

THE SEISMIC SERVICEABILITY ANALYSIS OF WATER SUPPLY NETWORKS

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The serviceability analysis of Water supply networks (WSNs) under earthquakes has significant importance for estimating the probable losses and the impact of diminished functionality on the affected community. In this paper, a physical-based analytic strategy is suggested. Following the stochastic seismic response of buried pipe networks, the transient flow analysis of WSNs with or without leakage is derived. Then the dynamic serviceability of WSNs under earthquakes is analyzed based on the probability density evolution method (PDEM). At last, an assessment of the serviceability of a small WSN illustrates the approach.

Keywords: Water supply networks; serviceability; stochastic ground motion; probability density evolution method

1 Introduction

Water supply networks (WSNs) are important components of lifeline systems, which include buried pipes, pumps, valves etc. to deliver water from sources to customers, and reservoirs, tanks and cisterns etc. to meet the needs of fire protection, emergency relief and some other infrastructures. However, a great many previous earthquake investigations prove that the seismic serviceability of WSNs is still very weak (O'Rourke 1996). In order to assess the dynamic serviceability of WSNs under earthquakes, a physical-based analytic strategy based on the probability density evolution method (PDEM) is suggested in this paper by combining the stochastic seismic response of buried pipe networks and the transient flow analysis of WSNs with or without leakage.

2 Seismic Response of Buried Pipe Networks

2.1 Seismic behaviors of buried pipe networks

In order to get the seismic behaviors of buried pipe networks, a finite element model proposed by Liu et al. (2015) is adopted in this paper. In this method, the motion equation of the pipe network can be expressed as:

$$[\bar{K}]_{\text{SYS}} \{\bar{u}\} = [\bar{K}]_L \{\bar{u}_g\} \quad (1)$$

where $[\bar{K}]_{sys}$ is the stiffness matrix of the system; $[\bar{K}]_L$ is transfer matrix of load. $\{\bar{u}\}$ and $\{\bar{u}_g\}$ are the displacement vector of the pipe elements and ground motion, respectively.

After solving the Eq. (1), we can get the joint deformation. For the joint i , the axial deformation u_j^i can be expressed as follows:

$$u_j^i = u_1^{i+1} - u_2^{i-1} \quad (2)$$

The method has been validated by an artificial earthquake test by Liu et al. (2017).

2.2 Leakage model of the pipe network

In order to determine the relationship between the leakage flow and the water pressure, the following point leakage model is used (Chen & Li 2004):

$$Q_L(t) = C_0 A_L(t) \sqrt{2gH_L(t)} \quad (3)$$

where $Q_L(t)$ is the leakage flow at the leakage point; $A_L(t)$ is the leakage area, $H_L(t)$ is the water pressure at the leakage point, g is the gravitational acceleration; t is time; C_0 is a model parameter. The leakage area of this joint can be estimated by the following equation:

$$A_{Li}(t) = \begin{cases} 0 & u_{jm}^i(t) < R_1 \\ \frac{u_{jm}^i(t) - R_1}{R_2 - R_1} \pi d_o \delta & R_1 \leq u_{jm}^i(t) \leq R_2 \\ \pi d_o \delta & R_2 \leq u_{jm}^i(t) \leq R_L \end{cases} \quad (4)$$

where d_o is the outer diameter of the pipe; δ is the maximum width of the joint gap; R_L is the depth of the spigot inserting into the socket. R_1 and R_2 are the limit value of elastic deformation and plastic deformation of the pipe joint, respectively. $u_{jm}^i(t)$ is the maximum tensile deformation of the pipe joint, which can be expressed as:

$$u_{jm}^i(t) = \begin{cases} \max |u_j^i(t)| & T_1 \leq t \leq T_2 \\ \max |u_j^i(T_2)| & T_2 \leq t \leq T_3 \\ 0 & T_3 \leq t \end{cases} \quad (5)$$

where T_1 and T_2 are the beginning and ending time of the earthquake, respectively; T_3 is the time when the broken joint has been repaired.

For simplicity, we assume that the leakage area of every joint in a pipeline can be focused in the two ends of the pipeline. Therefore, for node j in the pipe network, the total leakage area at this point should be:

$$AL_j(t) = \frac{1}{2} \sum_{k=1}^{N_p} \sum_{i=1}^{N_j} A_{Li}(t) \quad (6)$$

where N_j is the number of pipe joint of pipeline k ; N_p is the number of pipelines at node j in the pipe network.

2.3 Stochastic ground motion field

The ground motion field can be expressed as follows:

$$a(r, r_1, t) = -\frac{1}{2\pi} \int_{-\infty}^{+\infty} A_s(\xi, \eta, \omega, r, r_1) \cdot \cos[\omega t + \Phi(\xi, \eta, \omega, r, r_1)] d\omega \quad (7)$$

where $a(r, r_1, t)$ is the time history of ground motion in the engineering site; r is the distance from the hypocenter to the geometric center of the local site; r_1 is the simulated location of the ground motion in the local site; $A_s(\xi, \eta, \omega, r, r_1)$ is the amplitude spectrum of the ground motion; $\Phi(\xi, \eta, \omega, r, r_1)$ is the phase spectrum of the ground motion; $\xi = [A_0, T, \omega_g, \zeta_g, \alpha_0, c_g]^T$ is a stochastic vector representing the main physical factors that affect the ground motion; $\eta = [K, a', b', c', d']^T$ is a deterministic vector, which reflects the effect of transmission paths.

On the basis of the superposition method of narrow-band harmonic wave groups, Wang and Li (2011) suggested the time history samples of the ground motion in a local site can be derived as follows:

$$a(r, r_1, t) = -A_0 \cdot \sum_i A_i(r, r_1) \cdot F_i(r, r_1, t) \cdot \cos[\omega_i t + \Phi_i(r, r_1)] \quad (8)$$

where

$$A_i(r, r_1) = \frac{2}{\pi} \cdot \frac{\omega_i e^{-K\omega_i r}}{\sqrt{\omega_i^2 + (\frac{1}{T})^2}} \cdot \sqrt{\frac{1 + 4\zeta_g^2 (\frac{\omega_i}{\omega_g})^2}{[1 - (\frac{\omega_i}{\omega_g})^2]^2 + 4\zeta_g^2 (\frac{\omega_i}{\omega_g})^2}} \cdot e^{-\frac{\alpha_0 \omega_i r_1}{2}} \quad (9)$$

$$F_i(r, r_1, t) = \frac{\sin[(t - \frac{r}{c_i} - \frac{r_1}{c_g}) \cdot \Delta\omega_j]}{t - \frac{r}{c_i} - \frac{r_1}{c_g}} \quad (10)$$

$$\Phi_i(r, r_1) = \arctan(\frac{1}{T\omega_i}) - r \cdot d \ln[(a + 0.5)\omega_i + b + \frac{1}{4c} \cdot \sin(2c\omega_i)] - r_1 \cdot \frac{\omega_i}{c_g} \quad (11)$$

$$c_i = \frac{d\omega}{dk} \Big|_{\omega=\omega_i} = \frac{(a + 0.5)\omega_i + b + \frac{1}{4c} \cdot \sin(2c\omega_i)}{d \cdot [a + \cos^2(c\omega_i)]} \quad (12)$$

where A_0 and T are the random coefficients of the hypocenter; ω_g is the equivalent predominant circular frequency of the local site; ζ_g is the damping ratio of the local site. α_0 is the attenuation parameters of the local site; c_g is the apparent seismic wave velocity. a' , b' , c' and d' are the empirical coefficients for synthetizing ground motion fields.

3 The Transient Flow Analysis in the Pipe Network

For the one-dimensional transient flow in the pipe shown in Figure 1, its momentum equation and continuity equation can be respectively expressed as (Chaudhry 2014):

$$\frac{1}{gA} \left(\frac{\partial Q}{\partial t} + V \frac{\partial Q}{\partial x} \right) + \frac{\partial H}{\partial x} + fQ|Q|^{1-m} = 0 \quad (13)$$

$$\frac{\partial H}{\partial t} + V \frac{\partial H}{\partial x} + \frac{a^2}{gA} \frac{\partial Q}{\partial x} = 0 \quad (14)$$

where g is the acceleration of gravity; A is the cross sectional area of the pipe; Q is the flow rate in the pipe; V is the fluid velocity; H is the pressure head; f and m are two coefficients of friction resistance, which depend on different hydraulic loss models. Generally, a is the propagation velocity of small disturbances, such as sound, which can be expressed as (Chaudhry 2014):

$$a = \sqrt{\frac{K/\rho}{1 + \frac{Kd_i}{Ee}C_1}} \quad (15)$$

where K is the bulk modulus of the fluid; ρ is the fluid density; d_i is the inner diameter of the pipe; e is the wall thickness of the pipe; C_1 is a parameter which depends on the support conditions of pipe ends. When one end of the pipe is fixed while the other can lengthen and shorten freely, C_1 should be $1-\mu/2$, where μ is the Poisson's ratio; when the two ends are fixed, C_1 is $1-\mu^2$; when the pipe can lengthen and shorten freely at both ends, C_1 equals to 1.

Characteristic line method can be used to solve the above differential equations.

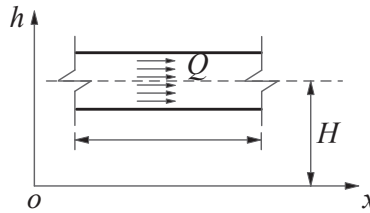


Figure 1. The one-dimensional fluid in pipes

4 Probability Density Evolution Method

In 2004, Li and Chen (2004, 2009) suggested a new method named the probability density evolution method (PDEM) for analyzing the nonlinear stochastic dynamical system with random parameters subjected to stochastic excitations. PDEM reveals the internal relationships between the probability density evolution and the physical system state evolution.

In general, the basic procedures of solving the probability density evolution function are as follows (Li and Chen 2009):

(i) Selecting some representative points θ_q in the random parameter space Ω_θ and then assigning corresponding probability P_q to the point θ_q , where $q=1,2,\dots,N$. N is the number of selected points. (ii) For the representative point θ_q , $q=1,2,\dots,N$, Eqs. (13)–(14) are solved to obtain the node head. (iii) Then for the selected point θ_q , the PDEE can be further written as:

$$\frac{\partial p_{z\theta}(z, \theta_q, t)}{\partial t} + \sum_{k=1}^m \dot{z}_k(\theta_q, t) \frac{\partial p_{z\theta}(z, \theta_q, t)}{\partial z_k} = 0 \quad (16)$$

Meanwhile, the initial condition can be expressed as:

$$p_{z\theta}(z, \theta, t) \Big|_{t=t_0} = \delta(z - z_0) P_q \quad (17)$$

(iv) Synthesizing the results will yield the instantaneous probability density function (PDF) as follows:

$$p_z(z, t) = \sum_{q=1}^N P_{z\theta}(z, \theta_q, t) \quad (18)$$

5 Case study

In order to further explain the method suggested in this paper, a small WSN is used as an example to analyze the function reliability of the pipe network when suffering the earthquake. The topological structure, the number of pipelines and nodes are shown in Figure 2. All pipes are grey cast iron pipes. The length of pipe segments is 6 m. The network is located in II type site based on the Chinese design code and the burial depth of all pipes is 1 m. The soil is soft clay with the undrained shear strength of 22.93 kPa. Therefore, the stiffness of axial and lateral soil springs can be got according to the ALA seismic guidelines (ALA 2005). Then based on the PDEM, the PDFs of the dynamic water head and flow rate, as well as the leakage flow rate at different nodes in the pipe network can be got. The analysis results of the node No. 7 in the pipe network have been shown in Figure 3 and Figure 4 as an example. Figure 3 is the PDF and the probability density contour of the dynamic water pressure while Figure 4 is the PDF and the probability density contour of the leakage flow.

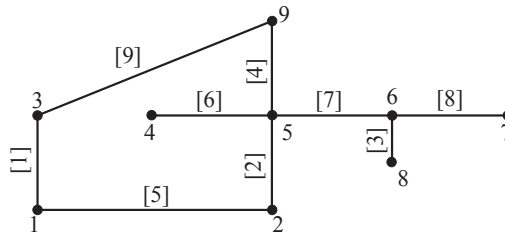


Figure 2. A small pipe network

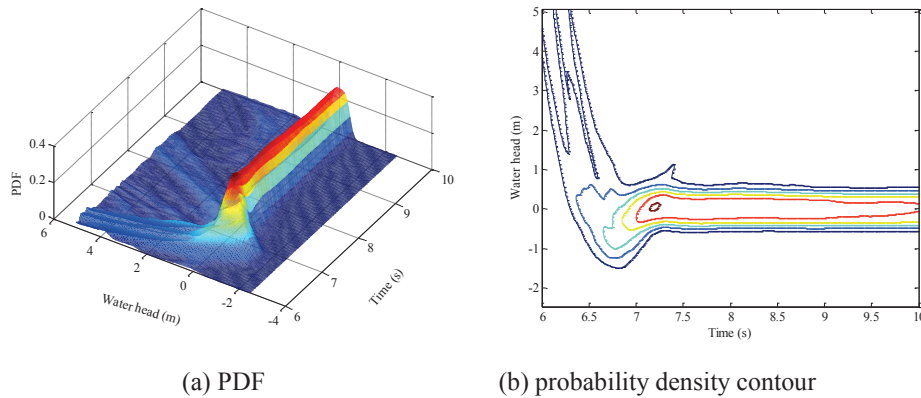


Figure 3. The water head at the node No. 7

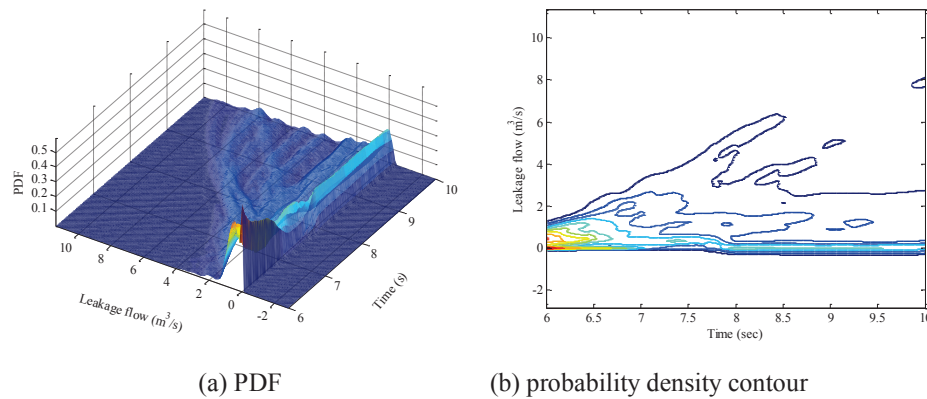


Figure 4. The leakage flow at the node No. 7

6 Conclusions

Based on the seismic response of buried pipe networks and the transient flow analysis in networks, this paper achieve dynamic real-time serviceability seismic function analysis of water supply networks based on physical mechanism. This model can be applied to the function analysis of water supply networks during and after earthquakes. The case study shows that during the earthquake, the water pressure will change sharply. When the leakage area of the network is constant, the water pressure and leakage flow rate will gradually stabilize.

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