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PROBABILISTIC ANALYSIS FOR FOUNDATION-SOIL SYSTEMS SUBJECTED TO VERTICAL EXCITATIONS

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This paper proposes an analytical method to calculate the probabilistic response of a rigid foundation on a soil medium subjected to vertical excitations. The uncertainty of the shear-wave velocity in soil is considered in this study. Simplified models are used to simulate the dynamic behavior of the unbounded soil. Numerical studies have been conducted to investigate the cumulative density function and the response spectrum for the foundation displacement. The numerical results show that the proposed method can generate the probabilistic response close to the Monte-Carlo simulation results. It is also found that the effectiveness of the simplified model significantly influence the computing accuracy of the probabilistic foundation responses in dynamic SSI analysis.

Keywords: Soil-structure interaction, simplified models, probability distribution, seismic excitation.

1 Introduction

In general, the seismic analysis of structural systems shall consider the inertial and kinematic interactions existing between structures, foundations and soil. The inertial interactions caused by the inertia from vibration of the structure and foundation while the kinematic interactions existing between the massless foundation and unbounded soil induced by free-field ground motions. Some studies have showed that the soil-structure interaction (SSI) significantly affects the dynamic responses of the structures subjected to ground excitations (Mylonakis and Gazetas 2000). To consider the uncertainty is pervasive in the measurement of soil properties, a few studies have recently investigated the probabilistic responses of SSI systems. Lutes et al. (2000) presented a modal-analysis-based approach to quantify the effect of uncertain soil–foundation and superstructure properties on structural response for seismically excited SSI systems. The stochastic response property of the SSI systems was investigated in frequency domain. In their study, a simple physical model proposed by Wolf (1997) was used to simulate the dynamic behavior of the unbounded soil. They found that the larger effects of uncertain soil properties are seen for the lower-frequency resonance peaks while the larger effects of uncertain structural properties are observed for the higher-frequency resonances. Chaudhuri and Gupta (2002) proposed a mode acceleration approach in frequency domain to analyze the seismic responses of secondary systems which are supported on flexible-base primary systems, while the shear-wave velocity and Poisson’s ratio of the soil medium are simulated to be statistically independent and normally distributed random variables. They found that uncertainty in shear-wave velocity and Poisson's ratio should be considered when the SSI is significant. Masoud (2011) used Monte-Carlo simulation to investigate the influence of foundation flexibility on the structural seismic response by considering the variability in the SSI system and uncertainties in the ground motion characteristics. The uncertainty in soil properties including shear-wave velocity, shear-wave velocity degradation ratio, Poisson's ratio and soil mass density are simulated in the numerical simulation. Their numerical results indicated that there is 30–50% probability for an increase in the total structural displacement of over 10% due to SSI effects, 10–30% probability for amplification of greater than 25% and 2–15% for an increase of over 50% in this response.

In the past, probabilistic SSI-related studies used theoretical impedance functions or simple physical models to simulate the dynamic interaction between the foundation and soil. The uncertainty of the soil properties seems to have been well considered in the existing studies; however, no attempt has been made to intensively investigate the effects of modeling the unbounded soil on the probabilistic SSI response. Therefore, this paper presents an analytical method to calculate the probability distribution of the dynamic response for a foundation-soil system while various simplified models can be used to simulate the unbounded soil. The accuracy of the proposed method is to be validated using the Monte Carlo simulation results. Moreover, this study also investigates the probabilistic response using the simplified models proposed by Chen & Shi (2006) and Wolf (1997).

2 Probabilistic Response of a Foundation-Soil System

This study aims to investigate the dynamic response of a rigid foundation on a soil medium, as shown in Figure 1. Consider that the rigid foundation is subjected to vertical excitation and the dynamic equilibrium equation of the foundation can be written as follows:

$$M\ddot{Y}_t(t)+K_z\left[Y_t(t)-Y_g(t)\right]=0 \tag{1}$$

where Y_t = total displacement of the foundation, Y_g = displacement of the input motion, M = foundation mass, K_z = dynamic impedance function. Suppose that the input motion is harmonic excitation, which is represented as $Qe^{i\omega t}$ with the displacement amplitude Q and the excitation frequency ω. The resulting foundation displacement can be then represented as $We^{i\omega t}$ with the displacement amplitude W which can be derived from Eq. (1), as shown below.

$$W=\frac{K_z}{K_z-\omega^2 M}Q \tag{2}$$

The dynamic impedance function can be further represented as follows:

$$K_z=kGR(k_z+ia_0c_z) \tag{3}$$

where k = static foundation stiffness, G = shear modulus of the soil, R = the characteristic length of the foundation, k_z = dynamic stiffness coefficients, c_z = dynamic damping coefficients, a_0 = $\omega R/V$ denotes the dimensionless frequency ratio with the shear-wave velocity of the soil V. The foundation displacement amplitude is then derived by substituting Eq. (3) into Eq. (2), i.e.,

$$W=Q\times\sqrt{\frac{k_z{}^2+(a_0c_z)^2}{(k_z-\frac{a_0^2b}{k})^2+(a_0c_z)^2}} \tag{4}$$

where $b = M/\rho R^3$ with the soil density ρ. In general, the dynamic impedances k_z and c_z are frequency-dependent and computed by analyzing the interaction between the foundation and soil. In Eq. (4), the dynamic impedances are assumed to be the function show below:

$$k_z = 1 - \mu a_0^2 \quad \text{and} \quad c_z = \gamma \tag{5}$$

Consider that the shear-wave velocity of the soil V is a random variate with a probability density function (PDF), $f_V(v)$. The derived PDF, $f_W(w)$,of the foundation displacement amplitude can be evaluated using the fundamental probability theory (Ang and Tang, 2007), as shown below.

$$f_W(w) = \sum f_V(g^{-1}) \left| \frac{dg^{-1}}{dw} \right| \tag{6}$$

where $g^{-1}(w)$ is the inverse function used to compute the corresponding shear-wave velocity when the foundation displacement $W = w$ is given, which can be further calculated from Eq. (4). This study will use Eq. (6) to compute the probability distribution of the foundation response subjected to vertical excitations while the existing simplified models are used to simulate the dynamic behavior of the unbounded soil.

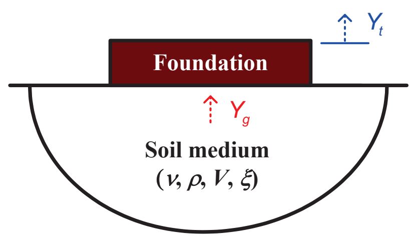

Figure 1. A foundation-soil system

3 Numerical Studies

This section analyzes the probabilistic response of a rigid circular foundation resting on a uniform elastic half-space subjected to vertical excitations. The parameters of the foundation-soil system are listed in Table 1. In this study, the shear-wave velocity of the soil is assumed to be a lognormal variate with a mean value 400 m/sec and a coefficient of variation 30%. The PDF of the rigid foundation are calculated using Eq. (6) while the simplified model proposed by Chen & Shi (2006) and Wolf (1997) are used to simulate dynamic impedance function of the uniform half-space. The simplified model used here is just a single-degree-of-freedom oscillator; however, Chen & Shi (2006) and Wolf (1997) proposed the curve-fitting technique and the equivalent model theory to evaluate the modeling parameters, respectively.

Table 1. Analysis parameters of the numerical study

Parameter	Value
Input displacement amplitude, Q	0.01 m
Radius of the foundation, R	5 m
Foundation mass, M	500,000 kg
Soil density, ρ	2000 kg/m^3
Poisson's ratio of the soil, ν	0.33

Figure 2 shows the cumulative density function (CDF) calculated from the derived PDF of the foundation displacement amplitude $f_W(w)$ while there are four excitation frequencies $\omega = 5$, 10, 15, and 20 Hz considered in the numerical study. To verify the accuracy of the proposed method, the CDF calculated from Monte Carlo simulation (MCS) with one million runs are also displayed. The theoretical impedance functions presented by Veletsos and Tang (1987) are used in MCS to make a rigorous comparison. It is found that the CDF analyzed by the proposed method consists much well with that by MCS when the excitation frequency is 5 Hz. When the excitation frequency increases to 20 Hz, the CDF calculated by the proposed method still agrees well with that by MCS when the Chen & Shi's model is used to simulate the unbounded soil. In contrast, when the Wolf's model is used, the CDF computed by the proposed method could have remarkable deviations in comparison to the MCS results. Moreover, the deviations would become larger when the excitation frequency increases. Besides, Figure 3 shows the CDF of the response for a rigid cylindrical foundation with an embedment ratio 0.25. Observe again that the results of the proposed method match well with that by MCS.

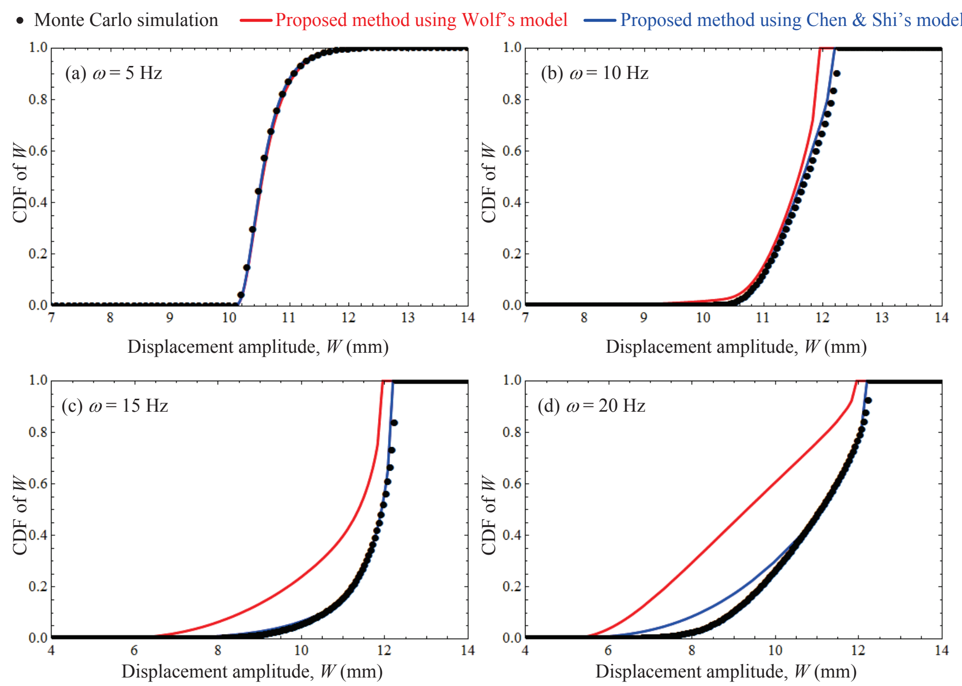

Figure 2. CDF of the displacement amplitudes for the surface circular foundation

This study also computes the probabilistic displacement response spectrum for the two cases considering the mean only and the mean plus one and two times its standard deviation, as shown in Figure 4. The excitation frequencies range from 1 to 30 Hz in this study. The maximum displacement responses are also computed using the proposed method and the MCS technique. It is observed that the maximum response computed from the proposed method using Chen & Shi's model consists much well with that by MCS when the excitation frequency is lower than 20 Hz. When the excitation frequency increases to 30 Hz, only a little deviation can be found between the results computed from the MCS and the proposed method. In contrast, when the Wolf's model is used in the proposed method, the mean of the maximum response ranging from 10 to 30 Hz will be considerably underestimated. Some similar results can be also found in the response spectrum considering the mean plus one and two times its standard deviation. These results show that the proposed method effectively calculate the probabilistic response for a foundation-soil system considering the uncertain shear-wave properties in soil. It is also found that the simplified model to simulate the unbounded soil significantly influence the accuracy of the probabilistic foundation response in dynamic SSI analysis. Besides, similar agreement is also found in Figure 5 for the embedded cylindrical foundation with an embedment ratio 0.25.

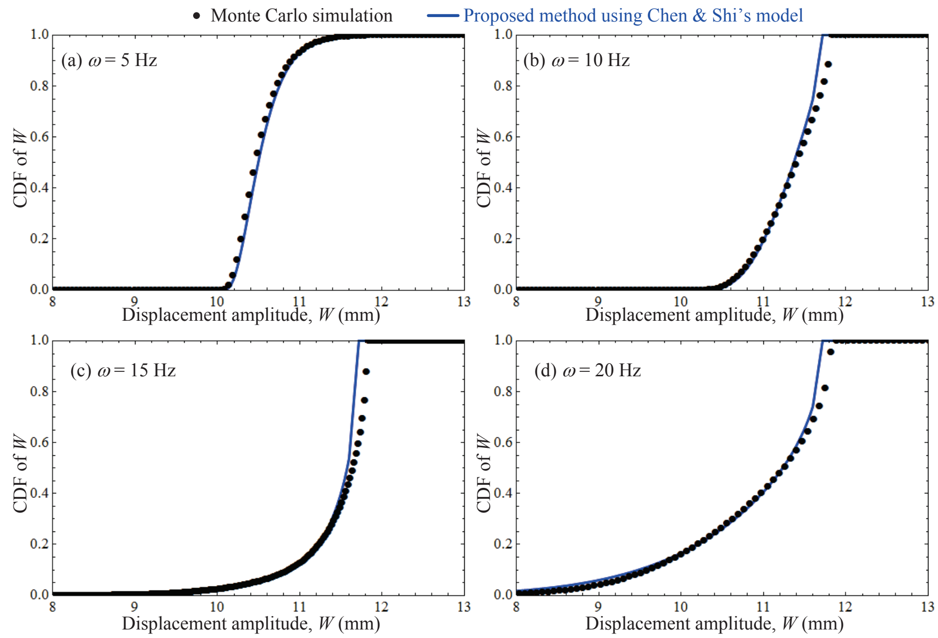

Figure 3. CDF of the displacement amplitudes for the embedded cylindrical foundation

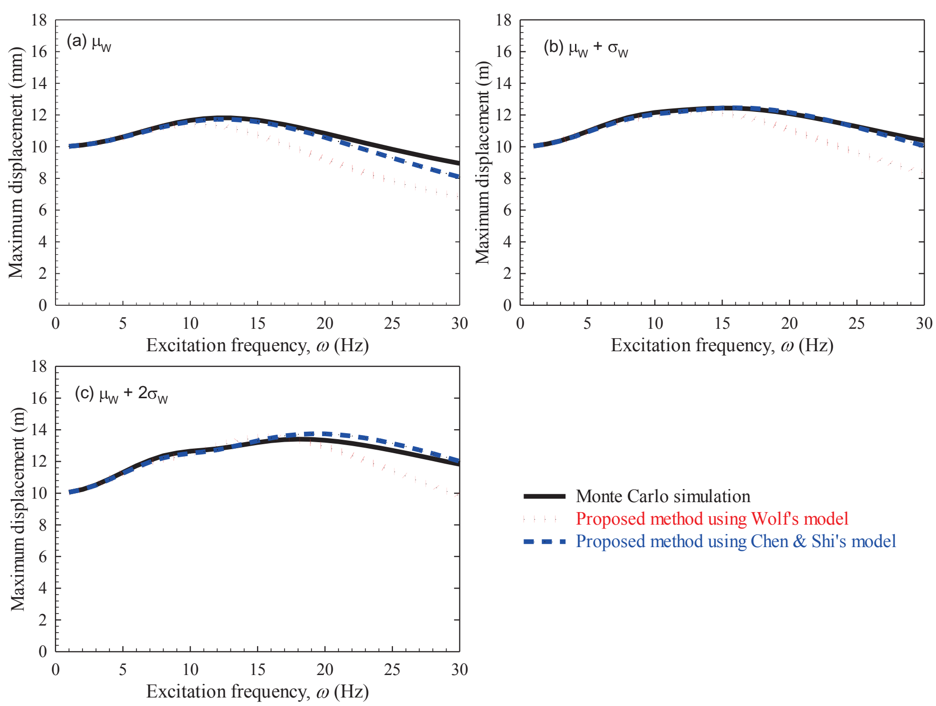

Figure 4. Displacement response spectrum for the surface circular foundation

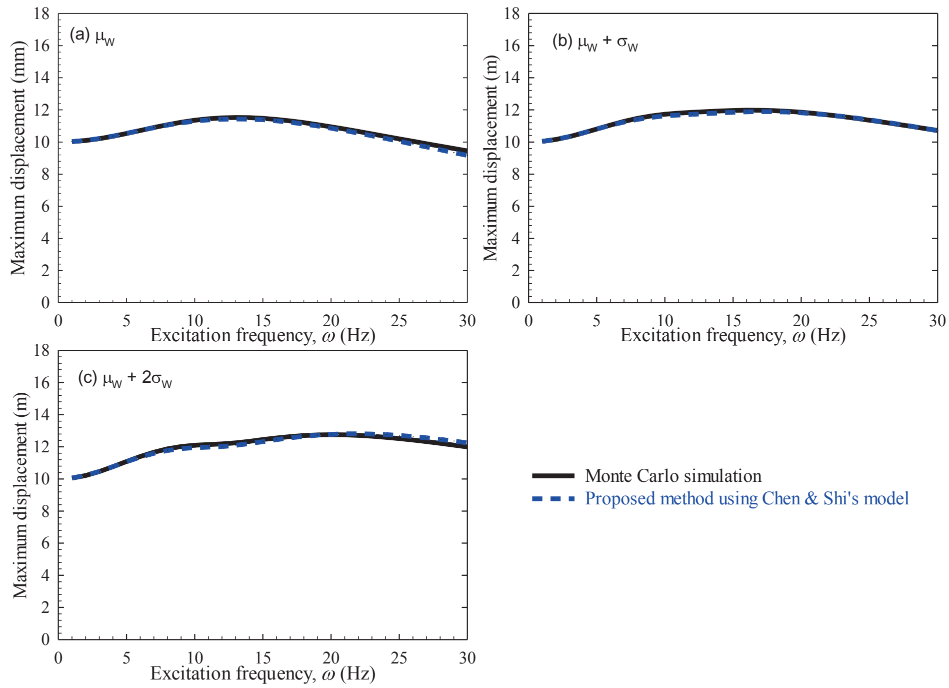

Figure 5. Displacement response spectrum for the embedded cylindrical foundation

4 Conclusions

A method is presented in this study to compute the probabilistic response of a rigid foundation on a soil medium subjected to harmonically vertical excitations. The proposed method uses existing simplified models to simulate the dynamic impedance function of the foundation-soil system. The probabilistic foundation responses are investigated through a numerical study considering the uncertain shear-wave velocity in soil. The cumulative density function and the response spectrum calculated by the proposed method are found to agree well with the results by Monte Carlo simulation. Therefore, the proposed method using Chen & Shi's model can effectively compute the probabilistic dynamic response in a foundation-soil system.

References

Ang, A. and Tang, W. H., Probability Concepts in Engineering: Emphasis on Applications to Civil & Environmental Engineering, Hoboken, NJ: John Wiley & Sons, 2007.

Chaudhuri, S. R. and Gupta, V. K, Variability in Seismic Response of Secondary Systems Due to Uncertain Soil Properties, *Engineering structures*, 24(12), 1601-1613, 2002.

Chen, S. S. and Shi, J. Y., Simplified Model For Vertical Vibrations of Surface Foundations, *Journal of Geotechnical and Geoenvironmental Engineering*, 132(5), 651-655, 2006.

Lutes, L. D., Sarkani, S., and Jin, S., Response Variability of An SSI System With Uncertain Structural and Soil Properties, *Engineering Structures*, 22(6), 605-620, 2000.

Moghaddasi, M., Cubrinovski, M., Chase, J. G., Pampanin, S., and Carr, A., Probabilistic Evaluation of Soil–Foundation–Structure Interaction Effects on Seismic Structural Response, *Earthquake Engineering & Structural Dynamics*, 40(2), 135-154, 2011.

Mylonakis, G., and Gazetas, G., Seismic Soil-Structure Interaction: Beneficial or Detrimental? *Journal of Earthquake Engineering*, 4(03), 277-301, 2000.

Veletsos, A. S., & Tang, Y. (1987). Vertical Vibration of Ring Foundations. *Earthquake engineering & structural dynamics*, 15(1), 1-21.

Wolf, J. P., *Foundation Vibration Analysis Using Simple Physical Model*, Prentice-Hall: Englewood Cliffs, NJ, 1994.