z)$ is the standardized data at depth $z_i = i\Delta z, i = 1, 2, \cdots, n$. The covariance function is obtained by the equation

$$C(\tau_j) = \frac{1}{n} \sum_{i=1}^{n-j} (x_i - \mu_x)(x_{i+j} - \mu_x) \ , \tag{2}$$

where $j = 0, 1, 2, \cdots, n-1$, lag $\tau_j = j\Delta z$, $\mu_x = \frac{1}{n} \sum_{i=1}^{n} x_i$.

The sample correlation is

$$\rho(\tau_j) = \frac{C(\tau_j)}{C(0)} . \tag{3}$$

Calculated by the above equations, the ACF of the soil parameter can be obtained.

The most commonly used autocorrelation model (ACM) is the single exponential model (SNX).

$$R(\tau) = e^{-k_{SNX} \cdot |\tau|}, \tag{4}$$

where k_{SNX} is a parameter capturing the correlation length. However, this model is non-differentiable at the origin of the spatial axis. Thus, Spanos *et. al* (2007) proposed a modified linear exponential model (LNX) by introducing a slight modification in the mathematical description of the single exponential model. The expression for the LNX is

$$R(\tau) = (1 + k_{LNX} |\tau|) e^{-k_{LNX} \cdot |\tau|} \ , \tag{5}$$

where, k_{LNX} are the parameters that capturing the correlation length.

A cosine-exponential model (CSX) has also been used which can accommodate the negative value of the ACF (Vanmarcke, 1977; Jaksa, 1995). The specific expression of the CSX is

$$R(\tau) = e^{-k_{CSX} \cdot |\tau|} \cos(k_{CSX} |\tau|) \ . \tag{6}$$

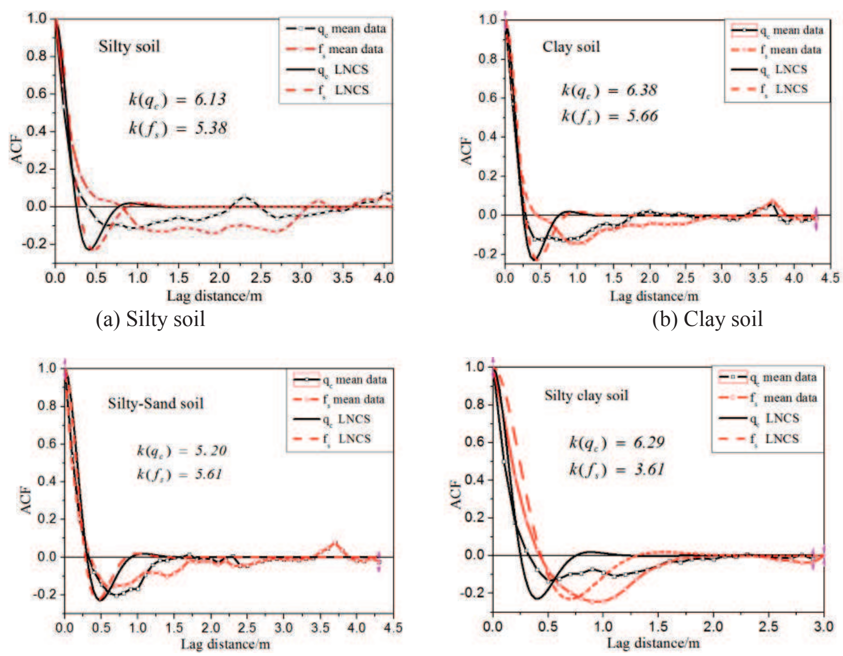
Note that, from the preliminary analysis of the autocorrelation of the soil data and many references (Baecher and Christian, 2003; Vanmarcke, 1977; Fenton, 1999), it has been found that the autocorrelation value will change from positive to negative after a certain lag distance, and back to zero again. In this regard, the differentiability of the model LNX and the alternating sign of model CSX are considered together. Thus, a new autocorrelation model is proposed herein involving a linear, an exponential, and a cosine terms. It is named as liner-exponential-cosine model (LNCS), and it is expressed as

$$R(\tau) = (1 + k_{LNCS}\left|\tau\right|)e^{-k_{LNCS}\cdot\left|\tau\right|}\cos(k_{LNCS}\left|\tau\right|) \;, \tag{7}$$

where k_{LNCS} is the corresponding parameter capturing the correlation length.

2.2.2 *The autocorrelation model simulation of the data*

Processing the CPT data of two parameters, cone tip resistance q_c and sleeve friction f_s, that are collected in the site, with the Eqs.(1)-(3). The autocorrelation can be obtained. Then, the ACF data are simulated with the autocorrelation model (LNCS) for the four different type soil types in the site by least square approximation method fitting. For enhanced clarity, the average autocorrelation data with the simulated model is used in this paper, shown in Figure 2.



(a) Silty soil (b) Clay soil

(c) Silty-Sand soil (d) Silty clay soil

Figure 2. Autocorrelation function comparison and fitting of the mean of q_c and f_s

3 Random Field Simulation with Karhunen-Loeve expansion

Next, the Karhunen-Loeve representation (Ghanem and Spanos, 2003), often used to capture uncertainty in engineering applications, is attempted here.

3.1 *Basic concept of the Karhunen-Loeve expansion*

A stochastic process $X(\tau,\theta)$ indexed on a bounded domain D, and having zero mean (for convenience) and finite variance, can be represented using a finite Karhunen-Loeve (K-L) series

$$X(\tau,\theta) = \sum_{i=1}^{\infty}\sqrt{\lambda_i}\,\xi_i(\theta)\phi_i(t) \,, \tag{8}$$

where $\xi_i(\theta)$ is a set of uncorrelated standardized random variables with zero mean and unit variance. If $X(\tau,\theta)$ is a Gaussian process, then an appropriate choice of $\xi_i(\theta)$ is a vector of uncorrelated standard Gaussian random variables; $\{\phi_i\}$ and $\{\lambda_i\}$ are the eigenfunctions and eigenvalues of the covariance function $C(\tau_1,\tau_2)$, respectively. They satisfy the homogenous Fredholm integral equation

$$\int_T C(s,t)\phi_i(t)dt = \lambda_i\phi_i(s)\,. \tag{9}$$

The truncated version of $X(\tau,\theta)$ can be expressed as

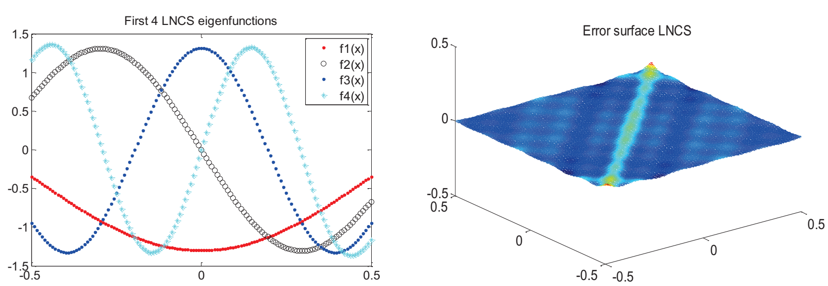
$$X(\tau,\theta) = \sum_{i=1}^{M}\sqrt{\lambda_i}\,\xi_i(\theta)\phi_i(t)\,. \tag{10}$$

The truncated Karhunen-Loeve expansion is optimal in the sense of a mean square error minimization. For a particular application, the number M to be chosen depends on the desired accuracy, and on complexity of the autocorrelation function of the random field. Ordinarily, in most engineering applications, less than 10 terms should suffice.

3.2 *Numerical solution of Fredholm integral equations using quadrature*

Based on the preceding perspective, the homogeneous Fredholm integral equation of the second kind, in Eq.(9), is first solved for the particular application of this paper to obtain the eighenvalues and eigenfunctions of the covariance function. In case the domain D the problem is the one-dimensional segment $[-a,a]$, the Fredholm integral equation can be solved analytically under some special circumstances(Ghanem and Spanos, 2003; Spanos, etc., 2007). In most cases, numerical methods are required. In this paper, the integral introduced in Eq.(9) was evaluated numerically by Simpson's quadrature scheme.

Figure 3 shows results of the linear exponential cosine model (LNCS) for *k*=6.29 of the parameter q_c of silty clay soil. Table 1 shows the first 10 eigenvalues.



(a)Frist 4 LNCS eigenfunctions (b) Error surface LNCS

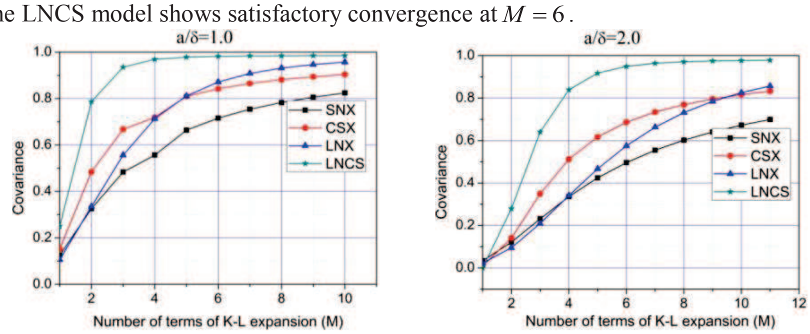
Figure 3. Numerical solution of model LNCS

Table 1. First ten eigenvalues

Number of eigenvalue	Numerical eigenvalue
1	2.777×10^{-1}
2	2.600×10^{-1}
3	1.891×10^{-1}
4	1.392×10^{-1}
5	6.782×10^{-2}
6	3.149×10^{-2}
7	1.505×10^{-2}
8	7.719×10^{-3}
9	4.250×10^{-3}
10	2.501×10^{-3}

3.3 Convergence of ACF models

Figure 4 shows the covaranice value at the zero lag for two cases, $a/\delta = 1, 2$, with different summation terms. It can be seen that as the terms increase, the covarince converges, while the models of SNX, CSX and LNX converge slowly, and the LNCS model converges faster. For example, in case of $a/\delta = 1$, the model LNCS shows convergence at $M = 4$, while the other three models still exhibit considerable errors. For the case $a/\delta = 2$, the LNCS model shows satisfactory convergence at $M = 6$.



(a) $a/\delta = 1$ (b) $a/\delta = 2$

Figure 4. Covariance converge speed of different models

3.4 *Two dimensional field simulation with K-L expansion for LNCS model*

Next, the new ACF model is used for a random field simulation of the soil site. Figure 5 shows a two-dimensional soil random field sample for silty clay soil. The size of the soil field is 10 × 4*m* and the mesh size is 0.1*m*. Herein, δ_h / δ_v =10, δ_h , δ_v are the scale of fluctuations in the horizontal and vertical directions, respectively. The midpoint discretization method is adopted.

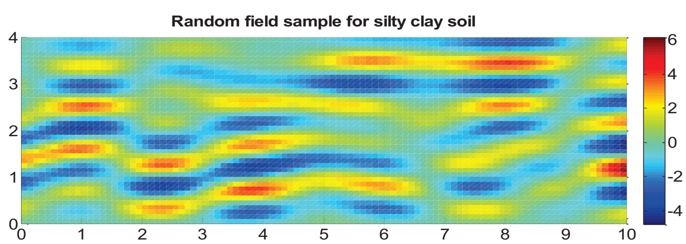
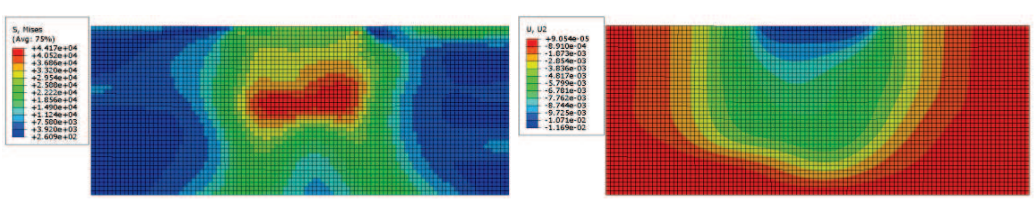


Figure 5. Random field sample for silty clay soil

4 Settlement analysis of soil with random field model

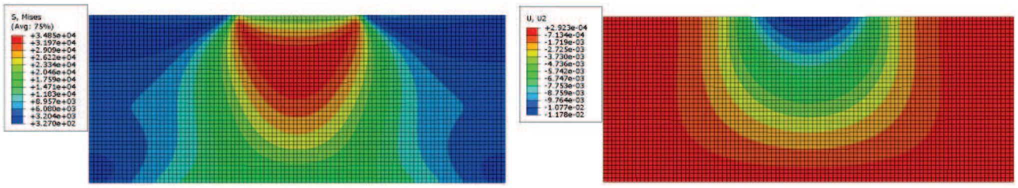
Further, the settlement of the site with silty clay soil is calculated. Here, the elastic modulus is considered to be as random field. The elastic modulus is assumed as lognormal distribution with the mean 10.87 MPa, and the scale of fluctuation of δ_{lnE} is 1.87 and 0.617 in horizontal and vertical direction, respectively. The Poisson ratio is taken as 0.3. It is assumed a uniformly distributed load, 50*KN/m*, acts on the surface of the soil with length 3*m*.

Figure 6 and Figure 7 show the comparisons of random field model with uniform random field (with mean value). It is seen the variation of the Mises stress is higher of the random field than uniform field, and the vertical settlement is a little smaller than the uniform field.



(a) Misess Stress (b) The foundation settlement

Figure 6. Results of random field



(a) Misess Stress (b) The foundation settlement

Figure 7. Results of uniform soil

5 Concluding Remarks

The ACF of the soil profile in a particular site in China has been studied using cone penetration test (CPT) data, and a new model has been proposed to ensure differentiability at zero lag axis, and to accommodate alternating signs of the ACF. The Karhunen-Loeve expansion method has been adopted to demonstrate the efficiency of the new model in simulating realistically actual soil deposits properties. The calculated model settlement has been juxtaposed with those corresponding to uniform field.

Acknowledgments

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