Proc. of the 6th Intl. Symposium on Reliability Engineering and Risk Management (6ISRERM)
31 May – 1 June 2018, Singapore
Editor(s) Xudong Qian, Sze Dai Pang, Ghim Ping Raymond Ong, Kok-Kwang Phoon

A New Approach for Structural Reliability Analysis of Offshore Drilling Riser System

X. ZHANG[1] Y. M. LOW[2] and C. G. KOH[2]

[1]*Department of Civil and Environmental Engineering, National University of Singapore, Singapore.*
E-mail: xiaodong.zhang@u.nus.edu
[2]*Department of Civil and Environmental Engineering, National University of Singapore, Singapore.*
E-mail: ceelowym@nus.edu.sg
[2]*Department of Civil and Environmental Engineering, National University of Singapore, Singapore.*
E-mail: cgkoh@nus.edu.sg

Offshore riser systems are subjected to wind, wave and current loadings, which are random in nature. The structural analysis of offshore riser systems requires long simulation time and high computational resources. Structural reliability method, as an analysis tool to quantify probability of failure of components or systems, can account for uncertainties in environmental conditions and system parameters. It is particularly useful in cases where limited experience exists, or a risk-based evaluation of design is required. Monte Carlo Simulation (MCS) method is the most widely accepted method and usually used to benchmark other proposed reliability methods. However, it is computationally demanding for predicting low failure probabilities, especially for offshore dynamic problems involving many types of uncertainties. To tackle the computational burden of MCS method for structural analysis, a new approach for structural reliability analysis is proposed. The proposed approach is a probability density estimation-based method, which could solve nonlinear and high-dimensional problems, particularly when involving multiple design points, it also does not require coupling with finite element analysis, which is difficult and sometimes impossible to implement. To avoid coupling, post-processing methods offer an attractive mean to efficiently recover the probability density function (PDF). The accuracy and efficiency of the proposed approach are demonstrated by an offshore drilling riser system example.

Keywords: Offshore, drilling riser, structural reliability, probability density function.

1 Introduction

Structural reliability is often measured by the probability that a component or system will perform a required function for a given period of time when used under the stated operating conditions (Ebeling, 1997). Structural reliability analysis primarily deals with how to estimate the failure probabilities of a structure for a certain threshold within a specific period, taking all the uncertainties into consideration. The well-known limitation of structural reliability methods is the computational cost.

Probability density estimation based structural reliability analysis methods, where the estimated PDF of structural response variable or performance function (limit state function) could be used to calculate reliability or failure probability, have many advantages: Probability density estimation methods are structure independent and are thus applicable to multiple design points, nonlinear and high-dimensional problems. Most importantly, the methods need only simulation results (typically by means of finite element analysis software) thus do not need to couple with structural analysis. Decoupling from structural analysis renders parallel computing possible to greatly improve the computational efficiency. For structural design or optimization problems, the thresholds are often not known as a priori but are to be determined to achieve a certain reliability level. This requires calculating multiple failure probabilities for different thresholds. When thresholds change, probability density estimation based methods do not need to repeat structural analysis (which can be time consuming for dynamic problems) if the relevant results from structural analysis are kept. Similarly, there is no need to conduct separate reliability analysis coupled with structural analysis for different failure modes, unlike in methods such as FORM, importance sampling and subset simulation.

(Li & Zhang, 2011) combined the PDF model obtained from maximum entropy method and dimension reduction method to perform structural reliability analysis. Instead of integer moments, (Zhang & Pandey, 2013) used fractional moments (evaluated by dimension reduction method) and maximum entropy to conduct structural reliability analysis. (Xu, 2016) also used fractional moments (evaluated by dimension reduction method) and maximum entropy to perform structural reliability analysis and provided a procedure to find better initial values for the optimization problem. (Alibrandi & Mosalam, 2018) combines maximum entropy, fractional moments and kernel density functions to evaluate the failure probabilities.

In this paper, we also use maximum entropy and fractional moments as our main tool to recover the PDF of response variable. Our aim is to identify and resolve the key issues of transferring the tool as a structural reliability method. A new moment method based on entropy theories and genetic algorithm (MEGA) is proposed and applied on a drilling rise system problem. The results show that the proposed MEGA is much faster than MCS.

2 The Proposed Method

The PDF model of MEGA originates from Shannon's Entropy theory and Jaynes' Maximum Entropy principle. Shannon's Entropy theory measures entropy as a negative logarithm of the probability distribution. Jaynes' Maximum Entropy principle provides the PDF model by maximizing the entropy of the distribution. The detailed description of entropy theories could be found in (Kapur & Kesavan, 1992).

The approximate PDF $f_M(x)$ found by entropy theories is:

$$f_M(x) = \exp(-\sum_{r=0}^{M} \lambda_r x^{\alpha_r}) \quad (\alpha_0 = 0) \tag{1}$$

where

$$\lambda_0 = \ln\left[\int_0^{\infty} \exp\left[-\sum_{j=1}^{M} \lambda_j x^{\alpha_j}\right] dx\right] \tag{2}$$

The PDF model in Eq. (1) has unknown model parameters $\boldsymbol{\alpha}$ and $\boldsymbol{\lambda}$. To find the model parameters, Kullback's Minimum Cross-Entropy principle (Kullback, 1959; Kullback & Leibler, 1951) converts the recovering PDF problem to an optimization problem with the objective function:

$$L(\boldsymbol{\alpha},\boldsymbol{\lambda},M)=\sum_{j=0}^{M}\lambda_j E\left(X^{\alpha_j}\right) \tag{3}$$

where $E(X)$ is the expectation operator of X and $E\left(X^{\alpha_j}\right)$ is the statistical moments of order α_j. The model parameters $\boldsymbol{\alpha}$ and $\boldsymbol{\lambda}$ could be found by minimizing Eq. (3).

The population moments $E\left(X^{\alpha}\right)$ in Eq. (3) are approximated by sample moments to exploit the advantages of probability density estimation based reliability methods as described previously. When the first four integer sample moments are used, i.e., $\boldsymbol{\alpha}=[1,2,3,4]$, (Chen, Hu, & Zhu, 2010) proposed a hybrid algorithm to compute the decision vector $\boldsymbol{\lambda}$. The hybrid algorithm could be altered to replace the integer order with fractional order without loss of accuracy. The generalized hybrid algorithm could be served to reduce the decision from vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\lambda}$ to $\boldsymbol{\lambda}$, where the number of decision variables reduces to half.

By using the hybrid algorithm, Eq. (3) could be redefined with one decision vector $\boldsymbol{\alpha}$ and a decision variable M :

$$L(\boldsymbol{\alpha},M)=\sum_{j=0}^{M}\lambda_j E\left(X^{\alpha_j}\right) \tag{4}$$

As a summary, Shannon's Entropy theory and Jaynes' Maximum Entropy principle provide the PDF model, the Kullback's Minimum Cross-Entropy principle converts the parameter estimation to an optimization problem and the adapted hybrid algorithm reduces the number of decision variables to half. With all the necessary theoretical background presented, the key issue of solving the optimization problem remains unresolved. $L(\boldsymbol{\alpha},M)$ in Eq. (4) is the objective function for the optimization problem with fractional decision vector $\boldsymbol{\alpha}$. This is a non-convex and non-continuous optimization problem, which is hard to solve. The accuracy of the decision variables will directly determine the accuracy of the PDF, thus determine the failure probabilities predicted. The global optimization algorithm Genetic Algorithm (GA) is selected to find the global optimum. We propose herein a global optimization algorithm GA to derive an accurate and efficient structural reliability method referred as MEGA.

3 Summary of the MEGA

To recover probability density function using the proposed MEGA is to solve an optimization problem with a system of fractional moment constraints, where the objective function of the optimization problem is derived from entropy theories. The main steps of MEGA are summarized below:

(i) Normalize the random variables to $(0,1]$.
(ii) Identify the objective function of the optimization problem in Eq. (3).
(iii) Solve the optimization problem to get the optimal decision variables $\boldsymbol{\alpha}$ and $\boldsymbol{\lambda}$;
(iv) Substitute the optimal decision variables $\boldsymbol{\alpha}$ and $\boldsymbol{\lambda}$ back to Eq. (1) to obtain the approximate PDF.
(v) Convert the PDF $f_Y(y)$ defined on $(0,1]$ to the PDF $f_X(x)$ defined on the domain of X:

$$f_X(x) = \frac{dy}{dx} f_Y(y) = \frac{1}{X_{\max} - X_{\min}} f_Y(y) \tag{5}$$

(vi) Use the PDF to perform structural reliability analysis.

4 Applications

MCS is carried out for benchmark purpose, and multiple samples (independent sets) of simulation results are selected from the MCS. For each sample of simulation results, a tail distribution is obtained from MEGA. The mean and coefficient of variation (CV) of the tail distributions are obtained. MCS has an explicit expression for describing the CV of its estimator (Ditlevsen & Bjerager, 1986), as follows:

$$CV_{MCS} = \sqrt{(1-p)/(pN)} \tag{6}$$

where p is failure probability and N is the sample size. To assess the performance of a structural reliability method, bias error and variance error should both be investigated. The speed up calculation in this paper takes into both the bias and variance error into consideration. The speed up is $N1/N2$, where $N1$ is the sample size of MCS and $N2$ is the sample size of MEGA. For certain failure probability p, the CV of MEGA is obtained through statistical analysis. $N1$ could be back calculated based on the same p and CV using Eq. (6). The sampling variability should be considered to calculate the speed up. Getting good result from one sample does not mean the same level good result could be gotten from other examples.

Offshore structures are constantly experiencing random environmental loadings from wind, wave and current. Structural reliability analysis of offshore riser systems must consider the effects of uncertainties in environmental loading and riser system. Due to the complexity of the offshore dynamics, the time domain analysis is quite time consuming and computational efficiency is an important consideration.

A drilling riser system is mainly used for well control containing functional fluids. The operability analysis of offshore drilling riser system is essential for safe and cost-effective design, for which the flex joint angel is key failure mode. The maximum flex joint angle is thus selected as the failure mode for illustration herein. The drilling riser system is modeled in a finite element analysis software Flexcom (Kenny, 2016).

The water depth considered is 7500ft (2286m). The vessel has an initial upstream offset of 125ft (38m), the top of the riser being connected to the vessel. The bottom of the riser is fixed at the seabed. The drilling riser system is subjected to wave and current loads. The significant wave height (H_s) is modeled by Weibull distribution:

$$f_{H_s}(h) = \frac{\beta_{H_s}}{\alpha_{H_s}} \left(\frac{h}{\alpha_{H_s}} \right)^{\beta_{H_s}-1} \exp\left\{ -\left(\frac{h}{\alpha_{H_s}} \right)^{\beta_{H_s}} \right\} \tag{7}$$

where $\beta_{H_s} = 2.255$, $\alpha_{H_s} = 2.156$. The zero-crossing wave period (T_z) conditional on H_s is modeled by a lognormal distribution:

$$f(T_z / H_s) = \frac{1}{\sigma T_z \sqrt{2\pi}} \exp\left\{ -\left(\frac{\ln(T_z) - \mu}{2\sigma^2} \right)^2 \right\} \tag{8}$$

where

$$\mu = 1.568 + 0.092\mathrm{h}^{1.005} \tag{9}$$

$$\sigma = 0.165 + 0.290\exp\left(-0.477\mathrm{h}\right) \tag{10}$$

The random variables are wave loading parameters H_s, T_z and phase angles of wave components. As 100 harmonics are used in the spectral discretization, the dimension for this structural reliability problem is 102 (H_s, T_z and 100 phase values). The simulation time is 1000s for each nonlinear dynamic analysis. This study conducted 1.0×10^6 structural dynamic analysis with random inputs. In total, 100 independent samples of size 2000 are randomly selected from the MCS results for MEGA to perform structural reliability analysis of the riser system. The mean tail distribution and speed up of the 100 tail distributions are presented in Figure 1.

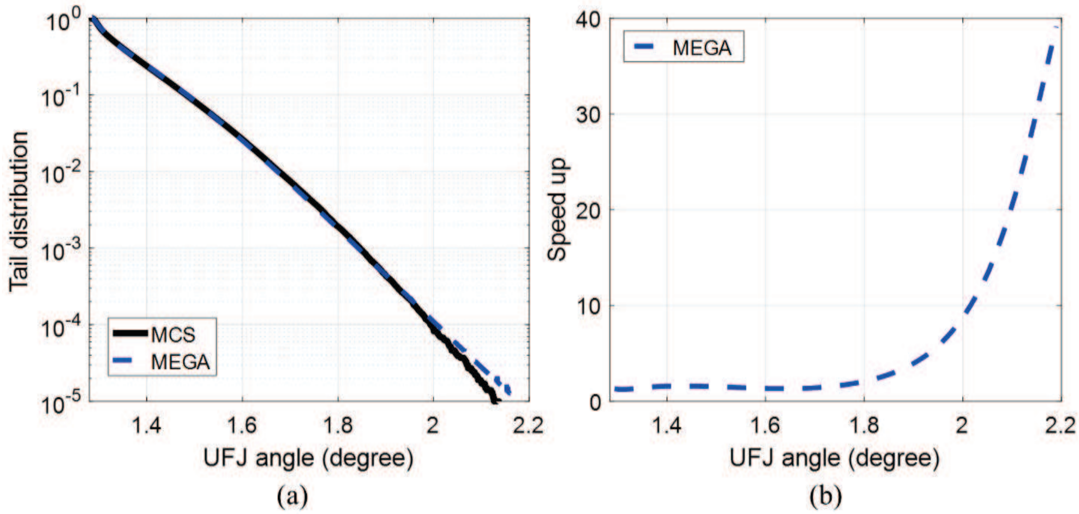

Figure 1. Mean tail distribution and speed up

Figure 1(a) shows comparison between the mean failure probability of 100 samples recovered by MEGA the MCS results from 1.0×10^6 simulation. Figure 1(b) shows the corresponding speed up. As can be seen from the figure, the mean failure probability of MEGA agrees quite well with MCS which shows the small bias of MEGA. The speed up is around 30 times faster than MCS when predicting failure probability 1.0×10^{-5}.

Noting that MEGA is a post-processing method for probability density estimation, it does not require coupling with FEM software. Thus, parallel execution of finite element analysis can be readily implemented. The finite element analysis of drilling riser was conducted on a 16 threads workstation parallelly, which saves a lot of simulation time. The result of MEGA is PDF or tail distribution, for a structural design or optimization problem which requires failure probabilities are different threshold, no repetitive calculation is required. If different failure mode is required, no repetitive FEM analysis is required. The speed up is calculated by taking variance error due to sample data into consideration, which is neglected and avoided in most of the existing literature.

5 Conclusions

This paper presents a new moment method based on entropy theories and GA. The paper shows that to recover PDF using MEGA is to solve an optimization problem with a system of fractional moment constraints, where the objective function of the optimization problem is derived from entropy theories. The population moments are approximated by sample fractional moments to reduce the variance error. The non-convex and non-continuous optimization problem is solved by different heuristic algorithms. GA is recommended for solving the global optimization problem. The main contribution of this paper is to have successfully solved the optimization problem, which is a big obstacle for applying entropy theories with fractional moments to structural reliability methods. By applying the MEGA to an offshore drilling riser example and a wind turbine load simulation problem, it demonstrates that the MEGA is robust, accurate and efficient.

Acknowledgements

The authors thank the National Research Foundation, Keppel Corporation and National University of Singapore for supporting this work done in the Keppel-NUS Corporate Laboratory (Research grant R-261-507-012-281). The conclusions put forward reflect the views of the authors alone, and not necessarily any of the institutions within the Corporate Laboratory.

References

Alibrandi, U., & Mosalam, K. M. Kernel Density Maximum Entropy Method with Generalized Moments for evaluating probability distributions, including tails, from a small sample of data. *International Journal for Numerical Methods in Engineering*. 2018.

Chen, B., Hu, J., & Zhu, Y. Computing maximum entropy densities: A hybrid approach. *Signal Processing: An International Journal (SPIJ), 4*(2), 114. 2010

Ditlevsen, O., & Bjerager, P. Methods of structural systems reliability. *Structural Safety, 3*(3), 195-229.1986.

Ebeling CE. An introduction to reliability and maintainability engineering. New York: McGraw Hill; 1997.

Kapur, J. N., & Kesavan, H. K. *Entropy optimization principles with applications*. Boston: Academic Press. 1992.

Kenny, W. G. Flexcom (Version 8.6.1). Retrieved from http://www.mcskenny.com/software-solutions/flexcom.html. 2016.

Kullback, S. (1959). *Information theory and statistics*: Wiley.

Kullback, S., & Leibler, R. A. On information and sufficiency. *The Annals of Mathematical Statistics, 22*(1), 79-86. 1951.

Li, G., & Zhang, K. A combined reliability analysis approach with dimension reduction method and maximum entropy method. *Structural and Multidisciplinary Optimization, 43*(1), 121-134. doi:10.1007/s00158-010-0546-2. 2011.

Xu, J. A new method for reliability assessment of structural dynamic systems with random parameters. *Structural Safety, 60*, 130-143. doi:10.1016/j.strusafe.2016.02.005. 2016.

Zhang, X., & Pandey, M. D. Structural reliability analysis based on the concepts of entropy, fractional moment and dimensional reduction method. *Structural Safety, 43*, 28-40. doi:http://dx.doi.org/10.1016/j.strusafe.2013.03.001. 2013.