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DIMENSION-REDUCTION SIMULATION OF MULTIVARIATE NON-STATIONARY GROUND MOTIONS

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It is found that the randomness degree (the number of random variables) of multivariate stochastic processes could be greatly reduced by introducing appropriate constraints correlating with the orthogonal random variables in the original spectral representation scheme. Based on this situation, a framework of spectral representation-based dimension reduction is developed for simulating multivariate non-stationary stochastic ground motions through introducing random function serving as a constraint. Furthermore, one random function form combining the trigonometric format and orthogonal polynomial format, is accordingly constructed for simulation purpose. As a result, the accurate representation of the original stochastic processes can be realized by introducing merely three elementary random variables, overcoming the principal challenge of the high-dimensional randomness degree faced by the classical Monte Carlo simulation method. Finally, numerical investigations involving the comparisons with the Monte Carlo simulation method are presented in order to demonstrate the superiority and effectiveness of the proposed methodologies in practical engineering applications.

Keywords: earthquake ground motions, multivariate non-stationary stochastic processes, spectral representation, dimension reduction, random function.

1 Introduction

Different from "point" structures, for structures with large size in horizontal direction, e.g., large-scale structures, bridges, dams and tunnels, adequate attention should be paid to the effects of spatial variability of earthquake ground motions. Since the supports of large-scale structures would undergo differential movements during a severe earthquake event (Saxena et al. 2000, Zerva and Zervas 2002), completely representing the spatially variable ground motions, generally described as spatio-temporal stochastic fields through time and spatial variables, would become significantly important (Wang and Li 2012). Consequently, the Monte Carlo simulation method (MC scheme), applied to generate spatio-temporal stochastic fields as seismic inputs, has been developed rapidly in recent years. In the families of the MC scheme, the spectral representation appears to be the most versatile and also the most widely used one (Cacciola and Deodatis 2011). In enforcement, the multi-dimensional univariate ($m\mathrm{D}-1\mathrm{V}$) spatio-temporal stochastic fields (commonly known as the continuous form) are usually transformed into the one-dimensional multivariate ($1\mathrm{D}-n\mathrm{V}$) stochastic vector processes (commonly known as the discrete form) when simulating the spatially variable ground motions.

By now, a large number of outstanding contributions have been made by Yang (1972), Shinozuka and Deodatis (1991, 1996), Deodatis (1996a, 1996b), Grigoriu (2004), and Liang et al (2007) in the latest forty years when the spectral representation develops fast.Though the simulation efficiency of the spectral representation has been improved dramatically, the extremely high-dimensional randomness degree involved in the MC scheme still remains a principal challenge for it to be applied in the probability density evolution analysis and reliability assessment of large-scale structures (Li and Chen 2009). Thus, the way to efficiently reducing the randomness degree in the spectral representation has become a research hotspot recently. Chen et al (2013, 2017) developed the stochastic harmonic function representation of both stationary and non-stationary stochastic processes, which could obtain the accurate target PSD through a small number of random harmonic components. Meanwhile, Chen and Li (2013) suggested the optimal determination of frequencies in the spectral representation of stochastic processes. Furthermore, Liu et al. (2016, 2017, 2018a, 2018b) proposed a dimension reduction approach by adopting random function for simulating $1\mathrm{D}-1\mathrm{V}$ stationary and non-stationary stochastic processes and $1\mathrm{D}-n\mathrm{V}$ (multivariate) stationary stochastic processes with only several elementary random variables. Moreover, another highlighted advantage of Liu's approach is that each sample generated by the proposed approach has definite probability information that enables it to be naturally combined with the probability density evolution method (PDEM) (Li and Chen 2009, Li 2016) to implement the dynamic response analysis and dynamic reliability assessment of complex structures.

This paper aims to extend the spectral representation-based dimension reduction to simulate the multivariate non-stationary stochastic ground motions. The random function form which combines the trigonometric format and orthogonal polynomial format is constructed in order to achieve the simulation purpose. Benefiting from this proposed scheme, the high-dimensional randomness degree is efficiently reduced to merely three.

2 Spectral Representation of Multivariate Non-stationary Stochastic Processes

Assume $\boldsymbol{X}(t)=\{X_1(t),X_2(t),\cdots,X_n(t)\}^{\mathrm{T}}$ is a real-valued, zero-mean, $1\mathrm{D}-n\mathrm{V}$ non-stationary stochastic process, given by (Priestley 1965):

$$\boldsymbol{X}(t)=\int_{-\infty}^{\infty}\boldsymbol{A}(t,\omega)\mathrm{e}^{\mathrm{i}\omega t}\mathrm{d}\boldsymbol{Z}(\omega) \tag{1}$$

where $\boldsymbol{A}(t,\omega)=\mathrm{diag}\{A_1(t,\omega),A_2(t,\omega),\cdots,A_n(t,\omega)\}$. $\boldsymbol{Z}(\omega)$ is a n-variate zero-mean complex vector process with its orthogonal increment satisfying the following basic conditions:

$$E\left[\mathrm{d}\boldsymbol{Z}(\omega)\right]=\boldsymbol{0}_{n\times1},\ E\left[\mathrm{d}\boldsymbol{Z}(\omega)\mathrm{d}\boldsymbol{Z}^{*\mathrm{T}}(\omega')\right]=\delta_{\omega\omega'}\boldsymbol{S}(\omega)\mathrm{d}\omega,\ \mathrm{d}\boldsymbol{Z}(-\omega)=\mathrm{d}\boldsymbol{Z}^*(\omega) \tag{2}$$

The coherence function matrix can be defined as follows:

$$\boldsymbol{\gamma}(\omega)=\begin{bmatrix}1 & \gamma_{12}(\omega) & \cdots & \gamma_{1n}(\omega)\\ \gamma_{21}(\omega) & 1 & \cdots & \gamma_{2n}(\omega)\\ \vdots & \vdots & \ddots & \vdots\\ \gamma_{n1}(\omega) & \gamma_{n2}(\omega) & \cdots & 1\end{bmatrix} \tag{3}$$

The coherence function matrix $\boldsymbol{\gamma}(\omega)$ can be decomposed adopting Cholesky's methodology into the following product:

$$\boldsymbol{\gamma}(\omega)=\boldsymbol{B}(\omega)\boldsymbol{B}^{*\mathrm{T}}(\omega) \tag{4}$$

Thus the EPSD matrix $\boldsymbol{S_X}(t,\omega)$ can be decomposed as follows:

$$\boldsymbol{S_X}(t,\omega)=\boldsymbol{A}(t,\omega)\boldsymbol{D}(\omega)\boldsymbol{B}(\omega)\boldsymbol{B}^{*\mathrm{T}}(\omega)\boldsymbol{D}^{\mathrm{T}}(\omega)\boldsymbol{A}^{\mathrm{T}}(t,\omega) \tag{5}$$

Then $\boldsymbol{X}(t)$ can be approximated through the following complex finite series:

$$\boldsymbol{X}(t) \approx \sum_{k=-N}^{N} \boldsymbol{A}(t,\omega_k)\boldsymbol{D}(\omega_k)\boldsymbol{B}(\omega_k)\boldsymbol{P}_k \mathrm{e}^{\mathrm{i}\omega_k t}\sqrt{\Delta\omega} \tag{6}$$

Let $\boldsymbol{B}(\omega_k) = \boldsymbol{\rho}(\omega_k) + \mathrm{i}\boldsymbol{\eta}(\omega_k)$ and $\boldsymbol{P}_k = \boldsymbol{R}_k + \mathrm{i}\boldsymbol{I}_k$, the ith component can be written as follows (Di Paola 2000):

$$X_i(t) \approx 2\sum_{r=1}^{i}\sum_{k=1}^{N} A_i(t,\omega_k)\sqrt{S_{ii}(\omega_k)\Delta\omega}\left\{\rho_{ir}(\omega_k)\left[R_{rk}\cos(\omega_k t) - I_{rk}\sin(\omega_k t)\right] - \eta_{ir}(\omega_k)\left[R_{rk}\sin(\omega_k t) + I_{rk}\cos(\omega_k t)\right]\right\},\ i = 1,2,\cdots,n \tag{7}$$

where R_{rk} and I_{rk} ($r = 1,2,\cdots,n$; $k = 1,2,\cdots,N$) are real-valued orthogonal random variables satisfying the following basic conditions:

$$E\left[R_{rk}\right] = E\left[I_{rk}\right] = 0,\ E\left[R_{rk}I_{jm}\right] = 0,\ E\left[R_{rk}R_{jm}\right] = E\left[I_{rk}I_{jm}\right] = \frac{1}{2}\delta_{rj}\delta_{km},\quad r,j = 1,2,\cdots,n;\ k,m = 1,2,\cdots,N \tag{8}$$

3 Dimension Reduction for Simulating Multivariate Non-stationary Stochastic Processes

The orthogonal random variable sets $\left\{R_{rk}, I_{rk}\right\}$ in Eq. (8) can be transformed into random function sets with elementary random variables, say $R_{rk} = g_{rk}(\boldsymbol{\Theta})$, $I_{rk} = h_{rk}(\boldsymbol{\Theta})$, where $g_{rk}(\cdot)$ and $h_{rk}(\cdot)$ indicate the deterministic orthogonal functions, respectively. $\boldsymbol{\Theta} = \left\{\Theta_1, \Theta_2, \cdots, \Theta_q\right\}$ indicates an elementary random vector with q-dimensional mutually independent elements, of which the probability distributions are assigned. Thus, the dimension reduction with just q elementary random variables substituting $2\times n\times N$ random variables in the original scheme is realized.

Hence, constructing appropriate random function forms holding the basic conditions defined in Eq. (8) is particularly important to realize the dimension reduction simulation of multivariate non-stationary stochastic processes. Inspired by Liu et al. (2017), one random function form combining the trigonometric format and orthogonal polynomial format with just three elementary random variables is constructed in this study. Then the orthogonal random variable sets $\left\{\bar{R}_{jm}, \bar{I}_{jm}\right\}$ ($j = 1,2,\cdots,n$; $m = 1,2,\cdots,N$) are defined as the orthogonal random functions with 3-dimensonal random vector $\boldsymbol{\Theta} = \left\{\Theta_1, \Theta_2, \Theta_3\right\}$, given by:

$$\begin{cases} \bar{R}_{jm} = \sqrt{2}\cos(j\times\Theta_1 + \alpha)\times T_m(\Theta_2) \\ \bar{I}_{jm} = \sqrt{2}\sin(j\times\Theta_1 + \alpha)\times T_m(\Theta_3) \end{cases},\ j = 1,2,\cdots,n;\ m = 1,2,\cdots,N \tag{9}$$

where α is a deterministic parameter in the interval $[0, 2\pi)$, which is valued by $\alpha = \pi/4$ in this study. Θ_1, Θ_2 and Θ_3 are mutually independent elementary random variables, and Θ_1 is distributed uniformly over the interval $(0, 2\pi)$. $T_m(\cdot)$ indicates the first family of Chebyshev polynomial function with its formula given by:

$$T_m(\Theta_q) = \cos(m\arccos\Theta_q),\ q = 2,3;\ m = 1,2,\cdots,N \tag{10a}$$

where Θ_q has the PDF as follows:

$$p_{\Theta_q}(\theta_q) = \frac{1}{\pi\sqrt{1-\theta_q^2}}, \ -1 < \theta_q < 1; \ q = 2,3 \tag{10b}$$

Next, the orthogonal random function sets $\{\bar{R}_{jm}, \bar{I}_{jm}\}$ defined in Eq. (9) should be mapped to the orthogonal random variable sets $\{R_{rk}, I_{rk}\}$ in Eq. (8) through a unique transformation, i.e., $\bar{R}_{jm} \to R_{rk}$, $\bar{I}_{jm} \to I_{rk}$ $(j, r = 1,2,\cdots,n;\ m,k = 1,2,\cdots,N)$. This procedure can be conveniently accomplished by means of the MATLAB tool box functions rand (‘state’, 0) and $randperm(\cdot)$. In this study, the number-theoretical method (NTM (Fang and Wang 1994) and Li and Chen (2007)) is employed to select the representative points (the number of representative samples).

4 Numerical Examples

Suppose that the acceleration time-histories at three location points are all along the line of main wave propagation on the ground surface, the ground motion can thus be considered as a 1D-3V non-stationary stochastic process. Figure 1 shows the configuration of these three points.

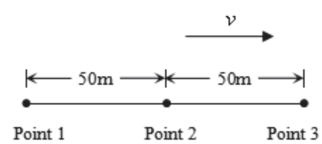


Figure 1. Configuration of the three points on the ground surface

These three points represent three typical local soil conditions, respectively. Specifically, the assumption in this study is that point 1 corresponds to stiff or soft rock soil, point 2 corresponds to medium-hard soil and point 3 corresponds to medium-soft soil. 828 representative samples are generated using the proposed scheme. The simulation results are shown in Figures 2 and 3. Comparisons of the average relative errors (AREs) upon the mean and standard deviation between the proposed scheme and the MC scheme are shown in Table 1. As a result, the accuracy of the proposed scheme is perfectly revealed.

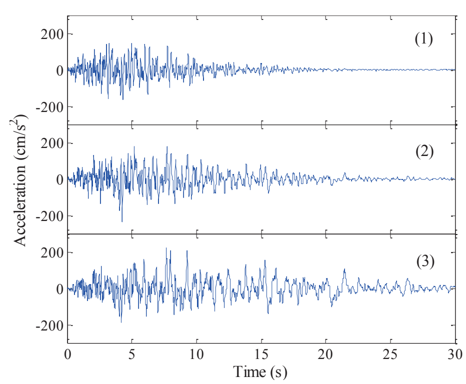
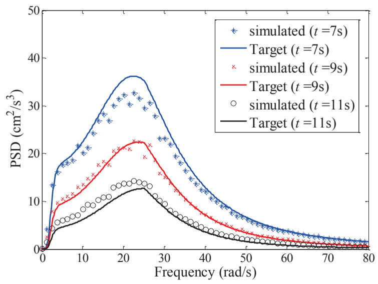
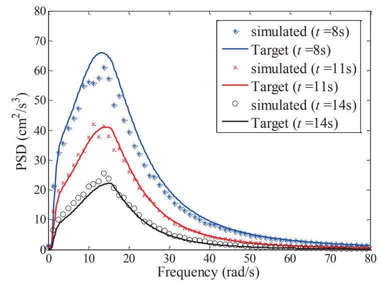


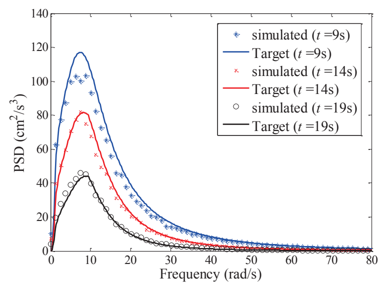
Figure 2. Representative samples generated by the proposed scheme at the three points



(a) Point 1



(b) Point 2



(c) Point 3

Figure 3. Comparisons of the PSD generated by the proposed scheme with the target values at typical instants for the three points

Table 1. Comparisons between the proposed scheme and the MC scheme

Error	Method	The number of representative samples	Soil conditions		
			Stiff or soft rock soil	Medium-hard soil	Medium-soft soil
Mean	The proposed scheme	828	7.72e-17	6.99e-17	6.32e-17
	The MC scheme		2.89%	2.93%	2.88%
Standard deviation	The proposed scheme	828	1.98%	2.05%	1.92%
	The MC scheme		1.99%	2.05%	2.14%

5 Conclusions

In this study, through introducing random function correlating with the orthogonal random variables in the original spectral representation scheme, a framework of spectral representation-based dimension reduction is developed for simulating the multivariate non-stationary stochastic processes. Benefiting from the proposed scheme, the extremely high randomness degree can be effectively reduced to merely three. Numerical investigations adequately reveal the availability of the proposed scheme in engineering practices.

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