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STRUCTURAL PHYSICAL PARAMETER IDENTIFICATION AND RELIABILITY UPDATING BASED ON GIBBS SAMPLING

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In order to ensure the safety, applicability and economy of the structure, the structural reliability analysis is needed in the design. But the traditional reliability analysis method cannot handle discrete variables. More to the point, structural state will dynamic change, but the traditional reliability analysis method cannot give response timely especially there is new information (usually measurements) about the structure. This is very important in structural health monitoring (SHM).In this paper, Gibbs sampling method was used to identify structural physical parameters for linear structural models. After some conversions to structural dynamic characteristic equations, a linear structural identification model was obtained, and the posterior distribution was obtained by the Bayesian updating theory. Using the model parameters and taking their randomness into consideration, the samples of physical parameters were obtained from the conditional posterior distribution of the linear structural identification model, the Gibbs sampling method is employed during the process, and structural reliability was updated based on the identified results. The approach is illustrated by applying it to a linear shear building, results show that the presented method can identify the damage level and locations, then the structural reliability was updated, and it has good accuracy.

Keywords: Gibbs sampling, Bayesian updating, physical parameter identification, reliability updating.

1 Introduction

Structural health monitoring originated in the aerospace field, In the 1970s, Yao et al, firstly introduced the concept of system identification into the field of civil engineering. With the development of computational technology and experimental technology, many researchers carry out extensive research on the basis of different forms of structural models. Testa and Heann studied the damage identification of welded structures. The damage was defined as the connection crack between the rod and the plate. The damage of the steel frame connection was identified by examining the rate of change of initial vibration mode and resonance frequency. Loh et al. used the ARMA model to simulate the system and obtained the system parameters of two 8-storey buildings by the method of least squares.

During the process of structural reliability status assessment, the measured data analysis can not abandon the traditional finite element simulation and exists independently. In fact, the two are mutually connected. Structural reliability analysis usually requires the use of finite element analysis to verify the credibility of the measured data analysis results. The system identification can obtain accurate structural modal and physical parameters, carry out model correction and guide the finite element modeling in order to obtain a more accurate finite element model. Structural system identification is the core technology of structural health monitoring, which has become an important research direction in this field. The primary task of structural health monitoring is to determine structural modal parameters from the measured data, describe the dynamic characteristics of the system as a whole, further identify structural physical parameters and provide a theoretical basis for fault diagnosis and reliability assessment.

In this article, Gibbs sampling method was used to identify structural physical parameters for linear structural models. After some conversions to structural dynamic characteristic equations, a linear structural identification model was obtained, and the posterior distribution was obtained by the Bayesian updating theory. Then we can update the model and its reliability index based on the identified results.

2 Method

2.1 The linear regression model

The linear regression model is generally represented as follows (Scott,2007):

$$Y = X\theta + e \tag{1}$$

$$e \sim N(0, \sigma_e^2 I) \tag{2}$$

where y_i is equal to a linear combination of a set of predictors, $X_i^T\theta$ plus error e_i, and that the error term is normally distributed with a mean of 0 and some variance σ_e^2,and I is an n-dimensional identity matrix. The diagonal elements of this matrix are all equal, and the off-diagonal elements of this matrix are 0s.

A Bayesian specification typically begins with a normality assumption on $y \mid x$ (often with the conditioning suppressed): $y_i \sim N(X_i^T\theta, \sigma_e^2)$. And an improper uniform prior over the real line is often specified for the regression parameters θ, namely:

$$P(\theta_i) = 1 \ (\theta_i \in (0, +\infty); i = 1, 2, \cdots m) \tag{3}$$

The prior probability distribution function (PDF) for σ_e^2 is taken to be the product of independent inverse gamma PDFs,

$$P(\sigma_e^2) \sim IG(\alpha, \beta) \ \propto (1/\sigma_e^2)^{\alpha-1} e^{-(\beta/\sigma_e^2)} \tag{4}$$

When $\alpha = \beta = 0$ the inverse gamma prior becomes the usual Jeffreys' non-informative prior, i.e.,

$$P(\sigma_e^2) \sim 1/\sigma_e^2 \tag{5}$$

This yields a posterior distribution that appears as:

$$P(\theta, \sigma_e^2 \mid X, Y) \propto (\sigma_e^2)^{-(n/2+1)} \exp\left\{-\frac{1}{2\sigma_e^2}(Y - X\theta)^T (Y - X\theta)\right\} \tag{6}$$

A Gibbs sampling for the linear regression model can be developed when the full conditional posterior distribution of θ and σ_e^2 is known. The full conditional posterior distribution for σ_e^2 is straightforward to derive from Eq. (6):

$$P(\sigma_e^2 \mid \theta, X, Y) \propto (\sigma_e^2)^{-(n/2+1)} \exp\left\{-\frac{1}{2\sigma_e^2}(Y - X\theta)^T (Y - X\theta)\right\} \tag{7}$$

This conditional posterior is easily seen to be an inverse gamma distribution with parameters $\alpha = n/2$ and $\beta = (Y - X\theta)^T (Y - X\theta)/2$ .While the conditional posterior distribution for θ is:

$$P(\theta \mid \sigma_e^2, X, Y) \propto (\sigma_e^2)^{-(n/2+1)} \exp\left\{ -\frac{1}{2\sigma_e^2 (X^T X)^{-1}} [\theta^T \theta - 2\theta^T (X^T X)^{-1} (X^T Y)] \right\} \tag{8}$$

It can be recognized that the conditional posterior distribution for θ is normal with a mean equal to $(X^T X)^{-1}(X^T Y)$ and a variance of $\sigma_e^2 (X^T X)^{-1}$.

2.2 Linear structural identification model

In terms of structural health monitoring, linear structural models are often used for model updating, since much vibration data of structures under investigation are obtained using low-amplitude excitation, such as ambient vibration and hammer. In this case, many structures (even damaged nonlinear structures) behave approximately linearly, so the linearity assumption of the approach is justified (Ching et al.,2006).

The i th natural frequency ω_i and mode shape vector ϕ_i of a n DOF system satisfy the following characteristic equation:

$$[K - \omega_i^2 M]\phi_i = \{0\} \tag{9}$$

The expanded form of the equation can be expressed as follows:

$$\omega_i^2 \begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & \ddots & \\ & & & m_n \end{bmatrix} \begin{Bmatrix} \phi_{i1} \\ \phi_{i2} \\ \vdots \\ \phi_{in} \end{Bmatrix} =$$

$$\begin{bmatrix} k_1\theta_1 + k_2\theta_2 & -k_2\theta_2 & & \\ -k_2\theta_2 & k_2\theta_2 + k_3\theta_3 & \ddots & \\ \ddots & & \ddots & \\ & & & k_n\theta_n \\ & & -k_n\theta_n & k_n\theta_n \end{bmatrix} \begin{Bmatrix} \phi_{i1} \\ \phi_{i2} \\ \vdots \\ \phi_{in} \end{Bmatrix} \tag{10}$$

Where $\theta_1, \theta_2, ..., \theta_n$ normalized no-dimensional parameters (calling the structural stiffness parameters in the following). θ_i indicates the contribution ratio of one structural member to the whole structure, and values range from 0 to 1. The i th structural member can be judged to be damaged when $\theta_i < 1$, and the damage degree can be seen through the value of θ_i .Note that the mass of the structural members are fixed values since it is less sensitive to the damage. If there is a need to identify the mass parameters, a similar transform to equation (9) will do.

Transform Eq. (10) and plus the error term, then the linear structural identification model can be expressed as follow:

$$\omega_i^2 \begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & \ddots & \\ & & & m_n \end{bmatrix} \begin{Bmatrix} \phi_{i1} \\ \phi_{i2} \\ \vdots \\ \phi_{in} \end{Bmatrix} =$$

$$= \begin{bmatrix} k_1\phi_{i1} & k_2(\phi_{i1}-\phi_{i2}) & & & \\ & -k_2(\phi_{i1}-\phi_{i2}) & k_3(\phi_{i2}-\phi_{i3}) & & \\ & & \ddots & \ddots & \\ & & & -k_{n-1}(\phi_{i(n-2)}-\phi_{i(n-1)}) & k_n(\phi_{i(n-1)}-\phi_{in}) \\ & & & & -k_n(\phi_{i(n-1)}-\phi_{in}) \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{Bmatrix} + e \tag{11}$$

2.3 Gibbs sampler algorithm

- Draw the initial sample $\{\hat{\theta}^{(0)}, \hat{\sigma}_e^{2(0)}\}$ from the prior PDFs and let $k=1$;
- According to Eq. (7)and(11), sample the error term $\hat{\sigma}_e^{2(k)} \sim P(\sigma_e^2 \mid \hat{\theta}^{k-1}, X, Y)$;
- According to Eq. (8)and(11), sample the structural stiffness parameters $\theta^{(k)} \sim P(\theta \mid \hat{\sigma}_e^{2(k)}, X, Y)$;
- Let $k=k+1$,go back to step two and cycle to obtain N samples $\{\hat{\theta}^{(k)}, \hat{\sigma}_e^{2(k)} : k=1,2,\cdots,N\}$.

When k gets large enough, the samples $\{\hat{\theta}^{(k)}, \hat{\sigma}_e^{2(k)}\}$ will follow the PDF $P(\theta, \sigma_e^2 \mid X, Y)$.

2.4 Reliability analysis method

According to the structure seismic reliability formula, in the serviceability limit [0,T],the probability of a structure that no damage occur under different levels of seismic action can be expressed as:

$$P(Y_i,T) = \sum_k P_E(I_K,T) \bullet P_{yi}(\delta < \delta_{\max} | I_K) \tag{12}$$

In which, $P_E(I_K,T)$ is the probability of encountering an earthquake whose intensity is I_K in the serviceability limit T, $P_{yi}(\delta < \delta_{\max} | I_K)$ is the no-damage probability of the ith story of the structure when ground motion of I_K intensity occurs.

3 Result

To examine the performance of the Gibbs sampling algorithm, studies are performed using simulated data from a 5-DOF linear shear structure, its physical parameters are as follows:
There are three damage patterns in this paper: (1)DP1: loss of 40% column stiffness in the first floor and loss of 30% column stiffness in the second floor; (2) DP2: loss of 50% column stiffness in the second floor and loss of 40% column stiffness in the third floor and loss of 30% column stiffness in the forth floor ;(3) DP3: the losses of column stiffness in the first ,second, third, fourth and top floor are 50%,40%,30%, ,20%,10% respectively.

Table 1. Physical parameter.

Floor	Mass (10^2kg)	Stiffness (10^5N/m)
1	6	1.2
2	5	1.1
3	5	1.1
4	5	1.0
5	4	1.0

In order to take the randomness of the modal parameters into consideration, in this paper, the first-order natural frequency used during the Gibbs sampling process is assumed to follow a normal distribution with mean taken to be the theory natural frequency value and proper variance, as shown in Table.2.

Table 2. Statistical properties of the first-order natural frequency.

Damage Patterns	Undamaged (UD)	DP1	DP2	DP3
Mean of ω_1 (rad/s)	4.4075	3.7635	3.5962	3.401
Variance of ω_1	0.3098	0.3000	0.3410	0.2899

For simplicity, the initial samples of the structural stiffness parameters vector θ of the four patterns are all taken to be $\theta = [0.8\ \ 0.8\ \ 0.8\ \ 0.8\ \ 0.8]^T$ .Following the Gibbs sampler algorithm, the Markov chain samples of the structural stiffness parameters are obtained. The results of the second floor are as follows:

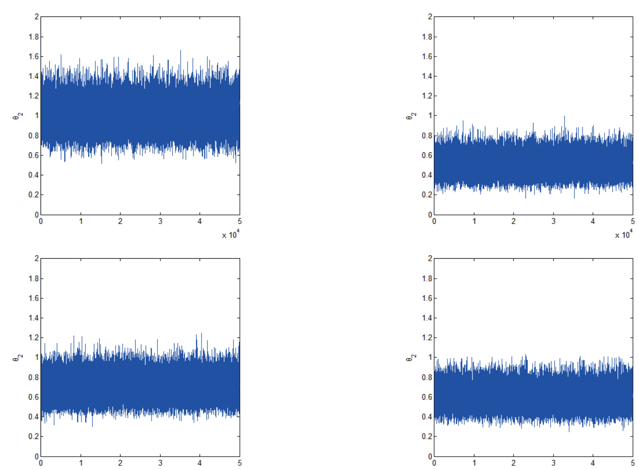

Figure 1. Sampling of structural stiffness from UD,DP1,DP2,DP3,respectively

Table 3. Posterior statistical properties of the stiffness parameters.

Damaged Patterns	θ_1			θ_2			θ_3		
	Mean	Var	Error(%)	Mean	Var	Error(%)	Mean	Var	Error(%)
UD	1.0054	0.1407	0.5400	1.0052	0.1407	0.5200	1.0062	0.1408	0.6200
DP1	0.6039	0.0957	0.6500	0.7051	0.1118	0.7286	1.0089	0.1600	0.8900
DP2	1.0083	0.1902	0.8300	0.5042	0.0951	0.8400	0.6053	0.1142	0.8833
DP3	0.5041	0.0858	0.8200	0.6049	0.1029	0.8167	0.7056	0.1201	0.8000

Damaged Patterns	θ_4			θ_5		
	Mean	Var	Error(%)	Mean	Var	Error(%)
UD	1.0054	0.1408	0.5400	1.0056	0.1409	0.5600
DP1	1.0096	0.1601	0.9600	1.0100	0.1608	1.0000
DP2	0.7059	0.1332	0.8429	1.0090	0.1907	0.9000
DP3	0.8072	0.1376	0.9000	0.9075	0.1554	0.8333

According the results, we can identify the degree of structural stiffness reduction and damage location, take the second picture of figure 1,we can known that the stiffness of the second floor is reduced approximately 30%.

In this paper, by means of *Newmark*-β method, 20 seismic loads are applied to the structure to obtain the inter-story displacement response, The distribution of structural response can be fitted to a logarithmic normal distribution, and then the failure probability of the structure under various conditions can be calculated by the Monte Carlo simulation method and the subset simulation method.

Table 4. Interlayer displacement response under DP0.

	6 degree	7 degree	8 degree	9 degree	10 degree
Logarithmic mean	-11.07661	-10.38343	-9.69033	-8.99718	-8.30400
Log standard deviation	1.49237	1.49235	1.49239	1.49238	1.49241
Failure probability by MC method	0.00068	0.00332	0.0121	0.0352	0.0905
Failure probability by SS method	0.000665	0.0030	0.0124	0.0377	0.0938

The reliability of the structure can be calculated separately using Monte-Carlo method and subset simulations and compared with each other:

Reliability probability by MC method(Number of samples :100000)：

$$Pr(MC)=0.485\times0.99932+0.334\times0.99668+0.1244\times0.9879+0.02579\times0.9648+0.00273\times0.9095 =0.9678$$

Total reliability index by MC method： $\beta=\Phi^{-1}(P_r)=\Phi^{-1}(0.9678)=1.8494$

Reliability probability by SS method (Number of samples:10000)：

$$Pr(SS)=0.485\times0.999335+0.334\times0.997+0.1244\times0.9876+0.02579\times0.9623+0.00273\times0.9062 =0.9678$$

Total reliability index by SS method： $\beta'=\Phi^{-1}(P_r)=\Phi^{-1}(0.9678)=1.8494$

The interlayer displacement response under DP1, DP2, DP3 are similar to above, the results are as follows:

Table 5. Physical parameter.

Damage Patterns	Probability index(MC)	Probability index(SS)
undamaged(DP0)	1.8494	1.8494
Damage Pattern 1(DP1)	1.7720	1.7672
Damage Pattern 2(DP2)	1.6706	1.8692
Damage Pattern 3(DP3)	1.5991	1.5953

From the result, it can be seen that compared with the initial structure, the structure updated after the physical parameters are identified, the reliability is updated and closer to the true state.

4 Conclusion

In this paper, we firstly make a series of changes to the dynamic characteristic equation of the structure to get the linear structure identification model, and then obtain the posterior distribution form of the model by Bayesian update theory. By using the modal parameters of the structure and taking into account its randomness, the Gibbs sampling method was applied to successfully identify the structural physical parameters and update the structural reliability index.

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