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Prediction of extreme responses of Floating Production Systems using Kernel Density Maximum Entropy

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The dynamic analysis of a deepwater floating production systems has many complexities, which can be captured by fully coupled time domain analyses. However they require an enormous computational cost, especially for the evaluation of the extreme values, which are of great interest for practical reliability design purposes. In this paper the non-Gaussian probability density functions of the responses are evaluated through a novel moment-based approach, based on the Maximum Entropy principle, called Kernel Density Maximum Entropy Method (KDMEM). The proposed method has several attractive features: (*i*) it gives a good approximation of the target distribution, including its tails, from samples of small size, (*ii*) when the number of samples increase, it is capable to converge asymptotically to the target distribution, (*iii*) it implements the principle of Maximum Entropy, so that it provides the least biased distribution given the available information, (*iv*) it does not require any coupling with the marine software, so that any commercial tool can be used, (*v*) it provides credible bounds of the uncertain performances, which is beneficial for risk-informed decisions. The accuracy and efficiency of KDMEM is shown through the application to a marine riser.

Keywords: Marine Riser, Kernel Density Maximum Entropy, Information Theory, Stochastic Dynamic Analysis, Machine Learning

1 Introduction

Floating production systems (FPS) have become an integral part of deepwater development in oil and gas exploration and production. Marine risers, mooring system and floater represent an integrated dynamic system responding to environmental loading due to waves, current and wind in a complex way. Recent research efforts have shown that the mooring lines and risers can have a significant dynamic influence on the platform. Therefore, in full rigor, a "coupled analysis" of the vessel and of all the collected lines in the time domain should be adopted to take into account all the dynamic interactions within the system. Moreover, the environmental loads are random, hence the need of stochastic dynamic analysis. The aim is the evaluation of the response statistics of dynamic systems subjected to stochastic excitations. Most existing approaches adopt the Monte Carlo Simulation (MCS), which is a robust approach, but it is too demanding for practical engineering purposes.

In this paper the stochastic dynamic analysis of the FPS is developed through the Kernel Density Maximum Entropy Method (KDMEM) recently proposed by the authors for a general dynamic system subjected to stochastic excitation (Alibrandi & Mosalam 2017). This is a statistical method providing a good reconstruction of the target Probability Density Function

(PDF), including its tails, from samples of small size. KDMEM is a data-driven approach, thus when the number of samples increases, it converges asymptotically to the target distribution. Moreover, it implements the principle of Maximum Entropy (Jaynes 1957) which provides the least biased distribution given the available information. KDMEM presents some attractive features with respect to many existing methods of structural reliability and stochastic dynamic analysis, since its performances are not affected by the number of random variables or degree of nonlinearity of the dynamic system. Moreover, it does not require any coupling with the structural analysis software. Thus, the results of any existing commercial software can be adopted. Also, when a reduced number of dynamic computations are considered, the joint adoption of KDMEM with the bootstrap technique, yields credible bounds of the uncertain parameters.

The method has been already successfully used for the determination of seismic fragility curves of a Reinforced Concrete building subjected to earthquake ground motion, modelled as a non-stationary Gaussian stochastic process (Alibrandi and Mosalam 2018). In this paper KDMEM is applied to a simplified model of riser proposed by Low and Langley (2008). This includes the stochastic modelling of loads, represented by the first- and second-order wave forces on the vessel, drag forces and inertia forces on the lines. The example shows the good performances of KDMEM as a practical design tool also for the stochastic dynamic analysis of FPS.

2 Kernel Density Maximum Entropy (KDMEM)

Let us consider a random variable X, whose PDF is $f_X(x)$ with support Ω. The target PDF, is expressed as a linear superposition of Kernel Density Functions (KDFs) as follows:

$$f_X(x) \cong f_{KD}(x;\boldsymbol{p}) = \sum_{i=1}^{N} p_i f_i^K(x;x_i,h) \tag{1}$$

where the coefficients p_i satisfy the constraints $0 \le p_i \le 1$, $\sum_i p_i = 1$, while $f_i^K(x;x_i,h)$ is the i −th basis KDF, centered in x_i with bandwith h. If the Gaussian distribution is chosen as KDF, then

$$f_i^K(x;x_i,\sigma) = \frac{1}{h\sqrt{2\pi}}\, exp\left\{-\frac{1}{2}\left(\frac{x-x_i}{h}\right)^2\right\} \tag{2}$$

where x_i and h are the mean value and the standard deviation σ of the Gaussian KDF, respectively. The centers x_i, $i = 1,2,\dots,N$, are uniformly spaced with a constant step $\Delta x = x_{i+1} - x_i$ in the range $[x_{min}, x_{max}]$. The bandwidth is $h = (2/3)\Delta x$, which is shown to be a suitable value under uniform spacing of the centers (Alibrandi and Ricciardi 2008, Alibrandi and Mosalam 2017). It is noted that when $N \to \infty$, then $h \to 0$, and Eq.(1) gives

$$f_{KD}(x;\boldsymbol{p}) = \sum_{i=1}^{N} p_i \delta(x - x_i) \tag{3}$$

where δ is the Dirac delta function. Therefore, the representation (1)-(3) can reconstruct any kind of distribution. Let us a consider now a set of M independent functions $g_k(x;\boldsymbol{\alpha})$ of parameters $\boldsymbol{\alpha}$, representing the available information. Multiplying both sides of Eq.(3) by $g_k(x;\boldsymbol{\alpha})$, $k = 1,2,\dots,M$, and integrating over the domain, the following relationship holds:

$$\begin{cases} \boldsymbol{1}^T\boldsymbol{p} = 1 \\ \boldsymbol{M}(\boldsymbol{\alpha})\boldsymbol{p} = \boldsymbol{\mu}(\boldsymbol{\alpha}) \end{cases} \tag{4}$$

where $\mathbf{1}$ is a vector of N unit entries, and $\boldsymbol{p}$ collects the N parameters $p_1, p_2, \dots, p_N$, while $\boldsymbol{M}(\boldsymbol{\alpha})$ and $\boldsymbol{\mu}(\boldsymbol{\alpha})$ are defined as

$$\begin{cases} M_{ki}(\boldsymbol{\alpha}) = \int_0^\infty g_k(x;\boldsymbol{\alpha})\delta(x - x_i)dx = g_k(x_i;\boldsymbol{\alpha}) \\ \mu_k(\boldsymbol{\alpha}) = \int_{\Omega_Z} g_k(x;\boldsymbol{\alpha})f_X(x)dx = E[g_k(x;\boldsymbol{\alpha})] \end{cases} \tag{5}$$

It is noted from Eq.(4) that $\boldsymbol{p}$ can be considered as the Probability Mass Function (PMF) of a discrete valued random variable X_δ, defined in the centers $x_1, x_2, \dots, x_N$ and corresponding probabilities $p_1, p_2, \dots, p_N$. The generalized moments of X_δ are $E[g_k(x_\delta;\boldsymbol{\alpha})] = \sum_{i=1}^N g_k(x_i;\boldsymbol{\alpha})p_i$ and are assumed to coincide with the corresponding moments of $f_X(x)$, i.e. $E[g_k(x_\delta;\boldsymbol{\alpha})] \equiv E[g_k(x;\boldsymbol{\alpha})] = \mu_k(\boldsymbol{\alpha})$, see Eq.(5). According to Jaynes (1957) the Maximum Entropy (ME) probability distribution, $\boldsymbol{p}_{ME}$, is the least biased distribution, given the satisfaction of the available information. It is obtained through the maximization of the Shannon's entropy $H(\boldsymbol{p}) = -\sum_{i=1}^N p_i ln(p_i)$ as follows:

$$\begin{cases} \max_p H(\boldsymbol{p}) \\ \mathbf{1}^T\boldsymbol{p} = 1 \\ \boldsymbol{M}(\boldsymbol{\alpha})\boldsymbol{p} = \boldsymbol{\mu}(\boldsymbol{\alpha}) \end{cases} \tag{6}$$

The optimization problem (6) is convex, which implies the uniqueness of the ME distribution of X_δ, expressed as follows:

$$p_i^{ME}(\boldsymbol{\lambda};\boldsymbol{\alpha}) = exp\left\{-\lambda_0(\boldsymbol{\lambda}) - \sum_{k=1}^M \lambda_k g_k(x_i;\boldsymbol{\alpha})\right\} \; i = 1, \dots, N \tag{7}$$

where $\boldsymbol{\lambda}$ collects the M Lagrange multipliers $\lambda_1, \lambda_2, \dots, \lambda_M$ of the dual optimization problem, while

$$\lambda_0(\boldsymbol{\lambda};\boldsymbol{\alpha}) = log\left\{\sum_{i=1}^N exp\left(-\sum_{k=1}^M \lambda_k \, g_k(x_i;\boldsymbol{\alpha})\right)\right\} \tag{8}$$

The Lagrange parameters $\boldsymbol{\lambda}$ can be determined as a solution of a linear system of M equations (Alibrandi and Mosalam 2017)

$$\boldsymbol{\Theta}(\boldsymbol{\alpha})\boldsymbol{\lambda} = \boldsymbol{\rho}(\boldsymbol{\alpha}) \tag{9}$$

From Eqs.(7)-(9) the parameters $\boldsymbol{p}_{ME}$ are determined, and substituted into Eq.(1), the Kernel Density Maximum Entropy (KDME) distribution is obtained, i.e. $f_{KDME}(x) = f_{KD}(x;\boldsymbol{p}_{ME})$.

2.1 *KDME distribution for given sample of data*

It is assumed that a set of n_s data $x^{(1)}, x^{(2)}, \dots, x^{(n_s)}$ drawn independently from the true but unknown target PDF $f_X(x)$ is available. The KDME distribution can be interpreted as a probabilistic model of parameters $\boldsymbol{\alpha}$ as follows:

$$f_{KDME}(x;\boldsymbol{\alpha}) = \sum_{i=1}^N p_{i,ME}(\boldsymbol{\alpha})f_i^K(x) \tag{10}$$

The model selection is pursued through the Maximum Likelihood Estimation (MLE) of $\tilde{f}_{KDME}$

$$\boldsymbol{\alpha}^* = \arg\max_{\boldsymbol{\alpha}} \frac{1}{n_s} \sum_{j=1}^{n_s} log\left[\tilde{f}_{KDME}\left(x^{(j)}; \boldsymbol{\alpha}\right)\right] \tag{11}$$

where $x^{(j)}$ is the j −th sample of X. The substitution of $\boldsymbol{\alpha}^*$ in Eq.(10) gives the required KDME PDF, i.e. $f_{KDME}(x) = \tilde{f}_{KDME}(x; \boldsymbol{\alpha}^*)$. When the number of data is small enough to yield overfitting, a strategy of regularization may be used.

In the model selection, an important issue is represented by the choice of the generalized functions $g_k(x; \boldsymbol{\alpha})$ yielding the generalized moments $\mu_k(\boldsymbol{\alpha})$. In Alibrandi and Ricciardi (2008) it is chosen $g_k(x) = x^k$, which provides the classical power moments $\mu_k = E[X^k]$. This choice is reasonable when the central part of the distribution is required. If conversely there is interest in the prediction of the extreme responses a good choice is $g_k(x) = x^{\alpha_k}$, providing the fractional moments $\mu_k = E[X^{\alpha_k}]$. Indeed, recent research has shown that a reduced number of fractional moments ($M = 2 - 6$) may provide a good description of the tails (Zhang and Pandey 2013, Xu 2016, Xu and Zhang 2016, Alibrandi and Mosalam 2017, 2018)

2.2 *Credible bounds of the KDME*

It is of interest to investigate the incurred error when the number n_s of analysis is reduced. To this aim, credible bounds of the KDME distribution are determined through the bootstrap resampling. Assume that a sample of size $n_s = 5$ of a chosen quantity has been determined such that $X \equiv \{x^{(1)}, x^{(2)}, x^{(3)}, x^{(4)}, x^{(5)}\}$, which are drawn from the unknown distribution $F_X(x)$. It is assumed that the sample represents the bootstrap population, whose distribution is modeled through a uniform discrete-valued distribution. Thus, each value of the sample has a probability of occurrence $\hat{p}_X^{(B)}(X = x^{(i)}) = 1/n_s$, $i = 1,2, \dots, n_s$, i.e. $\hat{p}_X^{(B)}(X = x^{(1)}) = \hat{p}_X^{(B)}(X = x^{(2)}) = \cdots = 1/5$. From the bootstrap discrete distribution $\hat{p}_{EDP}^{(B)}$, the bootstrap CDF $\hat{F}_{EDP}^{(B)}(edp)$ is determined. Bootstrap samples $EDP_{(B)}^{(s)}$ of size n_s can be so drawn from $\hat{F}_{EDP}^{(B)}(edp)$. The elements of $EDP_{(B)}$ are the same as those of the original data set, but repetitions may occur, i.e. some elements may appear only once, some may appear two or more times, and others may not appear. For illustration, two possible bootstrap samples are $X_{(B)}^{(1)} \equiv \{x^{(3)}, x^{(1)}, x^{(1)}, x^{(4)}, x^{(2)}\}$ and $X_{(B)}^{(2)} \equiv \{x^{(2)}, x^{(3)}, x^{(2)}, x^{(1)}, x^{(3)}\}$. For the s −th bootstrap sample, $X_{(B)}^{(s)}$, $s = 1,2, \dots, S$, the KDME CDF is evaluated as $F_{KDME}^{(s)}(x) = \sum_{i=1}^{N} p_i^{(s)} F_X^G(x)$. It is noted that $P_{KDME}^{(s)}(x) = 1 - F_{KDME}^{(s)}(x) = Prob[X^{(s)} \geq x]$. Therefore, S bootstrap samples provide S different values of $P_{KDME}(x)$. Therefore, the corresponding bootstrap distribution $P_{KDME}^{(B)}(x)$ is determined. If the mean value of $P_{KDME}^{(B)}(x)$ is considered, then the KDME solution of Eq.(10) is obtained. Moreover, from $P_{KDME}^{(B)}(x)$, the bootstrap confidence intervals can be determined by choosing two percentiles q_{LB} and q_{UB} of $P_{KDME}^{(B)}(x)$.

3 Numerical application

The KDMEM has been applied to a simplified FPS, presented in (Low and Langley 2008, Alibrandi and Koh 2017). It is composed of two lumped masses, of the vessel m_V and of the lines m_L, with generalized diplacements $x_V(t)$ and $x_L(t)$. The effective masses include added masses contributions. Without loss of generality, $x_V(t)$ and $x_L(t)$ can be seen as the surge motion of the vessel, and the first mode of vibration of the lines, respectively. The restoring force given by the lines is in general nonlinear due to the geometrical changes of the lines, and it is well approximated through a geometric nonlinearity $F_C(x) = k_1 x + k_3 x^3$. The lines are connected to a fixed boundary with an identical spring. The equations of the motion of the system are

$$\begin{cases} m_V \ddot{x}_V(t) + c_V \dot{x}_V(t) + F_C(x_{VL}) = F_V(t) \\ m_V \ddot{x}_L(t) + c_L \dot{x}_L(t) - F_C(x_{VL}) + F_C(x_L) = F_L(t) \end{cases} \tag{12}$$

where $x_{VL}(t) = x_V(t) - x_L(t)$, c_V and c_L are the damping of the vessel and lines respectively, k_1 and k_3 are the linear and non-linear contribution of the stiffnesses, $F_V(t)$ and $F_L(t)$ are the loads acting on the vessel and the lines, respectively. The structural damping of the lines is assumed equal to zero, $c_L = 0\ N\ sec/m$, while the viscous damping is given by the drag term in the wave force. The damping of the vessel c_V comes from several sources, such as viscous drag, radiation, aerodynamic, and so on. The random sea state is modelled through a discrete Fourier series

$$\eta(t, \boldsymbol{u}) = \sum_{i=1}^{n} \sqrt{G_{\eta\eta}(\omega_i)\Delta\omega}\,[cos(\omega_i t) u_i^c + sin(\omega_i t) u_i^s] = \boldsymbol{s}_\eta(t) \cdot \boldsymbol{u} \tag{13}$$

where n is the number of harmonic components, $u_1^c, u_2^c, \dots, u_n^c$ and $u_1^s, u_2^s, \dots, u_n^s$ are normal standard random variables. The correlation structure of $\eta(t, \boldsymbol{u})$ is given in terms of the underlying one-sided JONSWAP spectrum $G_{\eta\eta}(\omega)$, through the deterministic shape functions $s_{\eta,i}^c(t)$ and $s_{\eta,i}^s(t)$, collected in the $2n$ −vector $\boldsymbol{s}_\eta(t)$. The time dependent force on the vessel $F_V(t)$ is the sum of the following components:

$$F_V(t, \boldsymbol{u}) = F_w^{(1)}(t, \boldsymbol{u}) + F_w^{(2)}(t, \boldsymbol{u}) + F_{wind} + F_{curr} \tag{14}$$

where $F_w^{(1)}(t, \boldsymbol{u})$ and $F_w^{(2)}(t, \boldsymbol{u})$ represent the time-varying first- and second-order wave force, respectively, F_{wind} is the wind force, and F_{curr} is the force of the currents. Wind and current are assumed constant and collinear. The time dependent force on the lines $F_L(t)$ is the sum of the following components:

$$F_L(t, \boldsymbol{u}) = F_D(t, \boldsymbol{u}) + F_M(t, \boldsymbol{u}) \tag{15}$$

where $F_D(t, \boldsymbol{u})$ and $F_M(t, \boldsymbol{u})$ are the drag and inertia forces. All the data of this numerical application are reported in (Alibrandi & Koh 2017).

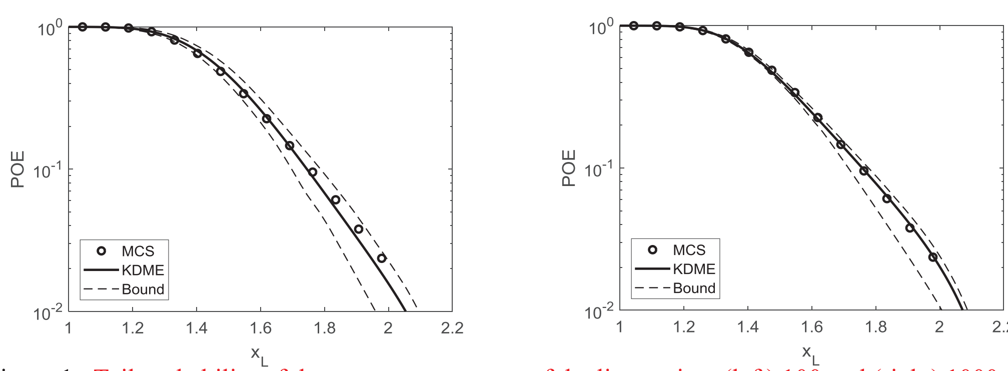

Figure 1. Tail probability of the extreme responses of the lines using: (left) 100 and (right) 1000 samples

Figure 1 shows the tail probability of the extreme response of the lines in semilogarithmic scale. Here we compare MCS with 50,000 samples (circle markers) and KDMEM (thick continuous line) together with its credible bounds (dashed lines), trained using respectively 100 and 1,000 samples. For sake of comparison it is noted that crude MCS would require approximately 1,000,000 analyses for estimating a tail probability of $P_f = 10^{-4}$. Conversely, KDMEM provides an excellent approximation with only 100 dynamic analyses, and it can therefore be adopted for practical design engineering. It is also noted that when the number of samples increase, the credible bounds become narrower, as expected.

3 Concluding Remarks

The dynamic analysis of deepwater floating production systems is a very complicated task to be accomplished, because of uncertainties in environmental loads, several sources of coupling between the vessel and the lines, and of nonlinearities. All these nonlinearities and couplings can be captured only through fully coupled time domain analyses. However they require a tremendous computational cost, especially for reliability design purposes. To reduce the computations, we applied to a simplified model of a floating production system a novel moment-based approach for the evaluation of the probability density function of the responses, called Kernel Density Maximum Entropy Method. It gives very good approximations of the PDFs of the quantities of interest, including the tails, from a very reduced number of samples. Thus, it can be used for practical design purposes.

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