

## COMPARATIVE STUDY ON RESILIENCY OF COMPLEX METRO NETWORK BETWEEN SHANGHAI AND PARIS

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In view of the high capability of relieving the pressure of public transportation via urban metro system, the safety of this complex network system is always the first concern required by the engineers and the government. The safety control becomes more important under the condition of disruption of the system caused either by unexpected breakdown of facilities or by intentional human attacks. Recently, a general framework of resiliency assessment model for metro network has been proposed by the authors in a systematic point of view covering both the vulnerability and recoverability. With this model in hand, this paper presents a comparative analysis for two typical metro systems in the world, i.e., Shanghai metro and Paris metro. The network connectivity is regarded to be the network performance indicator in this paper. From a topology analysis, the Paris metro network is found to be more compact than that for Shanghai metro. Both these two metro networks are small-world and scale-free network, which means that they are robust under random failure of stations but vulnerable under intentional attacks. These qualitative findings have been quantitatively verified by the resiliency analysis using the model proposed previously.

**Keywords:** Metro network, Network efficiency, robustness, resilience.

### 1 Introduction

Metro system has become popular for big cities because of its advantage of relief of ever-increased pressure of on-ground transportation. However, as a transportation system becomes larger, the vulnerability or the risk of such a large system subjected to any disruption becomes severer or sometime unacceptable to owners and publics. In particular with the network scaled metro system, the disruption of one station might not only affect the connectivity of one line, but also extend to the other lines or even the whole network. Hence, a clear understanding of the vulnerability of the metro network under disruptions should be of great importance to the safe operation. However, nowadays, only the vulnerability analysis for the network after disruption can not fully guide the repair work for the metro network. The repair sequence is sometimes decided without any rational assessment. The fast and efficient repair strategy should be urgently necessary especially for the case of multiple exchange station. In view of this situation, the resilience analysis combining both the robustness after disruption and the recovery after repair works can be helpful to the general assessment of the safety level of large metro system.

The authors have recently presented a general model of resiliency analysis including both the vulnerability and recovery analysis (Zhang, 2016; Zhang et al., 2018). This model has been

applied into the Shanghai metro system which has a 303 stations and 350 tunnels of the network size. The vulnerability and recoverability of Shanghai metro have been revealed by using this model. At this moment, a lot of big cities has a large size of metro network. It should be helpful to the metro operators to understand the safety level of metro network by using this analysis model. Hence, to follow up this research line, in this paper, the Paris metro network has been newly added and analyzed by using the same resiliency model. An interesting comparative study on the resiliency of the metro network system is discussed in details in this paper. Before the comparison is carried out, the resiliency model for metro network is briefly introduced.

## 2 Characteristics for Metro Networks

Basically, three steps should be carried out to generally analyze the resiliency of a metro network: a) basic mapping of a metro network into a topological graph; b) defining and measuring vulnerability and robustness of the metro network; and c) developing resiliency metrics. In this section, the topological mapping of the metro network is briefly explained. The metro station is simulated by the nodes, while the metro tunnel connecting two successive stations is simulated by link. An L-space type topological graph is used for metro network in this paper, i.e., the same as that used in the previously proposed model (Zhang et al., 2018). The typical topological maps of the Shanghai metro system and Paris system are plotted in Fig.1. The basic statistics of the two metro network are summarized in Table.1.

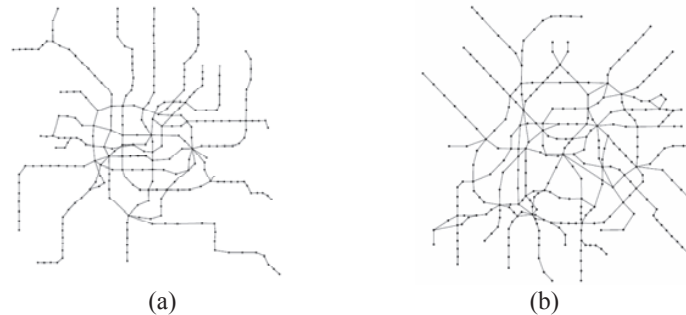


Fig.1 Topographic map of metro network for a) Shanghai; and b) Paris

Table 1 Characteristics indicators of Shanghai metro network

	Shanghai	Paris
Node Number $N$	303	302
Link	350	357
Characteristics path length $L$	14.87	12
Diameter of network $D$	41	34
Network cluster coefficient $C$	0.0082	0.0132
Limit state of $L$ ( $\ln N / \ln k^*$ )	6.82	6.7
Limit state of $C^*$ ( $k^* / N$ )	0.0076	0.0078

From Table 1, it is quite obvious to see the two networks are quite comparable in the size of nodes and links of the network. The Shanghai metro has 303 stations with 350 tunnels, while the Paris has 302 stations and 357 tunnels. But the performance of the network explained by the detailed characteristic parameters are quite different. The characteristics path length  $L$  is defined as the average over all path length  $d_{ij}$  for all pair of nodes in the network. The Paris network has a smaller value of  $L$  compared to that of Shanghai network, which means the Paris network is on average more efficient of the network between any two stations on the average point of view.

From the calculated network cluster coefficient  $C$ , both Shanghai and Paris appear to be the small-world network, as the cluster coefficient  $C$  is larger than the limit state of  $C^*$  both for Shanghai and Paris metros. From the definition of the small world for a network, the network is regarded to have an intensive connectivity locally and a good quality of operation for the whole network. A statistical analysis of the node degree for each station of the two metro networks is carried out. Fig. 2 plots the corresponding node degree  $k$  against the relative frequency of node degree ( $p(k)$ ) both for Shanghai (Fig.2a) and Paris (Fig.2b) in the logarithm form. It is obvious from both figures that the relationship between  $\ln k$  and  $\ln p(k)$  follows a sound linearity with a determination of coefficient  $R^2$  above 0.81. Hence, the network both for Shanghai and Paris metro can be regarded to be the scale-free network, which from its definition means that the network is robust under random failure but vulnerable under the intentional failure (Barabási and Albert, 1999).

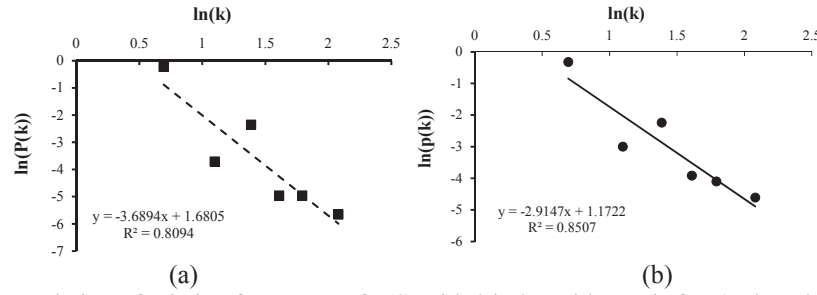


Fig.2 Correlation of relative frequency of  $p(k)$  with  $k$  in logarithm axis for a) Shanghai and b) Paris.

### 3 Vulnerability of Metro Network for Shanghai and Paris

To quantitatively assess the performance of a network, there are many of metric or indicators. As for a metro network, the network efficiency  $E_f$  is defined as the indicator to quantify the network connectivity as below:

$$E_f = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{d_{ij}} \quad (1a)$$

where  $N$  is the node number of a network and  $d_{ij}$  is the path length between node  $s_i$  and  $s_j$ . The value of  $E_f$  could range from zero to one, i.e., zero means no connectivity between any two nodes in the network, whereas one means any two nodes are connected. This specifies the lower and upper bounds for network connectivity.

Once the station or tunnel have been disrupted for some reason, the node or link is removed from the network. The network efficiency or network connectivity  $E'_f$  after the removal can be re-calculated following Eq. 1 with the left node number and path length. The parameter  $E'_f$  is regarded to be the robustness of the network while the difference between  $E_f$  and  $E'_f$  is the vulnerability of the network.

$$E'_f = \frac{1}{N'(N'-1)} \sum_{i \neq j} \frac{1}{d'_{ij}} \quad (1b)$$

$$V = \Delta E_f = E_f - E'_f \quad (1c)$$

Theoretically speaking, the connectivity of topographical metro network could be affected by failures both from metro stations (nodes) and metro tunnels (links). In this paper, the failure

of metro stations is specifically considered but not the links. The failure of a metro station could be classified into two types. The first failure type occurs randomly and is caused possibly by natural hazards, power and signal malfunction or even human errors. The probability of occurrence of such a failure can be assumed to be the same for all stations in a network. This assumption can be changed and varied values used without affecting the overall approach although the computationally complexity increases. The second type of failure is of an intentional nature as caused mainly on purpose by arson or terrorism. Usually, the most important station in a large-scale network, e.g., a multi-line interchange, could be intentionally targeted and disrupted. For topological analysis, the failure of a station can be modeled by removing the node from network. As for the random failure, the node is removed randomly following a specific probability distribution function (Crucitti et al., 2004). As for the intentional failure, assuming that attacks occur in the order of importance of the nodes, the nodes are removed sequentially following a descending order of magnitude of the node degree  $k$  (Crucitti et al., 2004).

Following the above vulnerability analysis by using two different removal strategies, Fig.4 plots the corresponding network connectivity  $E_f$  after the continuous removal of the nodes in the metro network. The removal of the nodes is conducted one by one gradually. The square dots line represents the random attack, while the circular dots line represents the intentional attack. Fig.4a is the vulnerability analysis for Shanghai metro, and Fig.4b is for Paris metro. For a clear comparison, the vertical axis in Fig. 4 is normalized by the initial network connectivity. Hence, all the network connectivity is decreased from unit after the removal of the nodes. By comparing the decreasing trend of random attack to the trend of intentional attack, it is obvious both from Fig.4a and 4b that the connectivity is decreasing slowly for random attack than that for intentional attack. It suggests that both Shanghai metro and Paris metro is much more robust for random attack while vulnerable for intentional attack. It is consistent with the qualitative analysis of scale-free network.

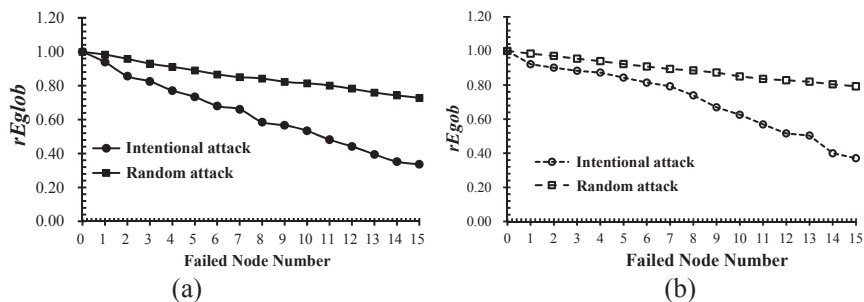


Fig.4 Robustness of metro network connectivity for different cities: a) Shanghai and b) Paris.

Fig.5 plots the comparison of the network connectivity decreasing trend between Shanghai and Paris both for random attack and intentional attack. Please note that Shanghai metro and Paris metro almost have the same number of nodes and links, which makes the comparison purely reflects the network efficiency or the network performance. From Fig.5, it is indicated that Paris metro is slightly more robust than Shanghai metro both under the random attack scenario and the intentional attack scenario. In particular with the intentional attack as shown in Fig.5b, Paris metro has a much higher residual connectivity as the number of removed nodes is not so many. It is consistent with findings from the network cluster coefficient  $C$  for Shanghai metro and Paris metro. The Paris metro has a much high cluster coefficient at 0.0132 compared to the value for Shanghai metro, i.e., 0.0082. As the network is much clustered, one node failure near the clustered node zone might not greatly affect the global network efficiency.

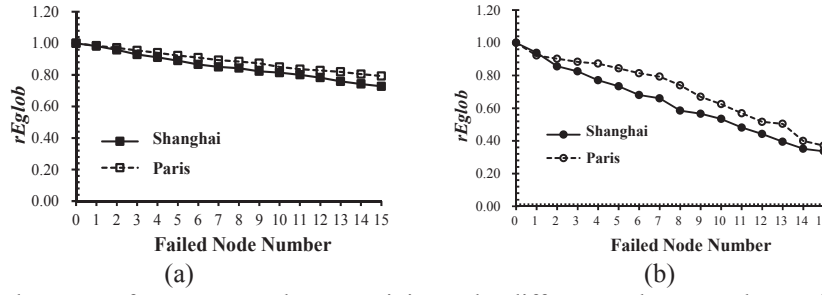


Fig.5 Robustness of metro network connectivity under different node removal type a) random and b) Intentional

#### 4 Resilience Analysis

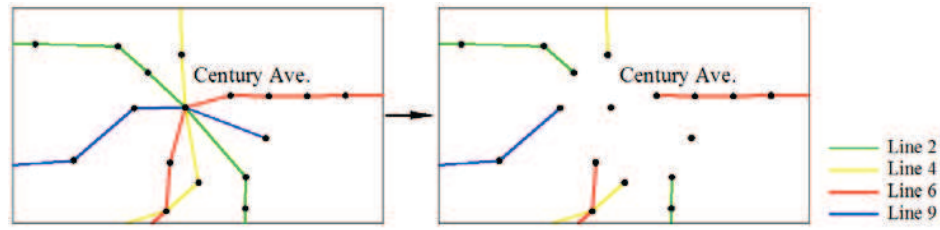


Fig.6 Four lines exchange station in Shanghai metro

As one of the evaluation metric for network resiliency, the resilience triangle is used here which essentially means lost performance curve wiped along with the disruption time  $t_h$  compared to the un-disrupted performance, which is represented by the following equation:

$$R_e = \frac{\int_{t_0}^{t_0+t_h} [E_f(t)] dt}{t_h E_{f0}} \quad (2)$$

where  $t_0$  is the time moment when the disruption occurs,  $E_{f0}$  stands for the un-disrupted connectivity and  $E_f(t)$  is the disrupted performance after the failure of nodes and before the final recovery of all the failed nodes. Due to page limit, this paper only illustrates a typical quantification of the resiliency ability of Shanghai and Paris metro network by using the four line exchange station as an example, i.e., Century Ave station in Shanghai and Bienvenue station in Paris. Fig.6 illustrates the Century Ave station connecting the metro line 2, 4, 6 and 9. If this station is failed and removed from the whole network, there are four steps to fully recover the station by assuming that each line can only be recovered by a single step. However, the question remained is how to select the best repair sequence as fast and efficient as possible. Potentially, there are about 24 choices for the repair sequence ( $24 = 4 \times 3 \times 2 \times 1$ ). By using Eq. 2 as shown above, each repair sequence could obtain a corresponding value of  $R_e$ . Obviously, the higher the value of  $R_e$  is derived, the best the repair sequence should be. By repeating the calculation of  $R_e$  for Century Ave station recovery, the largest value of  $R_e$  is equal to 0.9732 and associated with the best recovery sequence: 1) line 2  $\rightarrow$  2) line 6  $\rightarrow$  3) line 9  $\rightarrow$  4) line 4. Fig.7a shows the typical performance recovery following the above repair sequence. It seems from Fig.7a that the recovery of performance is almost linearly increased. By following the similar analysis procedure for Bienvenue station of Paris network, i.e., 4-line exchange station connecting line 4, 6, 12 and 13, the highest value of  $R_e$  is derived equal to 0.9828. The best sequence is first to

repair line 13, then to repair line 6, thirdly to repair line 4 and finally to repair line 12. The corresponding recovery curve of the disrupted performance is shown in Fig.7b. Compared to Century Ave station in Shanghai metro, the recovery curve for the best sequence in Paris appears to be more nonlinearly increasing with the repair steps. The increment of the performance is sharper compared to Century Ave station, which is better in the sense of fast recovery of the disrupted network.

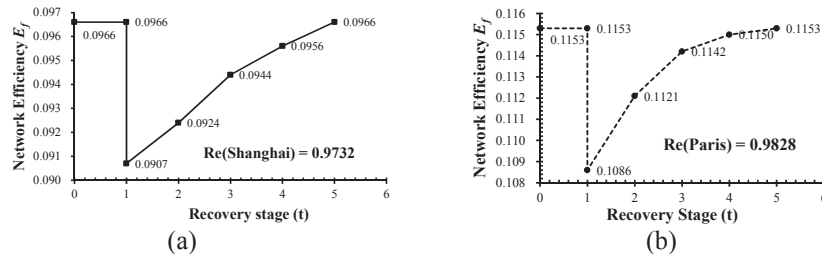


Fig.7 Network efficiency during the recovery stage for a) Shanghai; and b) Paris

## 5 Conclusion

Based on the previously proposed resiliency analysis model, this paper presents a comparative study on the network resiliency of Shanghai metro and Paris metro system. A detailed comparison including the basic network characteristics in normal condition, the vulnerability and the robustness after the disruption, and the recovery strategy during repair procedure is typically highlighted and analyzed. It is found that both Shanghai and Paris are the small-world and scale free network system. It means that the network has strong connectivity locally, strong robustness of network connectivity under random attacks; but serious vulnerability under intentional attacks. Although the number of station and tunnels are the same between Shanghai and Paris, the robustness of Paris metro network is higher than that of Shanghai metro in particular with the robustness under intentional attack. As for the resiliency ability of four line exchange station, the Bienvenue station of Paris network could be recovered much faster than the Century Ave station of Shanghai network. It is consistent with the high cluster coefficient  $C$  for Paris compared to the coefficient for Shanghai metro.

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