

## Review of flexibility measures for the design and operation of infrastructure

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Infrastructure systems, due to their long life spans, are subject to different sources of uncertainty that may have a negative impact, but could also create opportunities to increase the system's value. The contradictory effect of uncertainty makes designs that can reduce the undesirable outcomes, or position the system to take advantage of new circumstances, highly attractive. Flexibility, understood as the ability of a system to change and adapt, is a concept that allows designers and decision-makers to improve designs and management processes in face of uncertainty. However, there is neither a universal measure of flexibility nor a definitive method to quantify its cost and benefits, which limits its use by decision-makers. This paper presents a state of the art review of the most recent developments in measuring flexibility, its value and its application as a decision tool. We also present the development of a novel methodology that combines notions and ideas discussed in the literature review. The methodology focuses on how flexibility can be used to enhance the design and operation of infrastructure.

*Keywords:* Flexibility, changeability, adaptability, engineering systems.

### 1 Introduction

Engineering systems are always under the weight of uncertainty. Changes in the market preferences, alterations in the environmental demands and technological advancements are all difficult to predict, and all can affect the system's response and the value it provides to the stakeholders. For these reasons, long-lasting infrastructure systems that can be adapted easily to face new requirements are highly desirable. A flexible or changeable design should allow the system to reduce the negative impacts of uncertainty and take advantage of new opportunities (Cardin 2017). Despite the apparent advantages of introducing flexibility into a system, there is still a lack of rigorousness in the definition of the concept. In some instances, authors use flexibility interchangeably with changeability and adaptability; in others, one of the "-ilities" mentioned is a component of the rest. This lack of consensus in the terminology has been, expectedly, transferred to the mathematical representation of flexibility. The problem of how to quantify the "level" of flexibility in a system and how to assess the value provided by its introduction is still a question without a definitive answer. Furthermore, the main problem is not measuring flexibility, but rather finding if its introduction generates value (Fitzgerald 2012). The flexibility index presented in this paper is a small contribution that aims to capture the flexibility of the system and answer the question "which system is more flexible?". Once this question is answered, the next step is to compare the value generated by the alternatives, and determine how much a decision maker should pay to have a flexible system. If such value is positive, it would suggest that flexibility is indeed an effective measure to protect and even increase the value

delivered by the system. The paper is organized as follows: in the first section, a concise review of the most important developments regarding the valuation and application of flexibility in engineering contexts is presented. In the second section, a novel flexibility index is introduced with two application examples. In the final section, the results are discussed and some conclusions are presented

## 2 Literature Review

### 2.1 Concept analysis

The word *flexibility* is usually associated with the idea of "ease of change". A material is considered flexible if it bends (changes shape) under a small load, a schedule is flexible if it can be rearranged with no consequences to the project, and even a person can be regarded as flexible if they can consider different approaches when dealing with a problem. However, in academic contexts a much more detailed definition is needed. The problem is that many ideas are usually associated with the word *flexibility*, which promotes a liberal and gratuitous use.

For August-Brady (2000) flexibility is an "*integrative, evolving, resilient response to recognized change and uncertainty based on openness and willingness to change, that results in a greater diversity of choice, effectiveness and efficiency in outcomes*". The definition provided mentions four aspects that are the core of the concept in engineering systems. First, flexibility is a response to uncertainty, if there is no change or the possible changes in the external inputs to the system are completely determined, there is no need for a flexible design. In second place, *openness and willingness to change* associates the response to uncertainty with a change in the system, i.e., the system is designed to go through adaptations instead of being designed to ignore or being immune to external changes. In third place, *diversity of choice* is related with being able to take advantage of new opportunities, as mentioned by Cardin (2015). Finally, *effectiveness and efficiency in outcomes* is associated with the value provided to the system by flexibility.

Other authors share similar ideas: for Olewnik (2006) a flexible system must maintain a desired performance through real time changes in the configuration. For Niese (2014) and Ross (2006), flexibility depends on the number of paths a system can take at a certain time. For Fricke (2005), flexibility is the ability of a system to be changed easily due to changing environments. For Swaney (1985) and Pistikopoulos (1990), is an ability of the design to accommodate process variations. In conclusion, each author has their own definition, but the key word "change" is a constant in all of them.

### 2.2 Flexibility quantification

The first problem that comes into sight at analyzing a system, is how to measure the amount of flexibility it has. From a decision-maker point of view, knowing which design alternative is more flexible and by how much can help in the decision process. In this section, a literature review of some methodologies to quantify the flexibility in a system is presented.

Swaney (1985) developed a flexibility index for chemical plants. The index measures the parameter space over which feasible operation can be achieved by adjusting the control variables. The system is represented through a set of uncertain parameters  $\theta$ , a set of control variables (degrees of freedom)  $z$  and a set of design variables of the equipment  $d$ . For Swaney (1985), the flexibility index is the maximum deviation the uncertain parameters can have while satisfying the operation constraints.

In Ross (2006), Ross (2008) and Fitzgerald (2012), a different but novel approach is used to quantify the flexibility, or changeability in the terms preferred by the mentioned authors. They present the idea of *exploration of the tradespace* that allows to quantify the degrees of freedom a

particular design may have, and to compare the trade-offs between cost and utility in a set of design alternatives. From the relationships between the cost and utility of each design, they derivate different metrics (e.g. filtered out-degree, Fuzzy Pareto Shift), which can be used as a measure of how flexible or changeable a system is.

Spackova (2015) introduced a simple but powerful relation. The authors developed a flexibility index that compares the cost of building an initial capacity  $j$ , the cost of adaptation from a capacity  $i$  to a capacity  $j$  and the cost of building an initial capacity  $i$ , with  $i < j$ . If the system is perfectly inflexible, the index is 0 and it means that the adaptation cost between states  $i$  and  $j$  is the same than building the capacity  $j$  from the beginning. In a perfectly flexible system, the index takes the value of 1, which means that the adaptation process is path-independent.

There are other methods and indexes in the literature; however, the ones mentioned here have a solid conceptual background and are easily relatable with the common conception of flexibility. They also exemplify how different techniques can be applied to the same problem and highlight the difficulties when dealing with the multiple ideas associated with the concept of flexibility.

### 3 Flexibility Index Development

In this section, the development of a new flexibility metric is presented. To understand better the components of the metric it is important to have a description of the intended behavior to be measured. In first place, associated with the word flexibility there are always the notions of change and adaptation. A system must have the possibility to change or be changed to be considered flexible. Also, the number of available options or degrees of freedom that a system has at an instant of time, tells about how flexible the system is, and that the amount of flexibility may change over time. The idea of "changing easily" is also associated with flexibility. It is expected, then, that a flexible system must be able to change cheaper/faster than an inflexible one. This suggests that the ratio "change vs cost/time" must also be included in the definition. Finally, the adaptation must increase the utility provided by the system, i.e., the system does not change for the sake of changing, but responds to external pressures in order to keep or improve the previous level of performance.

Consider a system whose physical structure and properties are defined by the state vector  $\mathbf{X}(t) = \{X_1(t), X_2(t), \dots, X_n(t)\}$ . The elements of  $\mathbf{X}(t)$  are system design variables such as material, geometry, etc. For simplicity, it will be written when convenient as  $\mathbf{X}(t) = X_t$ . The system's performance at time  $t$  will be  $V(\mathbf{X}(t)) = V_t$ . The system state vector may change over time through different mechanisms that may improve its characteristics or cause some deterioration. The utility derived from the current system performance is  $u(V_t)$ . Every change occurs at a cost,  $c(\mathbf{X}(t))$ , which can be expressed in different ways; e.g., economic value, CO<sub>2</sub> emissions, etc. Furthermore, the nature of those changes may be the result of the system inner structure, or due to external decisions made, for example, by the owner. Through time, the system state  $\mathbf{X}(t)$  changes due to external demands, managerial decisions,  $M$ , and/or changes in the system internal structure (e.g., degradation). Let's define a function  $\phi = u(V_t)/c(\mathbf{X}(t))$ , in which both  $u(V_t)$  and  $c(\mathbf{X}(t))$  are continuous and derivable functions. Notice that  $\phi$  is a function that is proportional to utility and inversely proportional to cost. The differential of  $\phi$  is:

$$d\phi = \left( \frac{du(V_t)}{dV} \frac{dV(X_t)}{dX_t} \frac{1}{c(X_t)} - u(V_t) \frac{1}{c(X_t)^2} \frac{dc(X_t)}{dX_t} \right) \frac{dX_t}{dt} dt \quad (1)$$

The *instantaneous flexibility* is defined here as a relative change in  $\phi$ , which can be computed as:

$$f = \frac{d\phi}{\phi} = \left( \frac{u'(V_t)}{u(V_t)} V'(X_t) - \frac{c'(X_t)}{c(X_t)} \right) \frac{dX_t}{dt} dt \quad (2)$$

For  $u(V_t) \neq 0$ ,  $c(X_t) \neq 0$  and  $t > 0$ . Replacing  $dX_t/dt = m(t)$ , the final formulation of the instantaneous flexibility is:

$$f = \frac{d\phi}{\phi} = \left( \frac{u'(V_t)}{u(V_t)} V'(X_t) - \frac{c'(X_t)}{c(X_t)} \right) m(t) dt \quad (3)$$

The function  $m(t)$  indicates the changes in the system state. This function takes the value  $m(t) \neq 0$  if there is a decision to change the system state at time  $t$ ; and zero otherwise. The criteria that make  $m(t) \neq 0$  are of varying nature and depend upon the stakeholders' interests. The *accumulated flexibility* of a system can be evaluated within a specific time window  $[t_i, t_j]$ ; thus,

$$F(t_i, t_j) = \int_{t_i}^{t_j} \left( \frac{u'(V_t)}{u(V_t)} V'(X_t) - \frac{c'(X_t)}{c(X_t)} \right) m(t) dt \quad (4)$$

Note that flexibility values vary because they depend upon the problem at hand; furthermore, flexibility can take both positive and negative values. Negative flexibility implies that the cost of changing the system state overshadow the gains in utility. On the other hand, positive values mean that the costs of changing the system state are compensated by an increase in utility.

#### 4 Application Example

The following example shows how to use the flexibility index from “Eq. (4)” to compare alternative designs. Note Flexibility in absolute terms is only conceptual and, therefore, there is not an absolute measure. It can only be measured in comparison with other systems. The system used in the example is a simple structure that must attend some demand  $d(t)$  (e.g. a production facility, a bridge). The structure is built with an initial capacity  $C_0$ , at a cost  $K_0$ . The capacity of the structure can be expanded by building modules (*modularity* is a typical example of how to add flexibility into a system) with the same individual capacity  $C_i$  and cost  $K_i$ . The total capacity of the system at time  $t$  is simply  $C(t) = C_0 + n(t)C_i$ , with  $n(t)$  equal to the number of modules added. The system performance is measured as the ratio of the capacity and the demand  $V_t = C(t)/d(t)$ . The system utility is modeled with a single peaked preference function, with the maximum in  $u(V_t) = 1$ . This means that scenarios where the capacity is lower than the demand (unexploited market) and where the capacity is higher than the demand (uncovered production, maintenance and storing costs) are less preferable than the scenario where the system has exactly the capacity that the market demands.

Two sources of flexibility are analyzed: i) external flexibility, which comes from the decisions of an agent, and ii) internal flexibility, which is provided by the intrinsic system design features. In the first case, two strategies are compared: increase the system capacity at predefined points in time or increase it (or decrease it by disabling a module) when the performance exceeds certain threshold. In the second case, two systems with different adaptation costs are compared.

To evaluate the flexibility index, the following methodology was employed: first, a random demand scenario is created. The possible demand scenarios include lineal growth, logarithmic

growth and sinusoidal behavior. Second, the system performance, utility and cost history are calculated. These values are used to compute the accumulated index from “Eq. (4)”. The values are then normalized by comparing with a base case where the final capacity required was built from the beginning. Multiple scenarios (1000) are generated with the purpose of evaluating the system response under many circumstances. The distribution of the flexibility index is used to compare the systems and draw conclusions.

#### 4.1 Results

To evaluate the impact of the adaptation strategies on the flexibility index three scenarios are compared: one adaptation in the middle of the analysis period of 20 years, three adaptations at fixed time instants (5<sup>th</sup>, 10<sup>th</sup> and 15<sup>th</sup> year), and adaptations when the performance exceeds certain threshold. The resulting distributions of the flexibility index are presented in Figure 1. The results are consistent with the intuition that a system that can change when is needed is more flexible than one that changes at preset times or does not change at all. The median value of the distribution of the freely changing system is an order of magnitude higher than the system limited to three adaptations, while the adaptation of the system limited to one does not provide enough value to compensate for the costs (0.3912 vs 0.026 and -0.053).

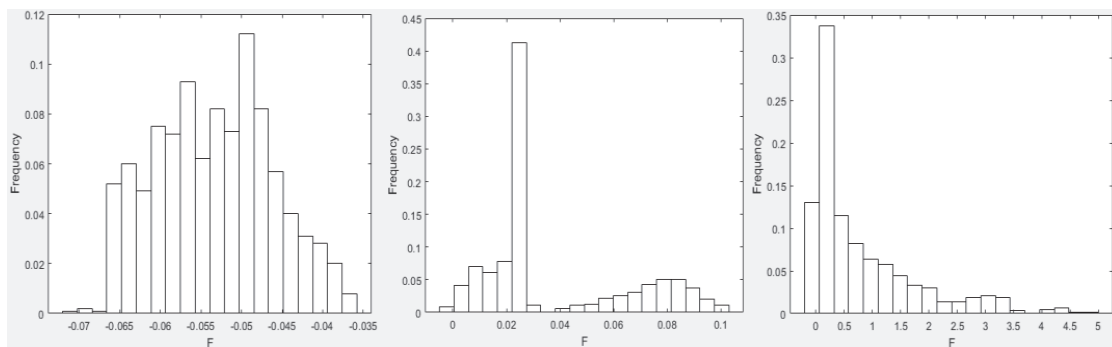


Figure 1. Distribution of flexibility index for one adaptation (left), three fixed time adaptations (center), adaptations when required (right).

The second analysis focuses in the intrinsic characteristics of the systems. If two systems share similar properties but in one case making an adaptation is more expensive than in the other case, one may think that the first system is more flexible than the second one. In Figure 2 the results of the experiment can be compared. The difference is not as pronounced as in the first experiment; however, by comparing the median values (0.413 vs 0.1742) and the mean values (0.7779 vs 0.5579) one can conclude that the index values are higher for the system that is cheaper to adapt.

#### 5 Conclusions

The development of a novel flexibility index was presented. The index compares the utility brought to the system by a change or adaptation, to the cost of those interventions, under a specific demand scenario. If the change improves the utility provided by the system, the index will be large and positive, if the change is too expensive or the utility gain too small, the index will be close to zero or negative. Two specific cases were analyzed, one where the flexibility

was provided by an external agent (management) and one where it depended on design characteristics. The results showed that the systems with the highest values of flexibility were the most easily changeable, which is consistent with the definition of Section 2. Future research should include the quantification of the value provided by the introduction of flexibility.

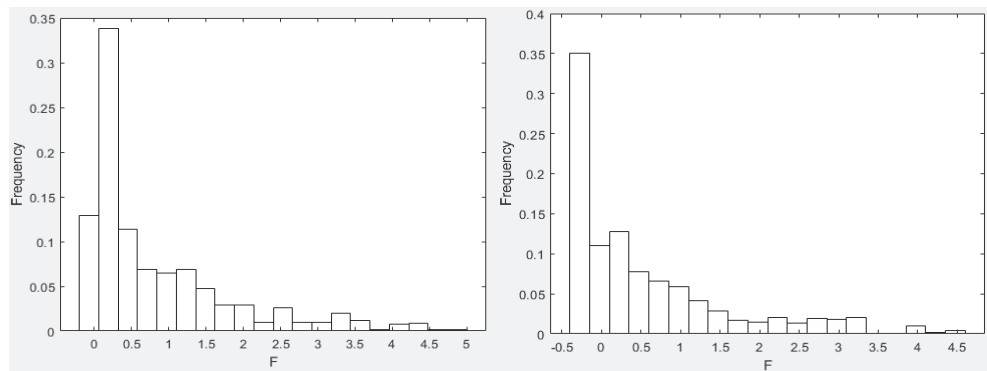


Figure 2. Distribution of flexibility index for system with adaptation cost  $k=18$  (left) and adaptation cost  $k=30$  (right).

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