

# Stochastic seismic stability reliability of high CFRD slopes based on generalized probability density evolution method

PANG RUI<sup>1</sup>, XU BIN<sup>2</sup>, KONG XIANG-JING<sup>2</sup>

<sup>1</sup>*School of Hydraulic Engineering, Dalian University of Technology, Dalian, China  
E-mail: pangruipangpan@163.com*

<sup>2</sup>*State Key Laboratory of Coastal and Offshore Engineering, Dalian, China  
E-mail: xubin@dlut.edu.cn, kongxj@dlut.edu.cn*

Earthquake ground motions have the characteristics of randomness and will lead to the stochastic behavior of high concrete face rockfill dam (CFRD) slopes; therefore, it is necessary to investigate the slope dynamic stability using stochastic analysis methods. A recently developed generalized probability density evolution method (GPDEM) coupling with spectral representation-random function method is presented to obtain the stochastic and evaluate the seismic reliability of dam slope stability based on the safety factor and the cumulative time of safety factor less than 1.0 ( $F_s < 1.0$ ) under stochastic seismic excitation. Comparison between the results of the Monte Carlo method (MCM) and GPDEM analysis demonstrates the effectiveness and high accuracy of the GPDEM. The reliability analysis of dam slopes can directly reflect the stability probability and degree of safety, indicating the novel approach to dam slope stability assessment from a stochastic viewpoint.

*Keywords:* high CFRD slopes, stochastic seismic excitation, GPDEM, stability reliability, cumulative time.

## 1 Introduction

Landslide of dam slope is one of the main forms of earthquake damage of high CFRDs according to the results of dynamic numerical analysis (Zhu 2011, Zou 2013), dynamic physical model tests (Liu 2016, Zhu 2011) and practical earthquake damage (Liu 2015, Zhang 2015). In addition, the ground motions have the characteristics of randomness whether in time or space, which will lead to the stochastic behavior of structural responses (Yazdani 2015). In recent years, seismic reliability analysis and dynamic stability assessments of earth dam slopes from a stochastic perspective have a practical significance for earthquake-induced landslide disaster reduction. Al-Homoud AS et al. (Al-Homoud 2001) developed different models for evaluating the probabilistic three-dimensional stability analysis of earth slopes and embankments under earthquake loading using both the safety factor and the displacement criteria of slope failure. Tsompanakis et al. (Tsompanakis 2010) developed fragility curves of embankments by MCS-based numerical approach and the commonly used lognormal empirical approach respectively. In addition, the aforementioned studies can't effectively model the stochastic dynamic systems, and a traditional MC method (Shinozuka 1972) significantly improves our understanding of stochastic vibration analysis in the field of earth and rockfill dam engineering; however, it is very difficult to apply this method in practical engineering because of the large amount of computing time, such as earth and rockfill dams (Schuëller 2006).

A generalized probability density evolution method (GPDEM) was developed by Li and Chen et al. (Li 2006, Li 2009), and provided a new way to analyze the nonlinear stochastic seismic response and the probability of large and complex engineering structures combined with a new spectral representation-random function method which was proposed by Liu et al. (Liu 2016) to simulate the non-stationary ground motions. Huang and Xiong (Huang 2017) evaluated the seismic liquefaction performance of an 8-m high earth dam and obtained the stochastic dynamic response processes and probability based on deformation using this method. Pang et al. (Pang 2018) investigated the seismic reliability of high earth-rockfill dam slopes under stochastic earthquake excitation considering strain-softening behavior of rockfill materials through this method. In order to evaluate the seismic performance of high concrete-faced rockfill dams (CFRDs), Pang et al. (Pang 2018) adopted the aforementioned method to determine the failure probabilities considering the randomness of earthquake ground motions based on three performance indices deformation, stability of dam slope and safety of face-slabs.

This paper investigates the stochastic seismic response and stability reliability of dam slopes under stochastic earthquake excitation using a recently developed GPDEM. This GPDEM combines the stochastic dynamic analysis and currently deterministic dam finite element time-history response analysis methods and it is completely different from the traditional stochastic probability analysis method. After a series of deterministic dynamic calculations based on a 242-m high CFRD, the statistical values and the stability reliability of minimum safety factor are obtained according to the GPDEM. Finally, the stability reliability of cumulative time of safety factor less than 1.0 ( $F_s < 1.0$ ) will be determined by constructed a virtual stochastic processes (Li 2009). This new stochastic seismic dynamic method GPDEM is also compared with the traditional stochastic reliability analysis method Monte Carlo method (MCM), and the accuracy and effectiveness of this new method are demonstrated.

## 2 GPDEM equation and seismic reliability

Without the loss of generality, the dynamic balance equation of a CFRD subjected to earthquake excitation can be written as follows:

$$\bar{\mathbf{M}} \ddot{\mathbf{X}}(t) + \mathbf{C} \dot{\mathbf{X}}(t) + \mathbf{K} \mathbf{X}(t) = -\bar{\mathbf{M}} \ddot{\mathbf{X}}_g(\boldsymbol{\Theta}, t) \quad (1)$$

More generally, any physical parameters (e.g., safety factor) can be chosen as the random variable depending on  $\boldsymbol{\Theta}$  in the GPDEM equation. In this study, for the seismic reliability analysis of dam slopes, the selected physical parameters are the time series of safety factors. The GPDEM equation for the dam slope analysis will be expressed as

$$\frac{\partial p_{F_s \boldsymbol{\Theta}}(F_s, \boldsymbol{\Theta}, t)}{\partial t} + \dot{F}_s(\boldsymbol{\Theta}, t) \frac{\partial p_{F_s \boldsymbol{\Theta}}(F_s, \boldsymbol{\Theta}, t)}{\partial F_s} = 0 \quad (2)$$

The initial condition is

$$p_{F_s \boldsymbol{\Theta}}(F_s, \boldsymbol{\Theta}, t)|_{t=t_0} = \delta(F_s - F_{s0}) p_q \quad (3)$$

and the joint probability density function (PDF) of  $F_s(t)$  is

$$p_{F_s}(F_s, t) = \int_{\Omega_{\boldsymbol{\Theta}}} p_{F_s \boldsymbol{\Theta}}(F_s, \boldsymbol{\Theta}, t) d\boldsymbol{\Theta} \quad (4)$$

To solve the GPDEM equation, 377 deterministic dynamic calculations subjected to different ground motions are performed. Eq. (4) is solved using the finite difference method based on the TVD scheme (Li 2006).

## 3 Generation of stochastic ground motions

The seismic reliability of dam slopes is obtained by following the numerical steps for solving the GPDEM equation. The acceleration time series of the stochastic seismic ground motions are generated based on the spectral representation of the random function method of non-stationary stochastic processes (Liu 2016):

$$\ddot{X}_g(t) = \sum_{k=1}^N \sqrt{2S_{\ddot{X}_g}(t, \omega_k) \Delta\omega} [\cos(\omega_k t) X_k + \sin(\omega_k t) Y_k] \quad (5)$$

where  $\omega_k = k\Delta\omega$ ,  $S_{\ddot{X}_g}$  is a bilateral evolutionary power spectral density function of the non-stationary acceleration time series of the seismic ground motion,  $\{X_k, Y_k\}$  ( $k=1, 2, \dots, N$ ) are the standard orthogonal random variables, interval frequency  $\Delta\omega=0.15$  rad/s, and the number of truncated items  $N=1600$ .

Suppose that any two sets of standard orthogonal random variables  $\bar{X}_n$  and  $\bar{Y}_n$  ( $n=1, 2, \dots, N$ ) are the functions of two independent random variables  $\Theta_1$  and  $\Theta_2$ , respectively. The stochastic functions can be described as

$$\bar{X}_n = \cos(n\Theta_1), \quad \bar{Y}_n = \sin(n\Theta_2) \quad (6)$$

The random variables  $\Theta_1$  and  $\Theta_2$  can be generated using the number-theoretical method (Hua 1981). 377 acceleration time history samples will be generated based on the newest hydraulic seismic code (NB 35047-2015), and the peak ground acceleration (PGA) is 0.4 g. A second-order statistical analysis (mean and standard deviation) between the sampling and the target confirm the effectiveness and accuracy of the spectral representation-random function method based on GPDEM, as shown in Figure 1.

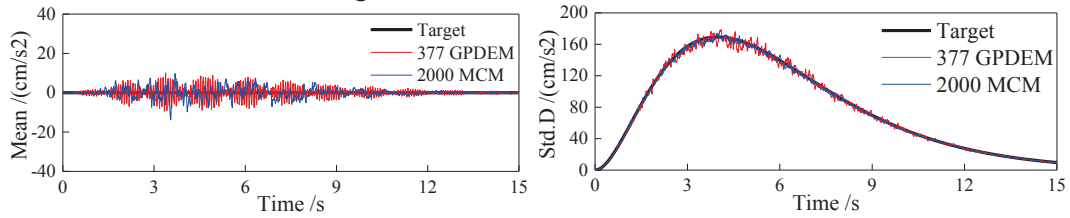


Figure 1. Comparison of mean and standard deviation between samples and target.

#### 4 Numerical case of a high CFRD dam

In this paper, a set of deterministic finite element calculations with static, dynamic and stability analysis are performed using the high-performance Geotechnical Nonlinear Dynamic Analysis Software GEODYNA and FEMSTABLE 2.0 (Zou and Kong 2005). A GuShui CFRD whose height is 242 m is used to carry out the two-dimensional stochastic numerical calculation, as shown in Figure 2. A Duncan-Chang *EB* constitutive model (Duncan and Chang, 1970) is adopted for static calculations of the rockfill, the dynamic calculations use a Hardin-Drnevich constitutive model (Hardin and Drnevich, 1972), and the model parameters are listed in Table 1. A modified Newmark method (Zou et al., 2012; Pang et al., 2018), which can obtain the safety factor and the cumulative time of  $F_s < 1.0$  simultaneously, is used to perform the stability calculation after static and dynamic analysis, and this method can also consider the softening characteristic of the rockfill which is used in this paper.

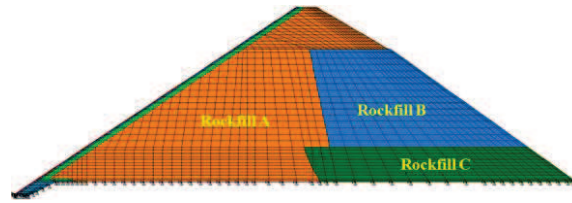


Figure 2. 2-D Finite element mesh of the CFRD.

**Table 1.** Parameters for Duncan E-B model and Hardin-Drnevich model.

Materials	$\rho$ (kg/m <sup>3</sup> )	$K$	$n$	$R_f$	$K_b$	$m$	$\varphi_0$ ( $^\circ$ )	$\Delta\varphi$ ( $^\circ$ )	$K$ (dynamic)	$n$ (dynamic)	$\nu$
Rockfill A	2150	1109	0.24	0.64	420	0.26	49.8	7.2	2660	0.444	0.33
Rockfill B	2100	800	0.32	0.64	490	0.30	49.8	7.2	3115	0.396	0.33
Rockfill C	2170	980	0.26	0.79	400	0.31	50.0	8.2	4997	0.298	0.33
Transition	2222	1250	0.31	0.78	500	0.16	53.5	10.7	3223	0.455	0.40
Cushion	2258	1200	0.30	0.75	680	0.15	54.4	10.6	3828	0.345	0.40

## 5 Stochastic dynamic response and seismic reliability of dam slopes

### 5.1 Stochastic dynamic response

The dynamic time history of safety factor is a primary aspect to analyze the stability of high CFRD slope. The minimum safety factor of dam slope will be obtained under a series of determined earthquake acceleration time history samples based on the GPDEM. In addition, a MCM, which seems to be the only universal method with adequate precision for arbitrary-dimension nonlinear stochastic dynamic system, is used to compare the dam stochastic response results of GPDEM for the purpose of demonstrating the accuracy of the GPDEM calculation. After a series of deterministic dynamic seismic time-series analysis, we substitute the minimum dynamic safety factor of the time series into the GPDEM equation as the response velocities, and solve the GPDEM equation by the finite difference method (Li 2009). The stochastic responses and the stability reliability of the desired physical parameters can then be obtained.

The mean and the standard deviation of the minimum safety factor are illustrated in Figure 3, where the results evaluated by the GPDEM and the MCM are compared. The comparison based on 377 times' GPDEM simulation with 2000 times of MCM simulation shows the accuracy and efficiency of the proposed method, but the computational samples will be much fewer and the expensive time is much shorter. In addition, the mean and standard deviation are of the same order of magnitude, which indicates that the seismic response physical quantities have a significant variation characteristic, and the seismic ground motions are stochastic processes and different seismic ground motions have great influence on the dynamic responses. More importantly, the GPDEM can capture the complete evolution process of PDF of the responses, as illustrated in Figure 4, which depicts the changes of the PDFs with time just like the mountain stretching to the distance. All these figures imply that the high CFRD's safety factor process is a complex stochastic evolution process. The PDFs of evolution demonstrates that the seismic response is very sensitive to the different seismic ground motions, and it is necessary to analyze the seismic responses of high CFRDs from a stochastic viewpoint because of the coupling effect of the randomness of ground motions and nonlinearity of rockfill.

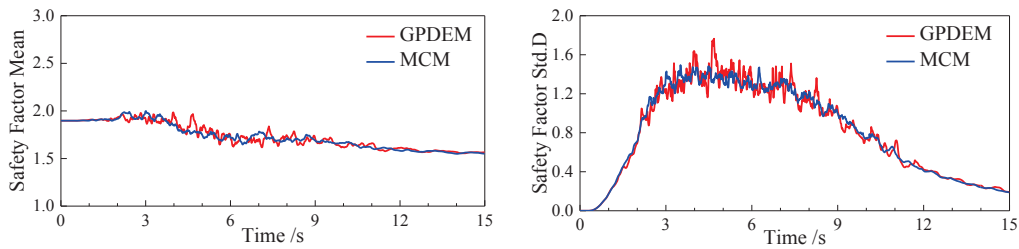


Figure 3. Mean and standard deviation time-histories of the safety factor.

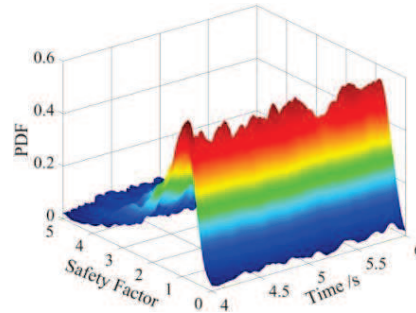


Figure 4. Time evolution PDF surface of safety factor.

## 5.2 Seismic stability reliability

In order to obtain the seismic stability reliability of dam slope, an equivalent extreme-value event (Li 2009) of the minimum safety factor is constructed, and the CDF (Figure 5(a)) is determined for minimum safety factor based on the GPDEM. The dynamic stability reliability corresponding to the safety factor of  $F_s=1.0$  is 0.0005 based on the GPDEM, and 0.0 based on the MCM. The cumulative time of the safety factor with  $F_s<1.0$  is a new way to assess the stability of dam slope of high earth and rockfill dams, and is proposed by some scholars and code (NB 35047-2015, Pang 2018). The CDF is illustrated in Figure 5(b) by constructing a virtual stochastic process (Li 2009), and the seismic reliability corresponding to 1 s and 2 s cumulative time of  $F_s<1.0$  are 0.0374 and 0.3407, respectively, based on GPDEM. However, the seismic reliability corresponding to 1 s and 2 s cumulative time of  $F_s<1.0$  are 0.0327 and 0.3134 based on MCM, which demonstrate the high accuracy of the GPDEM.

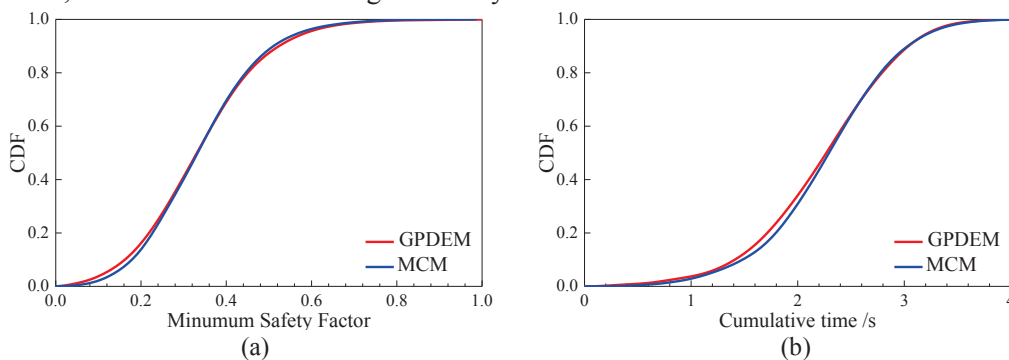


Figure 5. The CDF of (a) minimum safety factor and (b) cumulative time.

## 6 Conclusions

The stability of a high CFRD slope under earthquake excitation is a complex problem, especially considering the stochastic characteristic of ground motions, which leads to the safety factor is also random. The GPDEM, a recently developed stochastic method based on dynamic time series analysis, is introduced to obtain the stochastic dynamic safety factor of dam slopes subjected to stochastic ground motion excitation. We present a numerical case study to evaluate the stability of dam slope considering the dynamic reliability based on safety factor and cumulative time of  $F_s < 1.0$ . Compared with the MCM, the GPDEM is a cost-effective, efficient, and feasible analysis method for stochastic seismic analysis of high CFRD slope, and our results also demonstrate the benefits of analyzing high CFRDs from stochastic viewpoint. The methodology presented in this paper is a novel, efficient method and could also be used for stochastic vibration and reliability analysis by geotechnical engineers.

## Acknowledgments

This work was supported by National Key R&D Program of China (2017YFC0404904), the National Natural Science Foundation of China (Grant Nos. 51679029, 51508071 and 51779034) and the Fundamental Research Funds for the Central Universities (DUT17LK56). These financial supports are gratefully acknowledged.

## References

- Al-Homoud, A. S. and Tahtamoni, W. W., Seismic reliability analysis of earth slopes under short term stability conditions, *Geotech. Geol. Eng.*, 20(3), 201-234, Sep, 2002.
- Code for seismic design of hydraulic structures of hydropower project (NB 35047-2015), *National Energy Board of China (2015)*. [in Chinese]
- Duncan, J. M., Chang, C. Y., Nonlinear analysis of stress and strain in soils, *J. Soil Mech. Found. Div. ASCE*, 96(5), 1629-1654, Jan, 1970..
- Hardin, B. O., Drnevich, V. P., Shear modulus and damping in soils: design equations and curves, *J. Soil Mech. Found. Div. ASCE*, 98(118), 667-692, July, 1972.
- Hua, L. K., Wang, Y., *Applications of Number Theory to Numerical Analysis*, Springer, Berlin, 1981.
- Huang, Y., Xiong, M., Probability density evolution method for seismic liquefaction performance analysis of earth dam, *Earthq. Eng. Struct. Dyn.*, 46(6), 925-943, May, 2017.
- Li, J., Chen, J. B., The probability density evolution method for dynamic response analysis of non-linear stochastic structures, *Int. J. Numer. Meth. Eng.*, 65(6), 882-903, Feb, 2006.
- Li, J., Chen, J. B., *Stochastic dynamics of structures*, John Wiley & Sons, Singapore, 2009.
- Liu, H. L., Chen, Y. M., Yu, T., Yang, G., Seismic Analysis of the Zipingpu Concrete-Faced Rockfill Dam Response to the 2008 Wenchuan, China, Earthquake, *J. Perform. Constr. Facil.*, 29(5), 04014129, Oct, 2015.
- Liu, J., Liu, F. H., Kong, X. J., Large-scale shaking table model tests on seismically induced failure of Concrete-Faced Rockfill Dams, *Soil Dyn. Earthq. Eng.*, 82, 11-23, Mar, 2016.
- Liu, Z. J., Liu, W., Peng, Y. B., Random function based spectral representation of stationary and non-stationary stochastic processes, *Probab. Eng. Mech.*, 45, 115-126, July, 2016.
- Pang, R., Xu, B., Kong, X. J., Zou, D. G., Zhou, Y., Seismic reliability assessment of earth-rockfill dam slopes considering strain-softening of rockfill based on generalized probability density evolution method, *Soil Dyn. Earthq. Eng.*, 107, 96-107, Apr, 2018.
- Pang, R., Xu, B., Zou, D. G., Kong, X. J., Stochastic seismic performance assessment of high CFRDs based on generalized probability density evolution method, *Comput. Geotech.*, 97, 233-245, May, 2018.
- Schuëller, G. I., Developments in stochastic structural mechanics, *Archive of Applied Mechanics*, 75(10), 755-773, Oct, 2006.
- Shinozuka, M., Monte Carlo solution of structural dynamics, *Comput. Struct.*, 2(5-6), 855-874, 1972.

- Tsompanakis, Y., Lagaros, N. D., Psarropoulos, P. N., Georgopoulos, E. C., Probabilistic seismic slope stability assessment of geostructures, *Struct. Infrastruct. Eng.*, 6(1-2), 179-191, Feb, 2009.
- Yazdani, A., Salimi, M. R., Earthquake response spectra estimation of bilinear hysteretic systems using random vibration theory method, *Earthquakes and Structures*, 8(5), 1055-1067, May, 2015.
- Zhang, J. M., Yang, Z. Y., Gao, X. Z., Zhang, J. H., Geotechnical aspects and seismic damage of the 156-m-high Zipingpu concrete-faced rockfill dam following the Ms 8.0 Wenchuan earthquake, *Soil Dyn. Earthq. Eng.*, 76, 145-156, Sep, 2015.
- Zhu, Y. L., Kong, X. J., Zou, D. G., Dynamic Elastic-Plastic Analysis of Reinforcement Geogrids Applied in High Earth-Rockfilled Dam Slope, *Adv. Mater. Res.*, 243-249, 4520-4523, May, 2011.
- Zhu, Y. L., Kong, X. J., Zou, D. G., Li, Y. S., Shaking table test and numerical simulation on the effect of reinforcement on the seismic safety of dam slopes, *J. Harbin. Inst. Technol.*, 18(4), 132-138, 2011.
- Zou, D. G., Kong, X. J., Xu, B., *User manual for geotechnical dynamic nonlinear analysis*. Dalian: Institute of Earthquake Engineering, Dalian University of Technology, 2005.
- Zou, D. G., Xu, B. and Kong X. J., et al., Numerical simulation of the seismic response of the Zipingpu concrete face rockfill dam during the Wenchuan earthquake based on a generalized plasticity model, *Comput. Geotech.*, 49, 111-122, Apr, 2013.
- Zou, D. G., Zhou, Y., Ling, H. I., Dislocation of face-slabs of Zipingpu Concrete Face Rockfill Dam during Wenchuan Earthquake, *Journal of Earthquake and Tsunami*, 6(2), 1-17, June, 2012.