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Tail Probability Equivalent Linearization Method for stochastic dynamic analysis of marine risers

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The dynamic analysis of a deepwater floating production systems has many complexities, such as the dynamic coupling between the vessel and the riser, the coupling between the first-order and second-order wave forces, several sources of nonlinearities. Moreover, the sea state is random; hence the need of stochastic dynamic analysis. In this paper the evaluation of the non-Gaussian distributions of the responses of these systems is developed through a novel method of stochastic equivalent linearization, called Tail Probability Equivalent Linearization Method (TPELM). The Tail Probability Equivalent Linear System (TPELS) is the Equivalent Linear System (ELS) obtained by minimizing the difference between the tail probability of the equivalent system and the original nonlinear system. The TPELS is defined through a machine learning algorithm providing the hyperplane capable to classify correctly the highest number of sample data. TPELM has several attractive features: (*i*) it gives a good approximation of the tail probability with reduced computational cost, (*ii*) it works well in high dimensions and it is not affected by the presence of multiple design points, (*iii*) it gives information about the achieved accuracy, (*iv*) it does not require the evaluation of the design point, which is challenging in high dimensional spaces. The accuracy and efficiency of TPELM is shown through the application to a simplified model of a marine riser.

Keywords: Marine Riser, Stochastic Linearization, Tail Equivalent Linearization Method, Stochastic Dynamic Analysis, Secant Hyperplane Method, Support Vector Machine

1 Introduction

Floating Production Systems (FPS) have become an integral part of deepwater development in oil and gas exploration and production. Risers, mooring system and floater represent an integrated dynamic system responding to environmental loading due to waves, current and wind in a complex way. Moreover, the environmental loads are random, calling for the need of stochastic dynamic analysis. The aim is the evaluation of the response statistics of dynamic systems subjected to stochastic excitations. This task is relatively easy only when the input is Gaussian and the dynamic system is linear. This is unfortunately not the case in FPS, since some forces follow a non-Gaussian distribution and the dynamic system is inherently nonlinear due to large displacement, drag forces and second-order wave forces. The most robust procedure is represented by the Monte Carlo Simulation (MCS), however its computational cost is generally demanding, especially in the context of reliability analyses. Indeed, we are interested in the design of structures with high reliability, which implies very small failure probabilities. To reduce the computational cost, an attractive tool is represented by the Tail Equivalent Linearization Method (TELM), which is quite well established method, and it has been applied to many other problems, including marine structures (Garrè and Der Kiureghian, 2010). However, TELM has some known shortcomings for stochastic dynamic analysis of real-world engineering systems: (*i*) the evaluation of the design point is challenging in very high-dimensional problems, (*ii*) in some cases the TELM approximation is not accurate enough, (*iii*) the accuracy of TELM is not known in advance.

To this aim, in this paper we adopt the Tail Probability Equivalent Linearization Method (TPELM), recently proposed by the authors for a general dynamic system subjected to stochastic excitation (Alibrandi & Mosalam 2017). TPELM keeps all the attractive features of TELM, however it is capable to overcome all its shortcomings. For a comparison between the classical stochastic Equivalent Linearization Method (ELM), TELM and TPELM, see (Alibrandi and Mosalam 2017). The Tail Probability Equivalent Linear System (TPELS) is defined as the Equivalent Linear System (ELS) obtained by minimizing the difference in terms of tail probability between the TPELS and the original nonlinear system. Once TPELS is determined, the quantities of engineering interest can be evaluated with no further dynamic computation. The accuracy and efficiency of TPELM is shown through the application to a simplified model of FPS.

2 Modelling of the Floating Production System (FPS)

The equations of the motion for the coupled system composed by the vessel and the riser are (Low and Langley 2006)

$$\boldsymbol{M}\ddot{\boldsymbol{d}}(t) + \boldsymbol{C}\dot{\boldsymbol{d}}(t) + \boldsymbol{K}\boldsymbol{d}(t) + \boldsymbol{h}_C(\boldsymbol{x}) = \boldsymbol{F}(t) \tag{1}$$

where $\boldsymbol{d}(t) = \{\boldsymbol{d}_V(t) \quad \boldsymbol{d}_L(t)\}$ collects the displacement vectors of the vessel and the lines, respectively, $\boldsymbol{h}_C(\boldsymbol{x})$ is a vector defining the connection of the vessel with the riser, $\boldsymbol{F}(t) = \{\boldsymbol{F}_V(t) \quad \boldsymbol{F}_L(t)\}$ collects the wave forces on the vessel and on the lines, while $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the mass, damping and the time-dependent stiffness matrix of the FPS, respectively, defined as

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{M}_V & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{M}_L \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} \boldsymbol{C}_V & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{C}_L \end{bmatrix}, \quad \boldsymbol{K} = \begin{bmatrix} \boldsymbol{K}_V & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{K}_L \end{bmatrix} \tag{2}$$

The vessel is described as a rigid body with six degrees of freedom at the center of gravity, namely surge, sway, heave, roll, pitch and yaw. $\boldsymbol{M}_V$ contains the structural mass of the vessel including its added mass, $\boldsymbol{C}_V$ is the damping matrix including viscous skin drag, wave drift damping and radiation damping, while $\boldsymbol{K}_V$ is the linear hydrostatic stiffness matrix of the vessel. In practice the lateral deflection of the riser is kept within safe limits by appropriate top tensioning and vessel positioning. Thus, the riser is modelled through large deflections and small strains. The time dependent stiffness matrix $\boldsymbol{K}_L(t)$ of the riser is given by the sum of the bending stiffness matrix and of the time-dependent geometric stiffness matrix; $\boldsymbol{M}_L$ and $\boldsymbol{C}_L$ are the mass and damping matrix of the riser, respectively.

2.1 *Wave forces on the vessel*

The time dependent force on the vessel $\boldsymbol{F}_V(t)$ is the sum of the following components:

$$\boldsymbol{F}_V(t, \boldsymbol{u}) = \boldsymbol{F}_w^{(1)}(t, \boldsymbol{u}) + \boldsymbol{F}_w^{(2)}(t, \boldsymbol{u}) + \boldsymbol{F}_{wind} + \boldsymbol{F}_{curr} \tag{3}$$

where $\boldsymbol{u}$ is a vector of $2n$ normal standard random variables, $\boldsymbol{F}_w^{(1)}(t,\boldsymbol{u})$ and $\boldsymbol{F}_w^{(2)}(t,\boldsymbol{u})$ represent the time-varying first- and second-order wave force, respectively, $\boldsymbol{F}_{wind}$ is the wind force, and $\boldsymbol{F}_{curr}$ is the force of the currents.

The random sea state $\eta(t,\boldsymbol{u})$ is modelled through a discrete Fourier series (Jensen and Capul 2006, Alibrandi & Koh 2017)

$$\eta(t,\boldsymbol{u}) = \sum_{i=1}^{n}\sqrt{G_{\eta\eta}(\omega_i)\Delta\omega}\,[cos(\omega_i t)u_i^c + sin(\omega_i t)u_i^s] = \boldsymbol{s}_\eta(t)\cdot\boldsymbol{u} \tag{4}$$

where n is the number of harmonic components, $u_1^c, u_2^c, \dots, u_n^c$ and $u_1^s, u_2^s, \dots, u_n^s$ are normal standard random variables. The correlation structure of $\eta(t,\boldsymbol{u})$ is given in terms of the underlying one-sided wave spectrum $G_{\eta\eta}(\omega)$, through the deterministic shape functions $s_{\eta,i}^c(t)$ and $s_{\eta,i}^s(t)$, collected in the $2n$ −vector $\boldsymbol{s}_\eta(t)$.

The first-order wave forces $F_w^{(1)}(t,\boldsymbol{u})$ acting on the vessel may be expressed as

$$F_w^{(1)}(t,\boldsymbol{u}) = \sum_{i=1}^{n} T^{(1)}(\omega_i)\eta(\omega_i,\boldsymbol{u}) = \boldsymbol{s}_{F_1}(t)\cdot\boldsymbol{u} \tag{5}$$

where $T^{(1)}(\omega_i)$ are the first-order transfer functions and they are given by a linear diffraction analysis. In addition to the first-order forces, the vessel is subjected to the second-order forces coming from nonlinear hydrodynamic effects (Faltinsen 1990). These forces are determined from a second order diffraction analysis

$$F_w^{(2)}(t,\boldsymbol{u}) = \sum_{i=1}^{n}\sum_{j=1}^{n} T^{(2)}(\omega_i,\omega_j)\eta(\omega_i,\boldsymbol{u})\eta(\omega_j,\boldsymbol{u}) = \boldsymbol{s}_{F_2}(t)\cdot\boldsymbol{h}(\boldsymbol{u}) \tag{6}$$

where $T^{(2)}(\omega_i,\omega_j)$ are the Quadratic Transfer Functions (QTF), while $\boldsymbol{h}(\boldsymbol{u})$ is a quadratic function of the normal standard random variables.

2.2 *Wave forces on the riser*

Using the classical linear wave theory, at a given location $\boldsymbol{x}$, the horizontal wave particle velocity $\dot{\boldsymbol{d}}_w(t,\boldsymbol{x},\boldsymbol{u})$ and acceleration $\ddot{\boldsymbol{d}}_w(t,\boldsymbol{x},\boldsymbol{u})$are given as

$$\begin{cases}\dot{\boldsymbol{d}}_w(t,\boldsymbol{x},\boldsymbol{u}) = \dot{\boldsymbol{d}}_c(\boldsymbol{x}) + \boldsymbol{s}_v(t,\boldsymbol{x})\cdot\boldsymbol{u}\\ \ddot{\boldsymbol{d}}_w(t,\boldsymbol{x},\boldsymbol{u}) = \boldsymbol{s}_a(t,\boldsymbol{x})\cdot\boldsymbol{u}\end{cases} \tag{7}$$

where the $2n$ −vectors $\boldsymbol{s}_v(t,\boldsymbol{x})$ and $\boldsymbol{s}_a(t,\boldsymbol{x})$ collect the deterministic shape functions in terms of velocity and acceleration of the particle water, and they depend on the location $\boldsymbol{x}$, the wave elevation η, the wave number. In Eq.(7) $\boldsymbol{d}_w(t,\boldsymbol{x},\boldsymbol{u})$ and $\boldsymbol{d}_c(\boldsymbol{x})$ represent the displacement of the particle water and the time-independent velocity of the current, respectively. It is commonly assumed that the Morison's equation can be modified to model the hydrodynamic force per unit length on a flexible structure (Spanos et al. 1990)

$$f(t,\boldsymbol{x},\boldsymbol{u}) = f_D(t,\boldsymbol{x},\boldsymbol{u}) + f_M(t,\boldsymbol{x},\boldsymbol{u}) = \frac{1}{2}\rho_W C_D D(\boldsymbol{x})\dot{\boldsymbol{q}}(t,\boldsymbol{x},\boldsymbol{u})|\dot{\boldsymbol{q}}(t,\boldsymbol{x},\boldsymbol{u})| +$$
$$\rho_W A(\boldsymbol{x})\ddot{\boldsymbol{d}}_w(t,\boldsymbol{x},\boldsymbol{u}) + C_M\rho_W A(\boldsymbol{x})\ddot{\boldsymbol{q}}(t,\boldsymbol{x},\boldsymbol{u}) \tag{8}$$

where ρ_W is the water density, C_D and C_M are the drag coefficient and the inertia coefficient, $D(\boldsymbol{x})$ and $A(\boldsymbol{x})$ are the outside diameter and cross-sectional area of the riser, while $\boldsymbol{q}(t,\boldsymbol{x},\boldsymbol{u}) = \boldsymbol{d}_w(t,\boldsymbol{x},\boldsymbol{u}) - \boldsymbol{d}_L(t,\boldsymbol{x},\boldsymbol{u})$ is the difference between the displacement of the particle water and the riser. The fluid force acting on the lines, $\boldsymbol{F}_L(t,\boldsymbol{u})$, is then calculated from Eq. (8)

$$\boldsymbol{F}_L(t,\boldsymbol{u}) = \boldsymbol{F}_D(t,\boldsymbol{u}) + \boldsymbol{F}_M(t,\boldsymbol{u}) \tag{9}$$

where $\boldsymbol{F}_D(t,\boldsymbol{u})$ and $\boldsymbol{F}_M(t,\boldsymbol{u})$ are the nodal drag and inertia forces, respectively, obtained from $f_D(t,\boldsymbol{x},\boldsymbol{u})$ and $f_M(t,\boldsymbol{x},\boldsymbol{u})$ by setting $\boldsymbol{x} = \boldsymbol{x}_k$, with $\boldsymbol{x}_k$ the nodal coordinate.

3 Tail Probability Equivalent Linearization Method (TPELM)

Consider the response of the FPS to the stochastic excitation. Owing to the random variables $\boldsymbol{u}$, the response is stochastic and is denoted as $X(t,\boldsymbol{u})$ which can mean any component of $\boldsymbol{d}_V(t,\boldsymbol{u})$ or $\boldsymbol{d}_L(t,\boldsymbol{u})$ for the vessel and the lines, respectively. For a specified threshold x and time t, the tail probability is defined as $P_f(t,x) = Prob[X(t,\boldsymbol{u}) \geq x]$. To apply the structural reliability theory, we define the limit state function $g(t,x,\boldsymbol{u}) = x - X(t,\boldsymbol{u})$ so that the failure probability with respect to the limit state $P_f(t,x) = Prob[g(t,x,\boldsymbol{u}) \leq 0]$ is equal to the tail probability. In our case, we have clearly a nonlinear dynamic system since the second-order wave forces $F_w^{(2)}(t,\boldsymbol{u})$ are a non-Gaussian stochastic process, and the forces on the riser $F_L(t,\boldsymbol{u})$ are a non-Gaussian stochastic process depending on the non-Gaussian response of the riser. Consequently, the limit state is not linear. The most robust procedure for the evaluation of the tail probability is the Monte Carlo Simulation (MCS), but in its crude form, it is too demanding. This is especially so for the very small tail probabilities, which are the most crucial for a reliability analysis. To reduce the cost, in this paper we adopt the Tail Probability Equivalent Linearization Method (TPELM), recently proposed by the authors.

The Tail Probability Equivalent Linear System (TPELS) is defined as the Equivalent Linear System (ELS) obtained by minimizing the difference in terms of tail probability between the TPELS and the original nonlinear system. In a problem of structural reliability, any ELS is represented by a hyperplane. Therefore, the task is finding a linear surrogate model able to detect a tail probability as close as possible to $P_f(t,x)$. To this aim, TPELM adopts the Secant Hyperplane (SH) method, described in (Alibrandi et al. 2016). The SH is determined through a procedure based on the Support Vector Method (SVM) and it represents an optimal linear classifier from the point of view of the statistical learning theory. It has equation $g_{SH}(t,x,\boldsymbol{u}) = b_{SH}(t,x) - \boldsymbol{w}_{SH}(t,x)\cdot\boldsymbol{u}$, whose parameters $\boldsymbol{w}_{SH}(t,x)$ and $b_{SH}(t,x)$ define the hyperplane with minimum classification error and maximum capabilities prediction over the unseen data.

The secant approximation of the tail probability is $P_{SH}(t,x) = Prob[g_{SH}(t,x,\boldsymbol{u}) \leq 0] \cong P_f(t,x)$. The SHM provides also the bounding hyperplanes $g_L(t,x,\boldsymbol{u}) = b_U(t,x) - \boldsymbol{w}_{SH}(t,x)\cdot\boldsymbol{u}$ and $g_U(t,x,\boldsymbol{u}) = b_L(t,x) - \boldsymbol{w}_{SH}(t,x)\cdot\boldsymbol{u}$, with $b_L \leq b_{SH} \leq b_U$. In correspondence it is possible to determine credible bounds of the tail probabilty, i.e. $P_L \leq P_{SH} \leq P_U$, where $P_L(t,x) = Prob[g_L(t,x,\boldsymbol{u}) \leq 0]$ and $P_U(t,x) = Prob[g_U(t,x,\boldsymbol{u}) \leq 0]$. As a consequence, it is possible to know the accuracy of the achieved approximation, when a given number of samples is considered.

The parameters of the SH are obtained through a data-driven iterative procedure: at the $k-$th iteration the algorithm gives $\boldsymbol{w}_{SH}^{(k)}(t,x)$ and $b_{SH}^{(k)}(t,x)$. The iterative procedure implemented in SHM requires for any threshold the preliminary definition of an important direction. In (Alibrandi et al. 2016) the important direction is chosen coinciding with the design point direction. However, especially in high dimensions, the evaluation of the design point is challenging. Here, a different strategy is adopted. It is noted that for a low threshold value x_1, inside a set of a reduced number of sample data, it is likely that some of them belong to the failure domain. Thus, a first SH is determined, whose slope is $\boldsymbol{w}_{SH}^{(1)}(t, x_1)$, chosen as the important direction. For the following thresholds x_k, $k > 1$, the important direction is assumed coinciding with the slope of the SH corresponding to the previous threshold, i.e. $\boldsymbol{w}_{SH}^{(1)}(t, x_k) \equiv \boldsymbol{w}_{SH}(t, x_{k-1})$. In this way, the SH is evaluated for each threshold of the response at time instant t. Extensive numerical experimentation has shown that the secant approximation $P_{SH}(t, x) = Prob[g_{SH}(t, x, \boldsymbol{u}) \leq 0]$ to $P_f(t, x)$ requires on average 400-500 dynamic computations per threshold. But of course a reduced number of analyses can also be adopted, since in any case the accuracy of the approximation is provided by the credible bounds.

Once the SH is determined, the TPELS can be evaluated through $X_{TPELS}(t, x, \boldsymbol{u}) = \boldsymbol{a}_{TPELS}(t, x) \cdot \boldsymbol{u}$, where $\boldsymbol{a}_{TPELS}(t, x) = x(\boldsymbol{w}_{SH}/b_{SH})$ (Alibrandi and Mosalam 2017). It is easy to verify that $P_{TPELS}(t, x) = Prob[X_{TPELS}(t, x, \boldsymbol{u}) \geq x] \equiv P_{SH}(t, x)$ and consequently the difference $\left|P_{TPELS}(t, x) - P_f(t, x)\right|$ is minimized. The TPELS is uniquely defined by the coefficients $\boldsymbol{a}_{TPELS}(t, x)$, whose knowledge allows to determine each desired quantity of engineering interest.

4 Numerical application

A simplified model of a FPS presented in (Low & Langley 2008, Alibrandi & Koh 2017) is analyzed. The FPS is modelled through as a system with 2 degrees of freedom, represented by the generalized displacements of the vessel $x_V(t)$ and of the lines $x_L(t)$. In such case, the matrices of the equations of the motion [see Eqs.(1) and (2)] are

$$\boldsymbol{M} = \begin{bmatrix} m_V & 0 \\ 0 & m_L \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} c_V & 0 \\ 0 & c_L \end{bmatrix}, \quad \boldsymbol{K} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & 2k_1 \end{bmatrix}, \quad \boldsymbol{h}_C = \begin{Bmatrix} k_3(x_V - x_L)^3 \\ k_3[x_L^3 + (x_V - x_L)^3] \end{Bmatrix} \tag{12}$$

The random sea state is described by a JONSWAP spectrum of parameters H_s, T_z and γ. All the parameters of this numerical application are reported in (Alibrandi and Koh 2017). The spectrum has been discretized from $\omega = 0\ rad/sec$ to $\omega = 0.8\ rad/sec$, with a frequency step $\Delta\omega = (2\pi)/t_{fin} = 0.00488\ rad/sec$, leading to $2n = 360$ normal standard random variables. At first we apply Monte Carlo Simulation (MCS) with 50,000 samples, which is adopted to benchmark the results of TPELM. Figure 1 presents the tail probabilities of the vessel and the lines. Here we compare: (*i*) MCS with 50,000 samples (circle markers), (*ii*) Gaussian approximation whose variance is evaluated through MCS (dotted line), and (*iii*) TPELM solution (thick continuous line) together with its credible bounds (dashed lines). It is seen that the displacements of the vessel and especially of the lines are markedly non-Gaussian. The Gaussian solution fits the response well in the central region of the distribution but not in its tails, where it gives an unsafe approximation. Therefore for this example with significant nonlinearities, the application of a classical equivalent linearization method for highest thresholds is questionable. In contrast, TPELM fits the MCS solution very well and it reveals useful information about the achieved accuracy through credible bounds. In terms of numerical efficiency, it is underlined that TPELM is very effective especially in the range of the very small probabilities $P_f = 10^{-4} - 10^{-6}$, where it requires only 500 dynamic analyses per threshold while MCS would require on average more than $10^6 - 10^8$ analyses.

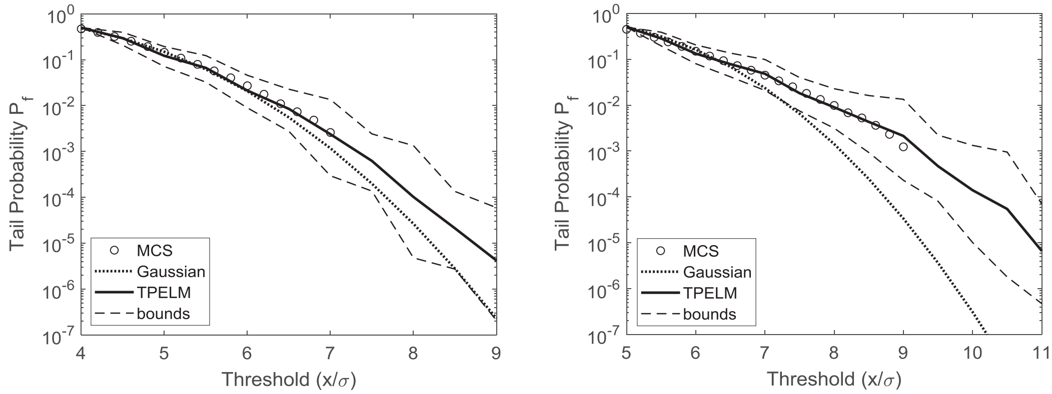

Figure 1. Tail probability of: (left) vessel, (right) lines

5 Concluding Remarks

This paper presents stochastic dynamic analysis of floating production systems, through the Tail Probability Equivalent Linearization Method (TPELM). The Tail Probability Equivalent Linear System (TPELS) is chosen such that its tail probability is as close as possible to the target. For engineering design purposes, a quantity of interest is the crossing rate or first passage probability, which can be easily determined with no further dynamic computation through the adoption of the TPELS. The developed numerical example has shown that, with a reduced computational cost (of several orders of magnitude compared to MCS), TPELS provides excellent approximations of very small probabilities.

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