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APPLICATION OF RESPONSE SURFACE METHOD TO RELIABILITY ANALYSIS AND DESIGN FOR UNDERGROUND ROCK TUNNELS

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The response surface method is regarded to be efficient for the problems, of which the limit state functions may not be available in explicit form and frequently require computational expensive numerical procedures, and builds a bridge between the existing reliability algorithms such as the first-order reliability method (FORM) and the second-order reliability method (SORM) and the standalone numerical packages. Focusing on practical reliability analysis and design of tunnels, the application of response surface method will be discussed. Different response surface models, including polynomial functions, artificial neural network and moving least square method, are introduced and compared. An iterative algorithm and an adaptive sampling technique are proposed. The efficient and practical procedures which combined the response surface methods with FORM and SORM are illustrated via examples of underground rock excavation. The application of response surface method to the reliability-based design optimization, in which two nested optimization procedures, i.e., the outer design optimization and the inner reliability analysis, are also presented for seeking an optimal design scheme for a circular tunnel.

Keywords: Reliability, Tunnel, Response surface method, Reliability-based design optimization.

1 Introduction

The first-order reliability method (FORM) and second-order reliability method (SORM) are popular in reliability analysis of geotechnical engineering involving uncertainties, which are well-documented in Ditlevsen (1981), Ang and Tang (1984), Baecher and Christian (2003) and Melchers (1999), for example. The spreadsheet-based procedures for FORM and SORM proposed in Low and Tang (2004, 2007) and Low (2014) provide an intuitive perspective and efficient tool for geotechnical reliability analysis. However, for most practical problems of underground rock tunnels, stand-alone numerical packages such as finite element method are usually involved, where the limit state functions are implicit. This will lead to some difficulties in carrying out reliability analysis with these easily conducted procedures. To overcome this obstacle, the response surface method (RMS) is commonly adopted to approximate the implicit functions with simple closed-form solutions (Lü et al. 2011, Lü and Low 2011).

In the traditional RSM technique, polynomial functions were usually adopted to fit the unknown limit state surface (LSS) in the vicinity of the most probable failure regions. However, some advanced models such as artificial neural network (ANN) (Lü et al. 2012) and moving least square method (MLSM) (Lü et al. 2012) have also been proposed and applied for reliability analysis of rock tunnel excavations. These surrogate models provide a variety of alternatives for reliability analysis under the framework of RSM. All models have their own advantages as well as disadvantages due to their unique characteristics, therefore, each of them plays a complementary role to others and would not be replaced. This paper will focus on practical reliability analysis and design of tunnels using RSM. Different response surface models, including polynomial functions, ANN and MLSM, are introduced and compared. In addition, the application of RSM to the reliability-based design optimization of a circular tunnel is also investigated.

2 Three considerations when perform reliability analysis using RSM

For the reliability analysis using response surface method, the computational accuracy depends on the fitting precision of the approximate response surface to real limit state function especially in the vicinity of the design point. Thus, there are three aspects which should be taken into consideration, i.e., the surrogate model type, the selection of sampling points and the algorithm of the procedure.

2.1 *Surrogate models*

The accuracy of the response surface in presenting the behavior of the actual limit state function largely depends on the surrogate model used for its generation. Brief introductions and comparisons to several surrogate models used in tunneling engineering are presented in this section.

2.1.1 *Polynomial function*

The polynomial function regarded as response surface is generally known as least square regression and are widely reported in literatures (Bucher and Bourgund 1990, Mollon et al. 2009). The polynomial-based response surface is generated to replace real limit state function from a set of sampling points. For example, a quadratic polynomial function has the form as below.

$$g(\mathbf{x}) \approx \tilde{g}(\mathbf{x}) = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n} \sum_{j=i}^{n} a_{ij} x_i x_j \tag{1}$$

where x_i is the basic random variables and n is the number of the random variables; And a_0, a_i, and a_{ij} are the unknown constant coefficients which are to be determined using the sampling points. Most commonly, least square is used to determine the coefficients that minimize the error of the approximation at the sampling points.

Generally, the widely used polynomial functions are linear and quadratic, in which the quadratic polynomial contains the quadratic polynomial function without and with cross terms. In Eq. (1), when all the coefficients of a_{ij} are zero, the polynomial function is then the linear polynomial.

Certainly, for most function fitting problems, a high-order polynomial will generate a relatively superior fitting accuracy, but may also yield the over-fitting result and may lead to an exponential increase in the required number of the sampling. Therefore, less attention has paid to the high-order polynomial response surface (Li et al. 2010).

2.1.1 *Artificial neural network*

Artificial neural network (ANN) is a powerful paradigm for mapping the relationship between a set of inputs and one or more outputs by means of training data obtained from either real experiments or numerical simulations. A number of applications of ANN in civil engineering have been reported (e.g., Cheng and Li 2008, Cho 2009, Zhang and Goh 2016).

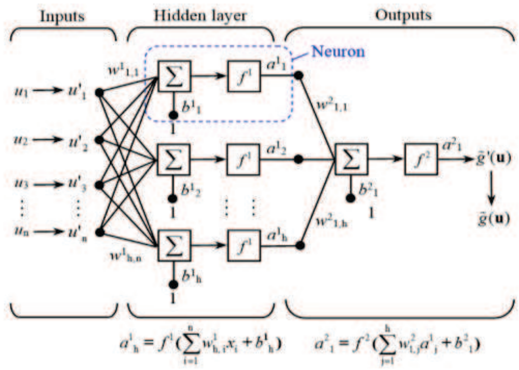

Figure 1. Architecture of the three-layer network with n inputs, h hidden neurons and one output.

The most widely used ANN is the multi-layer feed-forward back-propagation network (see Figure 1). The architecture of this ANN is composed of a number of neurons which are connected with weights and logically arranged into the input layer, the hidden layer and the output layer. However, the selection of the number of hidden neurons is more difficult because using too few neurons impedes the training process and prevents the correct mapping of inputs to outputs, while using too many neurons tends to overfit the data and requires more computation (Hagan et al., 1996). There is no general rule for determining the appropriate number of hidden neurons. In addition, the ANN network requires to be trained by adapting their weights and biases using optimization methods, which is a daunting task.

2.1.2 *Moving least square method*

In the moving least square method (MLSM), the approximated function can be written as:

$$\tilde{g}(\mathbf{x}) = \mathbf{p}(\mathbf{x})^{\mathrm{T}}\mathbf{a}(\mathbf{x}) \tag{2}$$

where **x** is the vector of random variables, $\mathbf{p}(\mathbf{x}) = [p_1(\mathbf{x}), p_2(\mathbf{x}), \cdots, p_m(\mathbf{x})]^{\mathrm{T}}$ is the basis function vector, **a**(**x**) is the undetermined coefficient which depends on the **x** coordinate.

The form of MLSM is analogous to the traditional least square method (LSM). But compared with the LSM, there are two significant improvements in MLSM: (1) The approximated function is not a simple polynomial or other general functions but consists of a coefficient vector **a(x)** and basis function **p(x)**, where both **a(x)** and **p(x)** are functions of **x** coordinate; (2) It considers that the value of fitted function $\tilde{g}(\mathbf{x})$ is only affected by the samples within the sub-domain of the **x** point and gives more weights to samples that are "nearby" rather than giving all samples equal weight. Therefore, the MLSM is a local approximation.

Due to the local approximation around each point through a weight function, the MLSM can achieve the reduction of the approximation error and the dependence on the type of basis function.

2.2 *Selection of training samples*

Generally, the accuracy of response surface not only rests on the type of surrogate model, but also depends on the design of experiments (DOE) for selecting the sampling point location and density. In reliability analysis, the random sampling method and central composite design method are commonly used to prepare sampling points. However, the random sampling method can not guarantee the distributed uniformity of samples in the domain of interest.

Recently, new experimental techniques are emerging for the DOE that provides the new alternatives for the selection of training samples, among which the Latin hypercube sampling (LHS) and uniform design (UD) are commonly employed sampling technique. These methods provide a space-filling technique that guarantees that the sampling points are scattered in the domain as uniformly as possible. In general, the UD is a deterministic method, whereas the LHS is a random method. Due to the stochastic nature of the LHS, its results are not the same for each sample, and the uniformity of space filling is not always satisfactory, which may create an unstable response surface, especially for a low number of sampling points. However, the use of LHS in computers is convenient and less limited by the increase in data dimension (Ji et al. 2017). And the application of UD can be efficiently performed with tabular procedures. A series of UD tables have offered great convenience to users. Using the existing UD table, the UD method can generate steady, uniform and representative samples conveniently. But additional computational effort may be required to obtain the tabularized uniform design data without available UD table, especially when the dimension is high and the number of samples is large.

2.3 *Procedures of reliability analysis using RSM*

After the determination of the surrogate model and the sampling points, a response surface can be readily established to approximate the implicit limit state function. Based on this obtained explicit response surface, the standard reliability method, such as the FORM, can be adopted to compute the reliability index and the corresponding design point conveniently.

It is essential to realize that the approximation of RSM should be as sufficiently well as possible to obtain the accurate reliability results. Thus, enough samples are needed to construct the response surface model. This approach that only built one response surface model can be found in the application of RSM in tunnel engineering (e.g., Li et al. 2016). However, one issue may be encountered that how to determine the appropriate number of samples.

In addition to above mentioned approach, the iterative algorithm is more commonly used in reliability analysis with RSM in which the response surface model is updated continuously based on the tentatively obtained design point. As the narrow region around the design point contributes most to the probability of failure, the iterative algorithm can improve the estimation accuracy of the response surface near the design point. For this reason, an adaptive sampling technique is proposed that the sampling range starts from a relatively large value and gradually reduced to a small value in the subsequent iteration. In the first iteration, the sampling points are around the mean value with a relatively large sampling range to roughly locate the position of the design point. Then new sampling points are added around the tentative design point with small range to gradually approach the true design point and improve the accuracy of response surface model around the design point. With such iterative procedure, the design point and the reliability index could be accurately seeked.

The practical procedure for tunnel reliability analysis which combined the response surface methods with FORM and SORM can be outlined as following:

Step 1: Prepare sampling points in U-space using UD.

Step 2: Build the samples set for response surface model by calculating the limit state function at each sampling point.

Step 3: Construct the response surface and compute reliability index and design point.

Step 4: Check convergence. Update the response surface model based on new added sampling points around design point until the convergence criterion is satisfied.

To illustrate the efficiency and accuracy of the proposed procedure, the circular rock tunnel example of Lü et al. (2017) is revisited here. Three types of response surface model, i.e., polynomial function, MLSM, and ANN, are compared. The results are presented in Table 1.

Table 1. Comparison of reliability results from different response surface methods

		MCS	MLSM-RS		RSM-1	RSM-2		RSM-3		ANN-RS	
			FORM	SORM		FORM	SORM	FORM	SORM	FORM	SORM
g_1	P_f(%)	0.089	0.104	0.092	0.104	0.104	0.085	0.104	0.087	0.104	0.087
	N	2×10^5	55	55	40	60	60	108	108	47	47
g_2	P_f(%)	0.451	0.441	0.557	0.441	Non-convergence		0.441	0.447	0.441	0.455
	N	2×10^5	55	55	56			144	144	55	55
g_3	P_f(%)	0.734	0.670	0.780	0.670	0.671	1.055	0.670	0.728	0.670	0.731
	N	2×10^5	71	71	48	75	75	144	144	55	55

Note: RSM-1, 2, 3 denote first-order polynomial-RS, second-order polynomial-RS with and without cross terms, respectively; N denotes the total number of the sampling points.

As shown in Table 1, the FORM results for different methods are equivalent. For the MLSM-RS, the total number of required sampling points is relatively small compared with the polynomial-RS and almost equivalent with the ANN-RS. Although the first-order polynomial-RS (RSM-1) can achieve the same reliability index with fewer samples for the case in hand, it cannot perform SORM analysis because the first-order polynomial is incapable of capturing the curvatures around the design point. Regarding the SORM results, the ANN-RS and the second-order polynomial-RS *with* the cross terms (RSM-3) show reasonable accuracy when compared with the MCS results. And MLSM-RS also shows good accuracy except that the error of one limit state function is relatively large. However, it is worthy to be noted that RSM-2 encounters the non-convergence issue, in which a pseudo response surface may occur during the iterative process.

3 Reliability-based Design Optimization Using RSM

In general, the mathematical formulation of a reliability-based design optimization (RBDO) problem can be defined as the form:

$$\begin{aligned} &\text{minimize} && f(\mathbf{d},\mathbf{x}) \\ &\text{subjec to} && P\left(G_i(\mathbf{d},\mathbf{x}) \le 0\right) \le P_i^{\text{Target}}, i = 1,2\cdots,nc \\ & && g_j(\mathbf{d},\mathbf{x}) \le 0 \quad , j=1,2\cdots,mc \\ & && \mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U \end{aligned} \tag{3}$$

In can be seen, the formulation of the RBDO methods involves two nested optimization procedures, i.e., the outer design optimization and the inner reliability analysis. According to the differences in the manner of dealing with the two optimization procedures, the RBDO algorithms can be classified into three categories, namely, the double-loop approach, the single-loop approach and the decoupled approach (Aoues and Chateauneuf 2010), among which the double-loop approach is the most direct method to solve the RBDO problem. The double-loop approach deals with reliability assessment in the inner loop and the design optimization in the outer loop, which leads to a nested optimization since the reliability analysis in essence is also an optimization procedure.

On the other hand, the limit state functions for tunnels are usually not available in their explicit forms, which leads to some difficulties in carrying out reliability analysis in the framework of optimization-based approaches. At this time, the RSM can again be adopted to deal with RBDO. The proposed procedure in section 2.3 with the iterative algorithm and adaptive sampling technique is used for computing the probabilistic constraints in RBDO. And here a linear polynomial is used as response surface model because of the less calculation cost for linear polynomial response surface.

A circular rock tunnel example presented in Lü et al. (2017) is analyzed to get an optimal design scheme. Two design variables, the shotcrete thickness and the distance from the shotcrete installation position to the tunnel face, are optimized to achieve the required reliability level with the minimum cost.

4 Conclusions

The application of RSM to the probabilistic analysis of rock tunnel excavations are introduced in this paper. Three types of surrogate model, i.e. polynomial function, ANN and MLSM, are presented and discussed. The efficient and practical procedure which combined the RSMs with FORM and SORM are illustrated via examples of a circular tunnel.

The RBDO, in which two nested optimization procedures, i.e., the outer design optimization and the inner reliability analysis, are also presented. And a procedure that the RSM is used to deal with RBDO is presented to carry out the optimization design of tunnels.

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