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Global Reliability Analysis of RC Frame Structures under Earthquakes Using Subset Simulation

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Abstract：Global reliability analysis (GRA) of structures based on system-level global limit state functions (GLSF) is an efficiently approximate method to overcome the difficulties in traditional failure mode approach of structural system reliability theory. To run the global reliability analysis of complex structures in practice, the crude Monte Carlo simulation as well as its different variants require substantial sample points and cost enormous computing resources for accurate estimations. In this paper, the global reliability analysis of reinforced concrete (RC) frame structures under seismic actions are calculated using the Subset Simulation (SS) method. With adaptive sampling schemes, the required number of sample points are reduced greatly, so that the global reliability index can be easily derived by the SS method. Two global limit state functions for RC frame structures are considered: global load-carrying capacity limit state and global deformation limit state. The two GLSFs of the RC frame structures are obtained through pushover analysis and capacity spectrum method, respectively. Then, the global reliability indices for the two system-level global limit states of RC structures under seismic actions are calculated by SS method. The accuracy and efficiency of the proposed method are verified and compared by the crude Monte Carlo simulation.

Keywords: global reliability, subset simulation, reinforced concrete frame, global limit state functions .

1 Introduction

Global reliability analysis (GRA) is an effective alternative to solving system reliability problems of real and complex structures in practice. Since the traditional failure mode approach to system reliability analysis of structures has two difficulties: search of significant failure modes, and calculation of joint failure probability. In the GRA methodology, the global limit state function is directly constructed through nonlinear structural analysis, instead of generation of safety margin corresponding to each failure mode. Obviously, the GRA approach provides another clear route to solve the system reliability problem of real and complex structures. In this method, Monte Carlo simulation (MCS) is usually employed to obtain the statistical moments of the global capacity of structures, and the accurate nonlinear structural analysis methods which consider more real material behaviors, such as nonlinear static analysis, or nonlinear dynamic analysis, are used to determine the global limit state function (GLSF) and the corresponding statistical moments of the global demands of structures. However, the MCS method requires quite intense computing resources. Considering the very small failure probability of engineering structures under rare earthquakes, an unaffordable computational effort involving 105~107 nonlinear simulations are required to obtain the statistical results of the global seismic capacity and demand of structures.

In order to solve this problem, subset simulation (SS) method is applied in this paper to seismic global reliability analysis of reinforced concrete (RC) frame structures. This efficient simulation method was first proposed by Au and Beck. At the beginning, Metropolis algorithm was used to generate sample points, but the correlation of sample points is higher. Then, Au and Beck used the improved Metropolis-Hastings algorithm which is a reasonable solution to the correlation of sample points. After that, Liu [8] used SS to analyze the multi-grid composite wall structure. Song [9] used SS for structural reliability sensitivity analysis. Recently, SS has been widely utilized in many fields. In this paper, SS is combined with stochastic pushover analysis as well as random capacity spectrum method to construct the global limit state functions (GLSF) of RC frame structures under seismic actions. The GLSFs of the RC frame structures for global load-carrying capacity limit state and global deformation limit state are obtained through stochastic pushover analysis and random capacity spectrum method, respectively, using the SS sampling schemes. Then the global seismic reliability indices of structures could be easily calculated based on the global limit state functions. The results are finally compared using the MCS method to validate the accuracy and efficiency of the proposed method.

2 Subset Simulation

2.1 The Basic Theory of Subset Simulation

SS has high efficiency to estimate high-dimension and small failure probabilities structures[7]. From the reliability problem, F was considered as the failure event in the uncertain X-space and g(x) is a response function or the performance function. So the failure event F is defined by

$$F=\{x:\mathrm{g}(x)\le 0\} \tag{1}$$

where x is the random vector which expresses the uncertain parameters in the system. Then the probability of the target failure event F can be indicated as P_F . Sometimes the failure probabilities in real structures is much small. The main idea in subset simulation is transfer the small probability into the product of a series of large conditional probabilities in sequence. Specifically, we use a nested sequence of failure regions $F_1 \supset F_2 \supset \cdots \supset F_m = F$, so that the target failure event can be defined by

$$F_m=\bigcap_{i=1}^{m} F_i \tag{2}$$

For each intermediate failure event $F_i(i=1,2,\cdots m)$, we denote

$$F_i=\{x:g(x)\le \mathrm{b}_i\}(i=1,2,\cdots,\mathrm{m}) \tag{3}$$

where b_i is a specific threshold and $b_1 > b_2 > \cdots > b_m = 0$. According to the multiplication theorem and the defined of the conditional probability in the probability theory , the failure probability P_F can be calculate by:

$$\begin{aligned} P_F = P\{F\} &= P\left\{\bigcap_{i=1}^{m} F_i\right\} = P\left\{F_m \middle| \bigcap_{i=1}^{m-1} F_i\right\} \cdot P\left\{\bigcap_{i=1}^{m-1} F_i\right\} \\ &= P\{F_m | F_{m-1}\} \cdot P\left\{F_{m-1} \middle| \bigcap_{i=1}^{m-2} F_i\right\} \cdot P\left\{\bigcap_{i=1}^{m-2} F_i\right\} \\ &\vdots \\ &= P\{F_1\} \cdot \prod_{i=2}^{m} P\{F_i | F_{i-1}\} \end{aligned} \tag{4}$$

We denote $P_1 = P\{F_1\}$, $P_i = P\{F_i | F_{i-1}\} (i = 2,3,\cdots,m)$. Then, the target event probability in Eq.(4) can be expressed by a more concise form:

$$P_F = \prod_{i=1}^{m} P_i \tag{5}$$

Consequently, the failure probability P_F can be converted into a product of several conditional probabilities. By selecting several intermediate events, the conditional probabilities can be made sufficiently large which could be more efficiently to calculate.

2.2 The process of Subset Simulation

In structural reliability analysis, Subset Simulation starts with evaluating P_1 by Monte Carlo simulation:

$$\hat{P}_1 = \frac{1}{N_1} \sum_{k=1}^{N_1} I_{F_1}(\mathrm{g}(\mathbf{X}_{1k})) \tag{6}$$

where $\mathrm{g}(\mathbf{X}_{1k})$ is the performance function of the random variables $\mathbf{X}_1 = \{X_{11}, X_{12}, \cdots, X_{1N_1}\}$, in which N_1 is the number of sample in the first simulation region, $\mathbf{X}_1$ are independent and identically distribution samples generated in the first step according to the probability density function(PDF) $q(\mathbf{x})$ and $I_{F_1}(\cdot)$ is the indictor function

$$I_{F_1}(\cdot) = \begin{cases} 0, \mathbf{X}_1 \in F_1 \\ 1, \mathbf{X}_1 \notin F_1 \end{cases}. \tag{7}$$

From Eq.(7), the region of the event F_1 need to be determined by the value of b_1. It hard to know b_1 directly; therefor, denoting a value p_0 set to P_1, then generate the samples $\mathbf{X}_1 = \{X_{11}, X_{12}, \cdots, X_{1k}\}, k = N_1$, calculate the performance function $\{\mathrm{g}(\mathbf{X}_i)\}$ and sort them in an increasing order $g(\mathbf{X}_1) \le g(\mathbf{X}_2) \le \cdots \le g(\mathbf{X}_{N_1})$. The value of b_1 is equal to $g(\mathbf{X}_{p_0 N_1})$ (if

$p_0 N_1$ is not an integer, take the maximum of the integers less than it).

The second step, calculate the subsequent conditional probabilities $P_i = P(F_i | F_{i-1}), i = 2,3,\cdots m$ which require the samples belonging to F_{i-1}. The conditional probability density function can be expressed as follow:

$$q_i(\mathbf{x}|F_{i-1}) = \frac{q(\mathbf{x})}{P(F_{i-1})} \tag{8}$$

Then, the failure probability of F_i can be shown:

$$\hat{P}_i = \frac{1}{N_i}\sum_{k=1}^{N_i} I_{F_i}(\mathrm{g}(\mathbf{X}_{ik})) \tag{9}$$

where $\mathbf{X}_{ik}(i = 2,\cdots,m; k = 1,\cdots,N_i)$ are identically distributed samples, N_i is the number of sample in the number i simulation region, $I_{F_i}(\cdot)$ is the indictor function

The samples $\{\mathbf{X}_i\}$ demand condition on F_{i-1}, so the generation of the samples can apply the Markov Chain Monte Carlo (MCMC) algorithm which could obtain the samples from each conditional PDF $q_i(\mathbf{x}|F_{i-1})$. After generating the conditional samples $\mathbf{X}_\mathrm{i} = \{X_{i1}, X_{i2},\cdots, X_{iN_i}\}$, compute the system response function $\{\mathrm{g}(\mathbf{X}_i)\}$, and sort them in an increasing order $\mathrm{g}(\mathbf{X}_1) \le \mathrm{g}(\mathbf{X}_2) \le \cdots \le \mathrm{g}(\mathbf{X}_{N_i})$. We donate a fixed value p_0 to equal to P_i. Then value of b_i is equal to $g(\mathbf{X}_{p_0 N_i})$ (if $p_0 N_i$ is not an integer, take the maximum of the integers less than it). Repeat the above procedure until the responses value of the threshold $b_i = g(\mathbf{X}_{p_0 N_i})$ is no more than 0.

The third step, calculate the conditional probability of the last failure domain F_m. Let $b_m = 0$ and estimate the probability of conditional failure event F_m as

$$\hat{P}_m = \frac{1}{N_m}\sum_{k=1}^{N_m} I_{F_m}(\mathrm{g}(\mathbf{X}_{mk})) \tag{10}$$

Finally, combining equation(6),(9),(10), the target failure domain F can be expressed as

$$P_F = \prod_{i=1}^{m} P_i \tag{11}$$

2.3 MCMC in Subset Simulation

Markov Chain Monte Carlo is a useful method to generate random samples in an arbitrary PDF applying in the intermediate event in subset simulation. The process of MCMC states with a few arbitrary samples $\mathbf{X}_t$ in the region of F_i, then a Markov Chain can be produced by the transition rules which definite that the next state samples $\mathbf{X}_{t+1}$ is only relating to the current state samples $\mathbf{X}_t$. The previous samples $\mathbf{X}_t$ and the new generated samples $\mathbf{X}_{t+1}$ getting together became a sequence of random variables $\{\mathbf{X}_1, \mathbf{X}_2, \cdots, \mathbf{X}_n\}$ called the Markov Chain. Let the chain in a stationary distribution $P(x)$. For each $\mathbf{X}_{t+1}$, choice a proposal PDF $Q(x)$ and the transition

PDF is $Q(\mathbf{X}_t|\mathbf{X}_{t+1})$ which is time-independent, then the chain satisfy the balance condition as

$$P(\mathbf{X}_t)Q(\mathbf{X}_{t+1}|\mathbf{X}_t) = P(\mathbf{X}_{t+1})Q(\mathbf{X}_t|\mathbf{X}_{t+1}) \tag{12}$$

The most extensive method in MCMC is the Metropolis-Hastings algorithm which can impose minimal requirements on the desired distribution. Using two steps transfers $\mathbf{X}_t$ to $\mathbf{X}_{t+1}$ in Metropolis-Hastings algorithm.

(i) Generate a sample $\mathbf{X}'$ from $Q(\mathbf{X}') = Q(\mathbf{X}'|\mathbf{X}_t)$, where $\mathbf{X}_t$ is the previous sample. Whether the new born sample $\mathbf{X}'$ could be accept or not depends on the acceptance probability $A(\mathbf{X}'|\mathbf{X}_t)$:

$$A(\mathbf{X}'|\mathbf{X}_t) = \min\left\{1, \frac{P(\mathbf{X}')\mathrm{Q}(\mathbf{X}_t|\mathbf{X}')}{P(\mathbf{X}_t)\mathrm{Q}(\mathbf{X}'|\mathbf{X}_t)}\right\} \tag{13}$$

$A(\mathbf{X}'|\mathbf{X}_t)$ is like a ratio of important sampling weights and ensures that the samples will come from the true distribution $P(x)$ after numerous calculations.

(ii) Determine the candidate sample $\mathbf{X}'$ to accept or reject. Draw an random uniform random number u from [0,1] which is defined as an acceptance threshold. The acceptation rules shown as

$$\mathbf{X}_{t+1} = \begin{cases} \mathbf{X}', & u < A \\ \mathbf{X}_t, & u \geq A \end{cases} \tag{14}$$

In Subset Simulation, the new state $\mathbf{X}_{t+1}$ should be judged that whether it lies in the failure domain F_i or not. If $\mathbf{X}_{t+1}$ satisfied all the conditions, it can be received in the Markov chain. Repeat several times to get a chain $\{\mathbf{X}_1, \mathbf{X}_2, \cdots, \mathbf{X}_n\}$,which is used in the second procedure in Subset Simulation.

When the MCMC solves some issues in high-dimensional spaces, the acceptance probability $A(\mathbf{X}'|\mathbf{X}_t)$ decreases dramatically. Numerous samples are rejected in this situation which reduces the computational efficiency. Au modified the Metropolis-Hastings algorithm which used one-dimensional sample to replace the random vector. Therefore, the acceptance probability is a ratio of one-dimensional PDF which increases the chance to accept. And the transition of each one-dimensional sample is independent.

3 Construction of Seismic Global Limit State Functions of Structures Using Subset Simulation

3.1 Seismic global limit state functions of structures

In this paper, we consider two global limit state function of structures under seismic actions: global load-carrying capacity limit state, and global deformation capacity limit state.

For the global load-carrying capacity limit state of a structure, the maximum base shear is taken as the global load-carrying capacity of the structure, and the total horizontal earthquake is taken as the global seismic demand for the structure. Then, the GLSF of the global load-carrying capacity limit state can be established as

$$Z = g(V_{\mathrm{S}}, F_E) = V_S - F_{\mathrm{E}} = h(X) - F_{\mathrm{E}} \tag{15}$$

The maximal base shear of a structure can be obtained by pushover analysis. The base shear of a structure is generally influenced by limit loading ability of structural components, the correlation among components, material' s constitutive models, and so on. These influencing factors are assembled into a random vector X, which is expressed as

$$V_{\mathrm{S}} = h(\mathbf{X}) = \mathrm{h}(X_1, X_2, \cdots, X_n) \tag{16}$$

where are n random variables.
The maximal base shear is assumed to follow lognormal distribution:

$$F_{v_{\mathrm{s}}}(v) = \Phi\left(\frac{\ln v - \lambda_{v_{\mathrm{s}}}}{\zeta_{v_{\mathrm{s}}}}\right) \tag{17}$$

where the λ_{v_s} is the logarithmic mean value of V_{S} , $\zeta_{v_{\mathrm{s}}}$ is the logarithmic standard deviation.
The horizontal seismic action F_{E} can be determined according to Chinese seismic design code of buildings:

$$F_{\mathrm{E}} = \alpha GD = \frac{A_m}{g}\beta(T, \xi)GD \tag{18}$$

where G is the equivalent gravity loads of the structure, D is the random factor, α is seismic influence coefficient, A_m is the additional random factors, g is the gravity acceleration.

For the global deformation capacity limit state of a structure, the maximum story drift u_D of the structure under seismic actions is taken as the global deformation demand of the structure. Correspondingly, the story drift threshold of the structure u_C can be treated as the global deformation capacity. Then, the GLSF for global deformation capacity limit state can be established as

$$Z = g(\mathbf{X}) = u_C - u_S(\mathbf{X}) \tag{19}$$

3.2 Construction of GLSFs of structures using subset simulation

In structural reliability analysis, the structures are established using OpenSees software, in which the corresponding statistical parameters of the random variables are considered. Then, the two GLSFs of the RC frame structures are obtained through pushover analysis and capacity spectrum method, respectively. Then, the global reliability indices for the two system-level global limit states of RC structures under seismic actions are calculated by SS method. In this study, the process need to calculate through the interaction between MATLAB and OpenSees, which is illustrated in Figure 1.

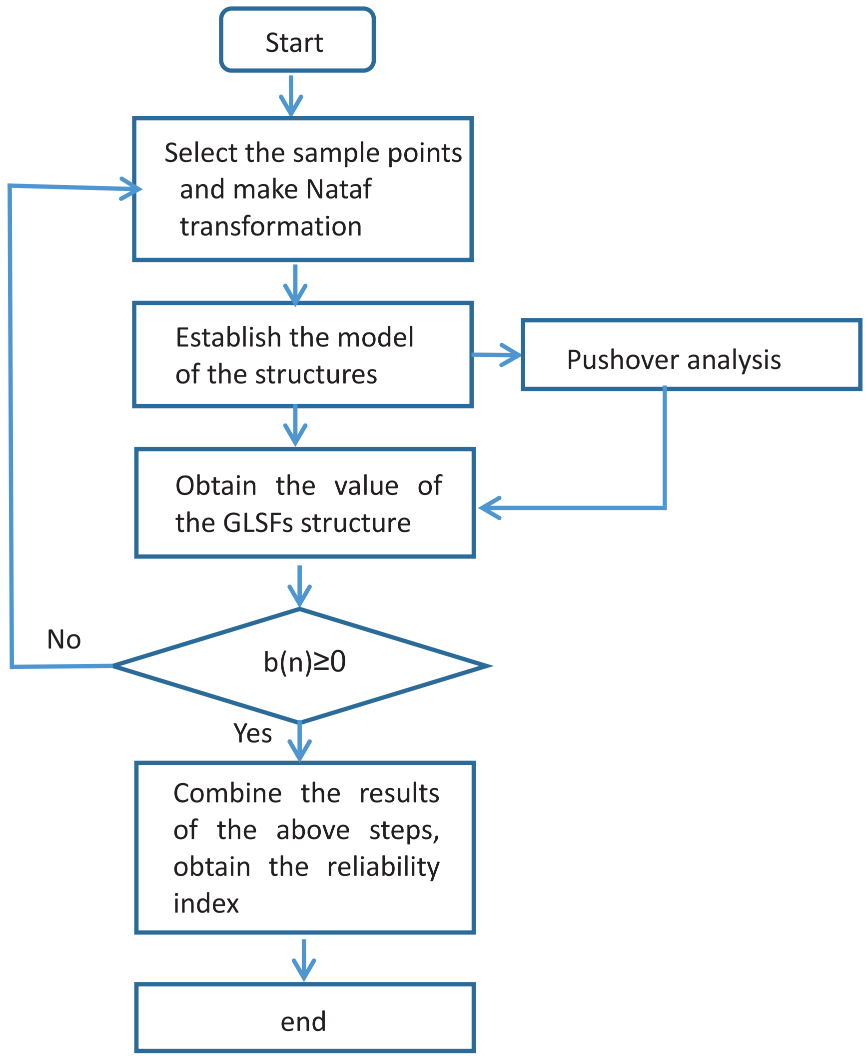


Figure1. The framework of the structural reliability calculation

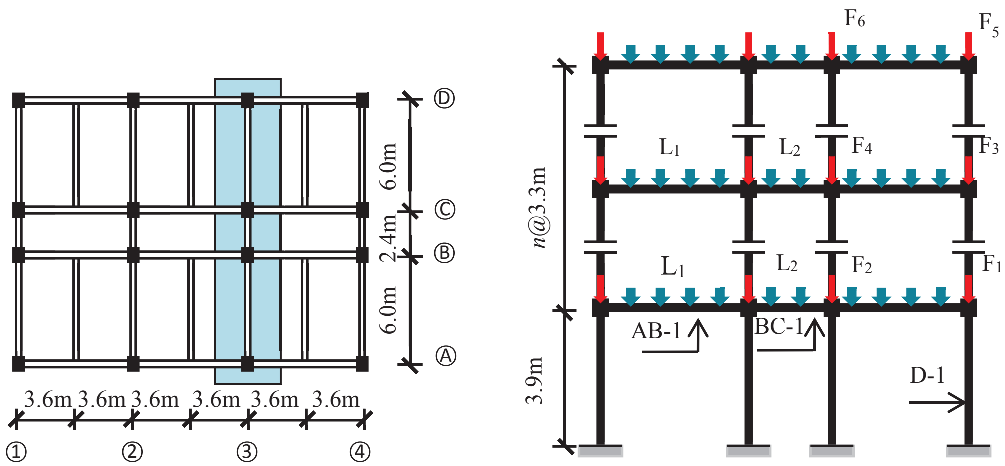
4 Case Study

4.1 Design and modeling of the RC frame structures

Model three reinforced concrete structures using OpenSees as the simulation platform with different storey but the same plan arrangement. These structures are three-storey, six-storey and nine-storey frame respectively. The design site classification of the structures are type Ⅱ and they are designed for seismic intensity of 8-degree earthquake zone. The strength grade of the main steel in beams and columns in the structures are HRB335. The strength of concrete used in the structures are C30 in three-storey frame and C35 in six-storey, nine-storey frame. The information about the load of the structures are shown in the table1. The plan and the elevation of the structures are illustrated in the Figure 2. These three structures are considered the structure uncertainty, material uncertainty and so on. The information of statistical parameters and distribution types of random variables can be shown in the table2.

Table1. Basic Load of RC Frame Structures[11]

Parameters	Value	Parameters	Value
Reference Wind Pressure	0.4kN/m^2	Standard Floor Live Load	2.0kN/m^2
Reference Snow Pressure	0.3kN/m^2	Standard Floor Dead Load	4.5kN/m^2
Thickness of Roof Slab without people	120mm	Roof Live Load	0.5kN/m^2



a) Plan view b) Elevation view

Figure2. Plan and elevation arrangements of RC frame structures

Table2. Basic Load of RC Frame Structures[11]

Source of Uncertainty	Random Variable	Mean Value	Coefficient of Variation	Relative Coefficient	Distribution Type
Concrete C30	$X_1(f_{c0,core})$	28.99 N/mm^2	0.20	0.3	Lognormal
	$X_2(f_{cu,core})$	17.91 N/mm^2			
	$X_3(\varepsilon_{c0,core})$	0.0023			
	$X_4(\varepsilon_{cu,core})$	0.0143			
	$X_5(f_{c0,cover})$	25.57N/mm^2	0.20	0.3	
	$X_6(\varepsilon_{cu,cover})$	0.0040			
Concrete C35	$X_1(f_{c0,core})$	32.57 N/mm^2	0.20	0.3	Lognormal
	$X_2(f_{cu,core})$	20.76 N/mm^2			
	$X_3(\varepsilon_{c0,core})$	0.0022			
	$X_4(\varepsilon_{cu,core})$	0.0124			
	$X_5(f_{c0,cover})$	0.0124 N/mm^2	0.20	0.3	
	$X_6(\varepsilon_{cu,cover})$	0.0040			
Steel HRB335 N/mm^2	$X_7(f_y)$	378	0.10	0.4	Lognormal
	$X_8(E_0)$	200000	0.05		

Dead Load (kN/m^3)	$X_9(\gamma)$	26.50	0.10	--	Normal
Live Load (kN/m)	$X_{10}(q)$	0.98	0.45	--	Gamma

4.2 Seismic reliability analysis for global load-carrying capacity limit state

Then, according to the design specification of Chinese architectural structure, the information about F_E can be shown in the table 3, considering the uncertainty of the seismic action F_E.

Table 3. Statistics of total horizontal seismic action

Structure	Mean Value (kN)	Coefficient of Variation	Distribution Type
F3	548.01	0.1~1.0	Extreme Value Type Ⅰ Distribution
F6	665.81		
F9	727.79		

Using subset simulation, the structure reliability based on global load-carrying capacity limit state can be obtained in the table 4. The results can be compared with the Monte Carlo Simulation (MCS). The results can be compared with the Monte Carlo Simulation (MCS).

Table 4. Global reliability indices of global load-carrying capacity

Reliability Index		Coefficient of Variation					
		0.0	0.2	0.4	0.6	0.8	1.0
F3	MCS	1.1957	0.5713	0.4084	0.3387	0.2903	0.2759
	SS	1.2243	0.5779	0.4125	0.3407	0.3186	0.2793
F6	MCS	0.5954	0.3638	0.2832	0.2492	0.2309	0.2198
	SS	0.5695	0.3580	0.2819	0.2508	0.2301	0.2240
F9	MCS	1.2308	0.5825	0.4155	0.3435	0.3005	0.2791
	SS	1.2081	0.5710	0.4193	0.3465	0.3121	0.2767

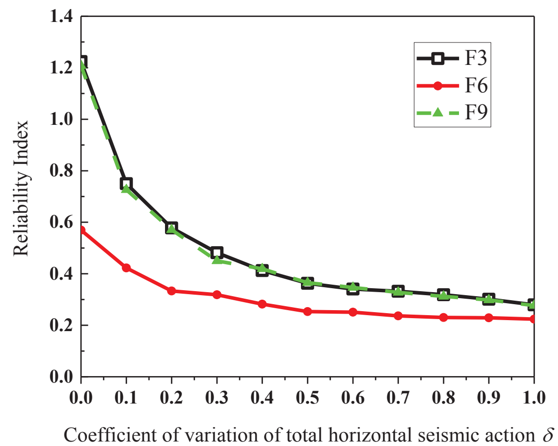


Figure3. Global reliability indices of global load-carrying capacity

The number of initial samples of subset simulation method is 1000, and the number in MCS is 10000. The results show that the reliability indices of the three structures are accurate based on global load-carrying capacity limit state calculated by the subset simulation method.

4.3 Seismic reliability analysis for global deformation capacity limit state

It is assumed that the threshold of the maximum story drift obeys the logarithmic normal distribution. The mean value of it is shown in table 5.

Table 5. The thresholds of the maximum structure story drift

Intact	Slight Damage	Medium Damage	Serious Damage	Collapse
1/550	1/250	1/120	1/60	1/50

The uncertainties of the seismic action need to be considered such as the damping ratio ζ , the maximum seismic effect coefficient $\alpha_{\max}$ and the design characteristic period of ground motion T_g . The mean values of these three random variables are 0.05, 0.90 0.35respectively. The coefficient of them are 0.3, 0.05, 0.1. ζ obeys lognormal distribution, $\alpha_{\max}$ and T_g obey normal distribution. Then use the capacity spectrum method to get the maximum story drift of the structures. The reliability indices can be calculated by SS. The results would be compared with the method of MCS in table 6.

Table 6. Global reliability indices of global deformation capacity

Reliability Index	structure		
	F3	F6	F9
MCS	1.8550	1.6276	1.7444
SS	1.8767	1.6540	1.7835

The SS method takes 1000 sample points for about 5.3 hours to get the results, and MSC spends around 23 hours on 10000 sample points. It is clear that SS method is much more efficient than the MCS method.

5 Conclusions

In this paper, SS method is used to calculate the global reliability indices of the structures based on global load-carrying capacity limit state and global deformation limit state. In this method, the number of the samples is reduced greatly because of the MCMC, which could improve the computation efficiency. The SS method costs only about 1/4 of the time to get the reliability index of the structure than the MCS. Moreover, after comparing the results with the MCS method, the accuracy of the subset simulation method is creditable.

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