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RELIABILITY ASSESSMENT OF EXCAVATIONS IN SPATIALLY VARIABLE SOILS USING SOBOL' SENSITIVITY INDEX

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This paper presents a method to assess the reliability of excavations in spatially variable soil by using sensitivity index known as the Sobol' index, which is extended to consider spatially correlated random variables. The Sobol' index can be interpreted as the variance reduction of model responses, and can be used to identify the most influential zones in subsurface soil to the response variability. The spatially variable subsurface domain is simulated by random field models with cross-correlated soil shear strength and stiffness parameters. Latin hypercube sampling (LHS) is adopted to generate the random fields, and coupled with sparse polynomial chaos expansion (SPCE) to obtain the probability density function of model responses, such as the maximum wall deflection and bending moments. The Sobol' index across the soil domain can then be evaluated with the most influential zone defined by the one with the largest index value. The approach also allows efficient evaluation of the mean and variance of system responses conditioned to the sample values at any location of the domain, and can aid the decision-making process for geotechnical investigation and risk assessment of excavation projects.

Keywords: Reliability analysis, Random field modelling, Wall deflection, Sobol' index.

1 Introduction

Geotechnical variability is a main source of uncertainty in many civil engineering projects, and its influence to the reliability and risk management should be carefully assessed. The uncertainty in the geotechnical process may be classified into three categories: inherent uncertainty such as spatial variability of soil properties, statistical uncertainty and transformation uncertainty. The random field theory is often adopted to investigate the influence of spatial variability, where the variance of soil properties at different locations is described by spatial correlation functions. For example, Jiang *et al*. (2015) investigated the influence of five different types of autocorrelation functions on the reliability index of slopes with spatially variable soils. On the basis of random field theory, random finite element method or random finite difference method can be applied to explore the effects of spatial variability of soil properties on the response of geotechnical structure (e.g. Fenton and Griffiths 2009; Cho and Park 2010; Sert *et al*. 2015).

Considering the variability of soil properties, the Monte Carlo simulation technique may be applied with random finite element/finite difference methods in probabilistic analysis, in order to obtain the probability density function of geotechnical responses. However, the process can be time-consuming as it requires the geotechnical analysis of many realizations, using finite element analysis or finite difference analysis methods, and the robustness of raw Monte Carlo simulations is not guaranteed (Lo and Leung 2017). To circumvent this problem, Blatman and Sudret (2010) proposed the sparse polynomial chaos expansion (SPCE), which is an extension of polynomial chaos expansion (PCE), to serve as a surrogate model to represent the system responses through an analytical equation. This helps to reduce the number of finite element/finite difference simulations, thereby reducing the computational demands of the probabilistic analyses.

The SPCE technique may be applied to conditional random field modeling, in order to evaluate the influence of sample information to the overall uncertainty of the system response. Alternatively, Lo and Leung (2018) incorporated SPCE into global sensitivity analysis by the Sobol' index (Sobol' 2001), in order to quantify the influence of sample values at various potential sampling locations. This was shown to be efficient in determining the optimal sampling points for different types of geotechnical problems, as there is no need to repeat multiple scenarios of conditional random field analyses.

To further demonstrate the capabilities of this methodology, this paper presents the probabilistic analysis of the response of a cantilever retaining wall during excavation. The internal friction angle and Young's modulus of the soil are simulated as random fields with cross-correlated spatial structure. The probability density function of the system response, including the maximum deflection and bending moment, is obtained from the SPCE metamodel. The Sobol' index analysis also quantifies the importance of sampling at various locations in the subsurface domain, and the influence to the response of the retaining wall.

2 Random Field Simulation

In general, the variations of soil properties in the subsurface domain may not follow Gaussian distribution. Yet, a correlated Gaussian random field may be first generated, and then transformed into a non-Gaussian random field. In this study, log-normally distributed random fields of soil properties are simulated, with spatial autocorrelation represented by the squared exponential function:

$$R_{ij} = \exp\left[-\left(\frac{\Delta x}{\theta_x}\right)^2 - \left(\frac{\Delta y}{\theta_y}\right)^2\right] \tag{1}$$

where Δx and Δy are the separation distances, and θ_x and θ_y are the autocorrelation distances in x and y directions. In the probabilistic analysis, the random fields are represented by vectors of random variables ($\boldsymbol{z}$):

$$z_i(x,y) = \exp[\tilde{z}_i(x,y)] = \exp\left[\mu_{\ln,i} + \sigma_{\ln,i}\left(\mathbf{H}\boldsymbol{\Lambda}^{1/2}\boldsymbol{\xi}\mathbf{L}_c^T\right)\right] \tag{2}$$

where $\sigma_{\ln,i} = \sqrt{\ln(1+(\sigma_i/\mu_i)^2)}$, $\mu_{\ln,i} = \ln\mu_i - \sigma_{\ln,i}^2/2$, with $\mu_{\ln,i}, \sigma_{\ln,i}$ and μ_i, σ_i representing the mean and standard deviation of Gaussian random variables and log-normal random variables, respectively; the subscript i may represent the Young's modulus (E) or friction angle (ϕ); $\boldsymbol{\xi}$ is the independent standard Gaussian sample matrix; $\mathbf{H}$ is the matrix that consists of columns of eigenvectors; $\boldsymbol{\Lambda}$ is the diagonal matrix with eigenvalue sorted in decreasing order; $\mathbf{L}_c^T$ represents the cross-correlation matrix. The matrices of eigenvector, $\mathbf{H}$, and eigenvalue, $\boldsymbol{\Lambda}$, are obtained by spectral decomposition of the matrix $\mathbf{R}$, consisting of R_{ij} components in Eq. (1).

3 Sensitivity Analysis by Sobol' Sensitivity Index

The Sobol' index was proposed by Sobol' (2001) to quantify the influence of various parameters to a physical model. For example, in geotechnical engineering, Al-Bittar and Soubra (2013) utilized the Sobol' index to study the most influential soil parameters for footing displacements. It should be noted that the original Sobol' index was developed to assess the contributions of independent random variables to system responses, without consideration of correlations between the parameters.

The focus of this study is on the influence of spatially correlated soil properties. To this end, Lo and Leung (2018) incorporated the Sobol' index into the implementation of SPCE, with consideration of spatially-correlated random variables:

$$S(e_i) = \frac{\mathrm{Var}[\mathrm{E}(g|e_i)]}{\mathrm{Var}(g)} \tag{3}$$

where e_i represents the residual of random variable z , g is the system response which may be represented by PCE. Therefore, $\mathrm{E}(\mathrm{g}|\mathrm{e_i})$ represents the expectation of system response conditioned on a fixed value $\mathrm{e_i}$. The detailed derivations of the extended Sobol' index evaluation can be found in Lo and Leung (2018), and the computation of Sobol' index mainly involves post-processing the results from unconditional random field models. Once the Sobol' index is computed at each soil location in the subsurface domain, a Sobol' index 'contour' can be generated to graphically represent the importance of each potential sampling location. In addition, if a sample is retrieved at the location, the information it provides will help reduce the uncertainty in system response. This can be represented as the reduction of COV for the conditional system response:

$$\text{COV reduction} = \mathrm{COV}(g) - \mathrm{E}[\mathrm{COV}(g|e_i)] = [1 - \sqrt{1 - S(e_i)}]\mathrm{COV}(g) \tag{4}$$

Moreover, the mean conditional response could be obtained based on the retrieved sample value:

$$\mathrm{E}(g|e_i) = r_0 + r_1 e_i + r_2 e_i^2 \tag{5}$$

and the coefficients r_0 , r_1 and r_2 can be derived from the SPCE following the procedures described in Lo and Leung (2018). Eqs. (4) and (5) together provide an efficient assessment on conditional reliability, without resorting to extra conditional random field analyses. It is also possible to evaluate the 'reliability index' using the mean and standard deviation of the response.

4 Illustrative Example: Cantilever Retaining Wall

The case of a cantilever retaining wall in spatially variable soil during excavation is presented here to illustrate the abovementioned methodology. The internal friction angle and Young's modulus of the soil are treated as perfectly correlated random fields in the reliability analyses. The mean value and COV for internal friction angle and Young's modulus are 30°, 60MPa (mean value) and 0.2, 0.15 (COV) respectively. Only isotropic spatial correlation is considered in this paper, which means the autocorrelation distances ($\theta_x = \theta_y = \theta$) are the same in all directions. Two separate cases are analyzed, with $\theta = 10$ m and 20 m, respectively. The cross-correlation parameter between the friction angle and Young's modulus is 1.

In this study, the finite difference code, *FLAC,* is used to conduct the geotechnical analyses. Considering the symmetry in the excavation, only half of the excavation is modelled. The height of the cantilever retaining wall is 10 m, with the lower 5 m embedded in the soil to provide passive pressure for the wall. The soil is modelled as linear elastic-perfectly plastic (Mohr-Coulomb) material, while the cantilever wall is modelled as linear-elastic material, with Young's modulus of 30 GPa and thickness of 0.6 m, and the interaction of soil-wall is simulated by interface elements. While the Young's modulus and friction angle of the soil are modelled as random fields, the other soil parameters are assigned as constants. These include cohesion (1 kPa), Poisson ratio (0.2) and soil density (1900 $\mathrm{kg/m^3}$). The number of elements in the numerical model is 1900, and further reductions on mesh size do not influence the analysis results.

To construct SPCE for autocorrelation distances of 10 m and 20 m, the corresponding number of principal components (to preserve 95% total variance) are 29 and 10, and the total number of coefficients retained in the second order SPCE expression are 465 and 66, respectively. To obtain these SPCE coefficients, 1000 realizations are generated. Thereafter, the probability density functions (PDFs) of the maximum bending moment and maximum horizontal displacement of the retaining wall are computed through the SPCE. Figure 1 shows the PDFs in the two scenarios of unconditional random field models with different autocorrelation distances. The curves in Figure 1 represent the responses obtained from SPCE, which generally match well with the histograms produced by the 1000 realizations. Comparing Figure 1(a) with 1(b) and Figure 1(c) with 1(d), different autocorrelation distances entail very similar mean system responses, but the variances of system responses increase with the autocorrelation distance, in the cases of unconditional random fields, with no sampling information. On the basis of Figure 1, the reliability index can also be calculated easily.

Based on the SPCE, the Sobol' index at all locations in the subsurface domain can be found. Figure 2 shows the Sobol' index map considering the maximum horizontal displacement as system response g, for the case when autocorrelation distances $\theta = 20$ m. The optimal sampling location is 14 m in depth and 1 m away from cantilever retaining wall, with the maximum Sobol' index equal to 0.8. This corresponds to percentage COV reduction of around 55%. For $\theta = 10$ m, the pattern of Sobol' index map is generally similar, where the maximum Sobol' index is equal to 0.45, and corresponds to percentage COV reduction of around 25%. The reduction of variability of system response conditional on a given sample corresponding to the maximum Sobol' index is more obvious for a larger θ. This phenomenon can be explained by the fact that for large autocorrelation distance, the soil parameters are more correlated with each other, which means the values of random variables do not change abruptly from one element to another. Once a soil sample is obtained from a certain location, the uncertainty in the neighboring regions are greatly reduced, which facilitates the reduction of uncertainty of the system response.

Conditional random fields are constructed to validate the Sobol' index approach. 1000 random fields were set up with internal friction angle and Young's modulus at optimal sample location equal to 36° and 69MPa, which are $+1\mathrm{SD}$ from the mean property value. The mean and standard deviation of maximum horizontal displacement from conditional random field analysis are 17.3mm and 0.9mm; while the mean and SD computed from Sobol' index approach (Eqs. (4) and (5)) are 17.4mm and 1.1mm. Therefore Sobol' index approach can estimate the conditional response quite accurately. Figure 3 compares the probability density obtained by conditional random field and Sobol' index analyses. The two curves match well with each other.

5 Conclusion

An extended Sobol' index approach, which considers the autocorrelation of soil properties, is presented to investigate the reliability of cantilever retaining walls in spatially variable soils.

The approach evaluates the relative significance of each sampling location to the variance of retaining wall response, based on which the corresponding COV reduction of the retaining wall response can be obtained. Since the Sobol' index is calculated using the analytical equation of SPCE, the sensitivity analysis for optimal sampling locations can be performed by post-processing the unconditional random field analysis results, without additional efforts in the simulation of conditional random fields. For the studied scenarios, the most influential sampling point is near the base of the retaining wall. While this study focuses on perfectly-correlated Young's modulus and friction angle, future studies will focus on other cross-correlation coefficients and how these affect the Sobol' index and the optimal sampling locations.

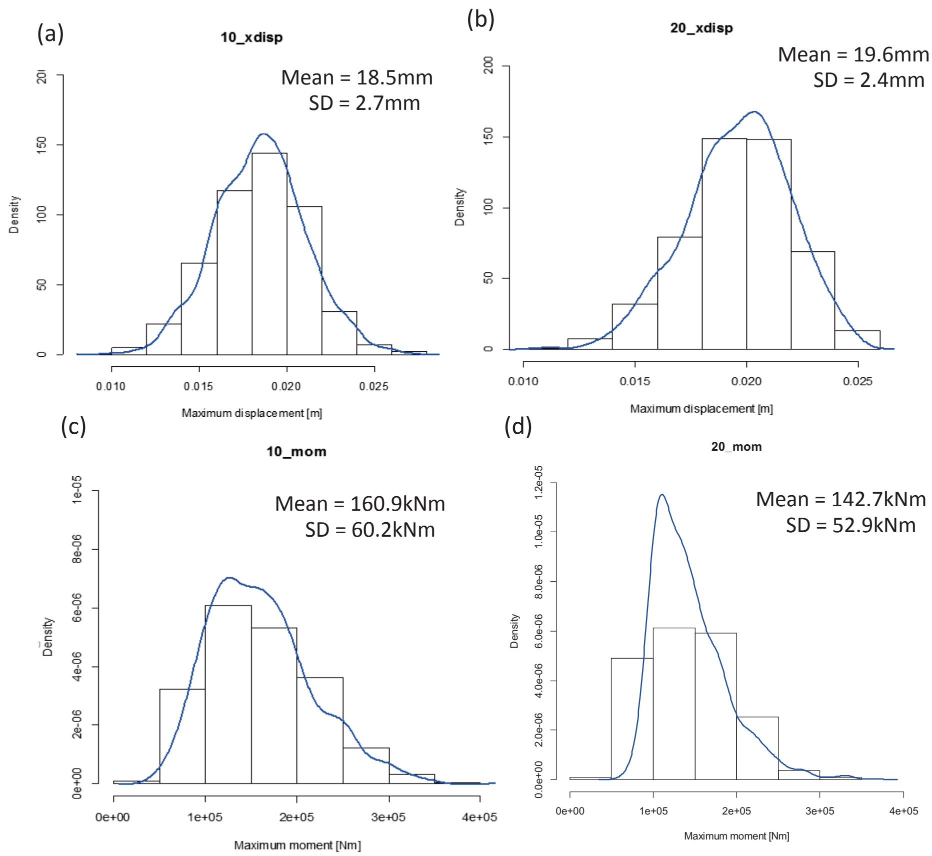

Figure 1. Maximum horizontal displacements and bending moments of retaining wall (left: $\theta = 10$ m; right: $\theta = 20$ m).

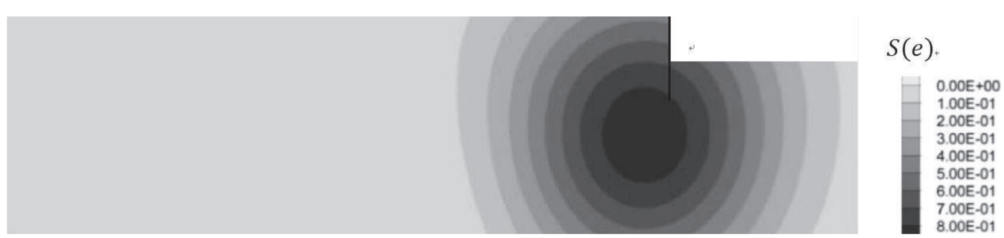

Figure 2. Sobol' index map for autocorrelation distance equal to 20 m

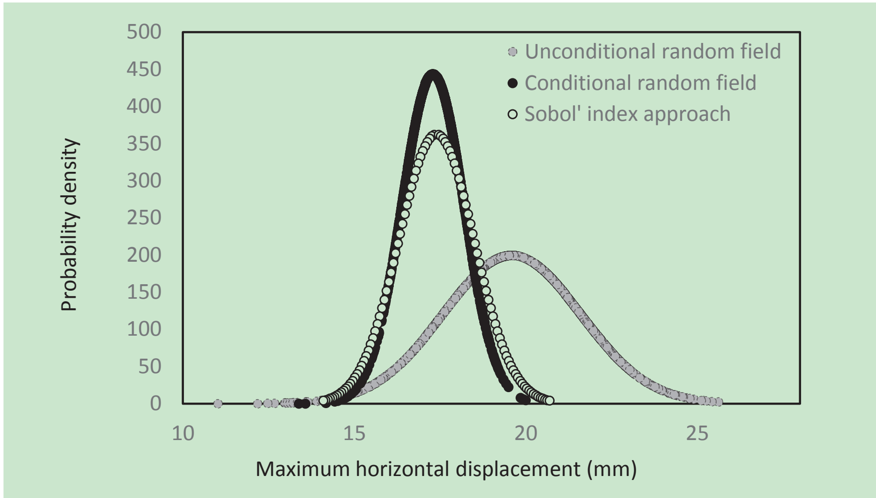

Figure 3. Probability density of maximum horizontal displacement under unconditional random field, conditional random field, and Sobol' index approach

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