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RELIABILITY-BASED DESIGN OPTIMIZATION CONSIDERING STRUCTURAL REDUNDANCY AGAINST FATIGUE-INDUCED FAILURE

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In structural design, reliability-based design optimization (RBDO) has been applied to obtain a set of design variables that maximize/minimize the given objective function and satisfy given constraints. In many previous studies, the objective function and constraints were formulated for component events, which represent the failure of structural members. However, a structure often has structural redundancy. For example, when a structure is subjected to cyclic loading, local fatigue-induced failures may initiate sequential failures, which may lead to structural collapse. Therefore, for the RBDO of such a structural system, it is essential to consider the risk of fatigue-induced sequential failure at a system level. To quantify the likelihood of fatigue-induced sequential failures and identify critical failure sequences, the Branch-and-Bound method employing system reliability Bounds (termed the B^3 method) was recently developed. This method identifies critical sequences of fatigue-induced failures in the decreasing order of their likelihood, and the method was successfully demonstrated through its application to several types of structural systems. This paper proposes a new RBDO approach that considers structural redundancy against fatigue-induced failure. To properly consider fatigue-induced sequential failure, the proposed approach employs the B^3 method and obtains system-level probabilities and sensitivities, which are required for the RBDO of structural systems. The proposed approach is tested and demonstrated by using a simple numerical example.

Keywords: Reliability-based design optimization, fatigue-induced failure, structural redundancy, system reliability.

1 Introduction

Various structural systems such as bridges, offshore platforms, and aircraft are often subjected to the risk of fatigue-induced failures caused by repeated loadings over their lifetime. Structural systems should be designed and maintained such that they have an adequate level of structural redundancy to prevent initial local fatigue-induced failures from causing exceedingly large damage (e.g., system collapse) that may result in catastrophic socio-economic losses. To quantify the likelihood of fatigue-induced sequential failures and identify critical failure sequences, the Branch-and-Bound method employing system reliability Bounds (termed the B^3 method) was recently developed. This method identifies critical sequences of fatigue-induced failures in the decreasing order of their likelihood, and the method was successfully demonstrated by using several numerical examples (Lee and Song 2011, 2012).

There have been studies on the optimized design of structures, and various reliability-based design optimization (RBDO) methods have been developed for this purpose. From the perspective of reliability analysis, these methods can be divided into two categories: component RBDO (CRBDO) and system RBDO (SRBDO). If a structure is designed to meet the failure probability requirements of a structural member, the employed optimization method is called CRBDO. On the other hand, if the design optimization is based on the system-level failure probability of a structure, the optimization method is called SRBDO. More details on these two approaches can be found in a previous paper (Nguyen et al. 2010).

However, design optimization for complex structural systems with structural redundancy is not an easy task. In this paper, a new SRBDO method that considers structural redundancy against fatigue-induced failure is proposed. To consider structural redundancy, the proposed method employs the B^3 method.

2 Proposed SRBDO Method

As a part of SRBDO, the B^3 method is introduced to the proposed method. In this section, the B^3 method is briefly explained; more details about this method can be found in the paper by Lee and Song (2011, 2012).

First, let us consider the following crack-growth model (Paris & Erdogan 1963):

$$\frac{da}{dN} = C\left(\Delta K\right)^m \tag{1}$$

where a denotes the crack length, N is the number of load cycles, C and m are the material parameters, and ΔK denotes the range of the stress intensity factor. Next, Newman's approximation (Newman and Raju 1981) is introduced and Eq. (1) is integrated from the initial condition to the current time point to obtain the relationship between the time duration T and the corresponding crack length a as follows:

$$\int_{a^0}^{a} \frac{1}{\left[Y(a)\sqrt{\pi a}\right]^m} da = C \cdot N \cdot S^m = C \cdot \nu_0 \cdot T \cdot S^m \tag{2}$$

where a^0 is the initial crack length, $Y(\cdot)$ is the geometry function, S is the stress range, and v_0 is the loading frequency. At the i-th structural member ("member" hereafter), a crack failure is assumed to occur when the crack length exceeds a critical length a_i^c. Then, the limit-state function for the member's failure within an inspection cycle $[0, T_s]$ is formulated as

$$g_i(\mathbf{X}) = T_i^0 - T_s = \frac{1}{C\nu_0\left(S_0\right)^m} \int_{a_i^0}^{a_i^c} \frac{1}{\left[Y(a)\sqrt{\pi a}\right]^m} da - T_s \tag{3}$$

where a_i^0 is the initial crack length at the i-th member, T_i^0 is the time required for the crack growth from a_i^0 to a_i^c, and **X** denotes the vector of r andom variables. To describe the failure sequences as *disjoint* events, the case in which the i-th member fails *first* (i.e., fails earlier than any other members) is formulated. The event probability is described as

$$P_i = P\left\{\left[\bigcap_{\forall l \neq i}\left(T_i^0 < T_l^0\right) \cap \left(T_i^0 < T_s\right)\right]\right\} \tag{4}$$

The event expressed in Eq. (4) is a parallel system event consisting of n component events. The probability can be computed by performing the component reliability analyses of the n events and a subsequent system reliability analysis. Similarly, the probability of a general failure sequence $\{1 \rightarrow 2 \rightarrow \ldots \rightarrow (i-1) \rightarrow i\}$ is described as

$$P_i^{1,\ldots,i-1} = P\left\{\left[\bigcap_{\forall j \neq 1}\left(T_1^0 < T_j^0\right)\right] \cap \left[\bigcap_{\forall k \neq 1,2}\left(T_2^1 < T_k^1\right)\right] \cap \cdots \cap \right. \\ \left. \left[\bigcap_{\forall l \neq 1,\ldots,i}\left(T_i^{1,\ldots,i-1} < T_l^{1,\ldots,i-1}\right)\right] \cap \left(T_1^0 + T_2^1 + \cdots + T_i^{1,\ldots,i-1} < T_s\right)\right\} \tag{5}$$

where $T_i^{1,\ldots,(i-1)}$ denotes the time required for the failure at the i-th member since the sequential failure $\{1 \rightarrow 2 \rightarrow \ldots \rightarrow (i-1)\}$. Unlike T_i^0 in Eq. (3), i.e., the time until the first failure for an undamaged structure, the time terms introduced for damaged structures should be computed while considering the effects of load redistributions. For this computation, Lee and Song (2012) derived a recursive formulation for a general failure sequence $\{1 \rightarrow 2 \rightarrow \ldots \rightarrow (i-1) \rightarrow i\}$ as follows:

$$T_i^{1,\ldots,i-1} = \frac{1}{C\nu_o (S_i^{1,\ldots,i-1})^m} \int_{a_i^0}^{a_i^c} \frac{da}{\left[Y\sqrt{\pi a}\right]^m} - \sum_{k=1}^{i-1} \left(\frac{S_i^{1,\ldots,k-1}}{S_i^{1,\ldots,k-1}}\right)^m T_k^{1,\ldots,k-1} \tag{6}$$

where $S_i^{1,\ldots,i-1}$ denotes the stress range at the i-th member after the occurrence of the failure sequence $\{1 \rightarrow 2 \rightarrow \ldots \rightarrow (i-1) \rightarrow i\}$.

It should be noted that the system failure sequences identified by using the event descriptions in Eqs. (4) and (5) are disjoint, i.e., mutually exclusive to each other. Therefore, one can simply add up or subtract the probabilities of the identified failure or non-failure sequences for obtaining the bounds imposed on the system failure probability without performing additional system reliability analysis to account for the statistical dependence between the identified sequences, i.e.,

$$\begin{aligned} P\left(E_{sys}\right)_{low} &= P\left(\bigcup_{i=1}^{N_C} C_i\right) = \sum_{i=1}^{N_C} P\left(C_i\right) \\ P\left(E_{sys}\right)_{upp} &= 1 - P\left(\bigcup_{j=1}^{N_L} L_i\right) = 1 - \sum_{j=1}^{N_L} P\left(L_i\right) \end{aligned} \tag{7}$$

where $P(E_{sys})_{low}$ and $P(E_{sys})_{upp}$ respectively denote the lower and upper bounds on the system failure probability, and C_i ($i = 1,\ldots,N_C$) and L_i ($j = 1,\ldots,N_L$) are the occurrences of the identified system failure sequences and non-failure sequences.

The B^3 method uses a systematic search scheme based on the disjoint event description of failure sequences. Fig. 1 illustrates the search process.

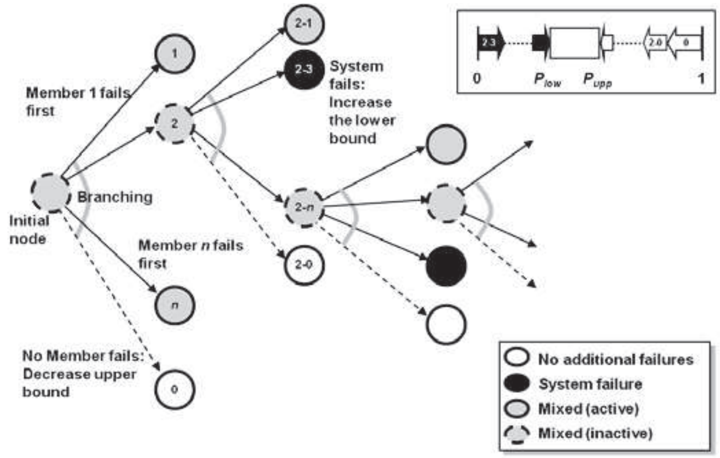

Figure 1. Search process by B^3 method (Lee and Song 2011, 2012)

This iterative procedure leads to a narrower gap between upper and lower bounds. It has been proposed that the procedure be terminated when the gap between the two bounds becomes smaller than 5% of the upper bound, and the application of this procedure to examples of an offshore platform (Lee and Song 2011) and an aircraft longeron (Lee and Song 2012) has shown that the upper bound at the termination point with a 5% gap is very close to the system failure risk obtained from Monte Carlo simulations.

In addition, one important advantage of the B^3 method is that it is very efficient, and hence, the system-level failure probability can be calculated using a relatively small number of structural analyses. This advantage makes it feasible to calculate the sensitivity of the objective function with respect to each design variable, which is required for a design optimization analysis, by simply using finite forward approximation.

The SRBDO problem in this research can be designed as follows:

$$\begin{aligned} &\min_{\mathbf{d}} f(\mathbf{d},\mathbf{X}) \\ \text{s.t. } &P\left[g_i(\mathbf{d},\mathbf{X}) \le 0\right] \le P_i^t,\ i=1,\ldots,n \\ &P_{sys} \le P_{sys}^t \end{aligned} \tag{8}$$

where $\mathbf{d}$ is the vector of deterministic design variables (such as the section area); $\mathbf{X}$ is the vector of random variables; $f(\cdot)$ is the objective function; $g_i(\cdot)$, $i = 1,\ldots,n$ is the i-th limit-state function representing the member failure; P_{sys} is the system failure probability; P_i^t is the target failure probability of structural members; and P_{sys}^t is the target failure probability of the structural system.

3 Numerical Example: Multi-layer Daniels System

3.1 *Problem description*

In this study, the risk of fatigue-induced sequential failures in a multi-layer Daniels system (Fig. 2) is investigated to demonstrate the proposed SRBDO method. In this example, system failure is defined as an event in which all the bars in one of the three stories fail.

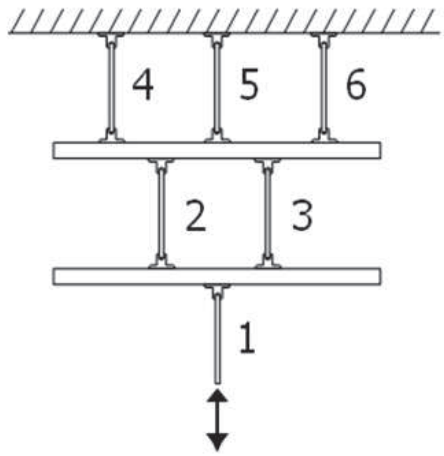

Figure 2. Multi-layer Daniels system (Lee and Song 2012)

As shown in Fig. 2, the multi-layer Daniels system consists of six bars that are assumed to be perfectly brittle and to have identical and deterministic elastic moduli. The cross-sectional areas of the bars are A_1 = 100 mm^2, $A_2 = A_3$ = 50 mm^2, and $A_4 = A_5 = A_6$ = 33.33 mm^2, and their widths are W_1 = 38.1 mm, $W_2 = W_3$ = 19.05 mm, and $W_4 = W_5 = W_6$ = 12.7 mm. In this example, the uncertainties of the initial crack length a_i^0, external load I, and material parameters C and m in the equation of Paris & Erdogan (1963) are considered as random variables with mean values of 0.11 mm, 17.2 kN, 1.36×10^{-13} mm/cycle/(MPa·mm)^m, and 3.0, respectively. It is assumed that the initial crack length follows an exponential distribution and that the other random variables follow a lognormal distribution. The coefficients of variation of a_i^0, I, C, and m are 1.0, 0.1, 0.533, and 0.02, respectively. For the sake of simplicity, all the random variables are assumed to be statistically independent of each other. In addition, the following deterministic parameters are used: loading frequency (ν_0) = 100 (cycles/h), inspection cycle (T_s) = 2,000 h, and critical crack lengths a_1^c = 30.48 mm, $a_2^c = a_3^c$ = 15.24 mm, and $a_4^c = a_5^c = a_6^c$ = 10.16 mm. Under these conditions, the B^3 method is used to calculate the system failure probability, which is found to be 9.767×10^{-3}. In addition, the most critical failure sequence and its probability are estimated to be "1" (i.e., member 1 fails first) and 4.871×10^{-3}, respectively.

However, if the area sum of the six members is to be minimized while keeping the system failure probability below a certain level, it is an SRBDO problem and the member areas are considered as design variables. In this paper, two SRBDO problems are introduced to test the proposed method. In Problem #1, the multi-layer Daniels system has to be designed such that the total area sum of the six members is minimized and the system failure probability is smaller than 5×10^{-3}. In Problem #2, in addition to the constraint for Problem #1, one more constraint is added: the probability of the most critical failure sequence should be smaller than 1×10^{-3}. For these two optimization problems, the abovementioned member areas (i.e., 100 mm^2 for member 1, 50 mm^2 for members 2 and 3, and 33 mm^2 for members 4, 5, and 6) are used as the initial values for the optimization, and the other conditions are assumed to be the same.

3.2 *Analysis results*

For Problems #1 and #2, the corresponding analysis results obtained using the proposed SRBDO method are listed in Table 1. First, the member areas obtained for Problem #1 increase compared to their original areas, and the total area also increases from 299.99 mm^2 to 313.5 mm^2. As a result, the system failure probability is estimated to be 4.996×10^{-3}, which satisfies the given constraint. The most critical failure sequence and its probability are found to be "1" and 1.969×10^{-3}, respectively.

However, for Problem #2, the probability of the most critical failure sequence should be less than 1×10^{-3}. As a result, the total area increases again (Table 1). In addition, the area of member 1 (i.e., A_1) increases significantly even though the areas of the other members slightly decrease. This is because the failure sequence "1" continues to be identified as the most critical failure sequence, and consequently, the probability is significantly reduced.

Table 1. Analysis results for Problems 1 and 2

Prob.	A_1 (mm²)	A_2 (mm²)	A_3 (mm²)	A_4 (mm²)	A_5 (mm²)	A_6 (mm²)	Total area (mm²)	System failure probability ($\times 10^{-3}$)	Most critical sequence probability ($\times 10^{-3}$)
#1	107.9	52.1	52.1	33.8	33.8	33.8	313.5	4.996	1.969
#2	113.8	51.8	51.8	32.8	32.8	32.8	316.1	4.989	0.966

4 Conclusion

This paper proposes a new approach for the RBDO of structural systems considering structural redundancy against fatigue-induced failure. To properly consider the fatigue-induced sequential failure at a system level, the proposed approach employs the B^3 method and calculates system-level probabilities and sensitivities that are required for the RBDO of structural systems. The proposed approach is tested through its application to a numerical example of a multi-layer Daniels system, and it has been successfully shown that the proposed approach can determine the system-level risk of fatigue-induced failure of a structure and perform SRBDO analysis.

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