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GAUSSIAN PROCESS-BASED BAYESIAN APPROACH TO CHARACTERIZATION OF BRIDGE STRAIN RESPONSES USING STRUCTURAL HEALTH MONITORING DATA

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In this paper, a Bayesian modeling approach with Gaussian processes (GPs) is adopted to characterize non-linear process model underlying the bridge strain response data. The present GP-based approach maintains the data-driven property which guarantees its high flexibility in modeling the complexity of a physical system. Considering that covariance function significantly affects the prediction performance of GP model (GPM), two commonly used categories of covariance functions, namely, squared exponential (SE) and Matern (MA) covariance functions, are extensively investigated. Their modeling performances are assessed through the real-time monitoring strain data of the long-span Ting Kau Bridge. It is found that the GPM with the MA covariance function outperforms that with the SE covariance function, which can be explained by their algebraic structures. This work presents an effective GP-based Bayesian approach to model the non-linear pattern of bridge strain responses, and also sheds some light on the selection of an appropriate covariance function for the specific modeling problems.

Keywords: Structural health monitoring, Gaussian process model, Covariance function, Structural Strain, Ting Kau Bridge.

1 Introduction

There are two main measurands from the instrumentation systems, namely, the fast-varying acceleration data and the slow-varying strain data. The former has obtained wide applications in structural health assessment over the past decades since it can be utilized to reliably extract the modal properties which the vibration-based methods rely on. As opposed to the acceleration data, the strain response, as a localized structural response, has gained growing interest for structural condition diagnosis and prognosis. In particular, online monitoring of strain can offer a way to derive the stresses experienced by the monitored structure during its service. Stress data can be directly used to indicate the safety reserve of a structural component or provide information about the load capacity of the whole structure; it would be better suited to characterize local damage of a structure than the vibration (acceleration) data. A lot of research activities have been concentrating on structural health assessment using successively accumulated strain data (Ni et al. 2012; Catbas et al. 2008; Xia et al. 2011; Zhu and Frangopol 2013).

Consequently, accurate characterization of the structural stress/strain responses is of great importance. For an in-service engineering structure experiencing various types of load effects such as live loads and environmental loads, the evolution of the structural strain responses is a typically non-linear dynamic process. In the literature, a variety of modeling tools are investigated by the SHM community, including but not limited to artificial neural networks (ANN) (Ni et al 2005; Hua et al 2007), and support vector machines (SVM) (Ni et al 2009; Zhou et al 2010), and Gaussian process model (GPM) (Worden and Cross 2018, Wan and Ni 2018). Among them, GPM has received substantial attention as a powerful Bayesian statistical technique for data-driven modeling (Rasmussen and Williams 2006). The predication capability of GPM is largely dependent on its covariance function which encodes our prior distribution over the underlying function which we wish to learn. This study proposes the use of the GP-based Bayesian approach for characterization of bridge strain responses, and in addition provides helpful guidance on choosing an appropriate covariance function for the construction of GPM.

2 Gaussian Process-based Bayesian Approach

A GPM is fully characterized by its mean function and covariance function. The choice of zero mean function is mainly because of the absence of the prior knowledge of the target function's overall trend (Neal 1999). Consider a n-pair observed data set $\mathcal{D} = \{(\mathbf{x}_i, y_i)|_{i=1}^{n}\}$, where $\mathbf{x}$ is the input vector input and y is the noisy scalar output. With the assumption of Gaussian noise, the observation model can be written as

$$y = f(\mathbf{x}) + \varepsilon, \quad \varepsilon \sim \mathcal{N}\left(0, \sigma_{\mathrm{n}}^2\right) \tag{1}$$

where $f(\mathbf{x})$ represents the latent function, and σ_{n}^2 denotes the variance of noise.

For notational convenience, we introduce the following notations: $\mathbf{X} = \{\mathbf{x}_i\}|_{i=1}^{n}$, $\mathbf{Y} = \{y_i\}|_{i=1}^{n}$, $\mathbf{f} = f(\mathbf{X})$, and $f_* = f(\mathbf{x}_*)$. Among them, f_* denotes the latent function realization to be predicted at an unobserved point $\mathbf{x}_*$. Under the assumption of Gaussian prior, one has the following joint Gaussian distribution

$$f(\mathbf{x}) \sim \mathcal{N}\left(0, C(\mathbf{x}, \mathbf{x}')\right) \tag{2}$$

$$p(\mathbf{f}, f_*) = \mathcal{N}\left(\begin{bmatrix} \mathbf{0} \\ 0 \end{bmatrix}, \begin{bmatrix} \mathbf{C} & \mathbf{C}_* \\ \mathbf{C}_*^{\top} & \tilde{C} \end{bmatrix}\right) \tag{3}$$

where $\mathbf{C} = C(\mathbf{X}, \mathbf{X})$, $\mathbf{C}_* = C(\mathbf{x}_*, \mathbf{X})$, and $\tilde{C} = C(\mathbf{x}_*, \mathbf{x}_*)$.

According to the Bayes' theorem, the joint posterior distribution of $\mathbf{f}$ and f_* conditioned on observations is equal to

$$p(\mathbf{f}, f_* \mid \mathbf{Y}) = \frac{p(\mathbf{f}, f_*) p(\mathbf{Y} \mid \mathbf{f})}{p(\mathbf{Y})}. \tag{4}$$

Since all terms in the above equation have the Gaussian form, the posterior distribution over predicted output after integration is also Gaussian and can be express as

$$p(f_* \mid \mathbf{Y}) = \mathcal{N}\left(\mathbf{C}_* \mathbf{K}^{-1} \mathbf{Y}, \tilde{C} - \mathbf{C}_*^{\top} \mathbf{K}^{-1} \mathbf{C}_*\right) \tag{5}$$

where $\mathbf{K} = \mathbf{C} + \sigma_{\mathrm{n}}^2 \mathbf{I}_{\mathrm{n}}$.

Among a wide variety of covariance functions, this paper considers two popular families of covariance functions, namely, SE and MA covariance functions, defined as

$$C_{\mathrm{SE}}\left(\mathbf{x},\mathbf{x}'\right)=\eta^2\exp\left[\frac{1}{2}r^2\left(\mathbf{x},\mathbf{x}'\right)\right], \qquad r^2(\mathbf{x},\mathbf{x}')=\sum_{k=1}^{d}\left(x_k-x_k'\right)/\ell_k^2 \tag{6}$$

$$C_{\mathrm{M}}\left(\mathbf{x},\mathbf{x}'\right)=\eta^2\left[1+\sqrt{5r^2\left(\mathbf{x},\mathbf{x}'\right)}+\frac{5}{3}r^2\left(\mathbf{x},\mathbf{x}'\right)\right]\exp\left[-\sqrt{5r^2\left(\mathbf{x},\mathbf{x}'\right)}\right] \tag{7}$$

where η^2 is the signal variance, and ℓ is the characteristic length scale. Apparently, the SE covariance function is infinitely differentiable, whereas the MA one is twice differentiable. This means that SE covariance function is powerful for modeling the very smooth processes, but in the meanwhile, it may be too restrictive for certain practical problems.

The hyperparameters denoted as $\boldsymbol{\Theta}=\{\ell_1,\ldots,\ell_d,\eta^2,\sigma_{\mathrm{n}}^2\}$ are commonly estimated by maximizing the marginal likelihood of training data in a Bayesian context. For more details, interested readers are referred to Wan and Ni (2018).

3 Illustrative Application: Ting Kau Bridge

3.1 *Bridge Description and Structural Health Monitoring System*

The Ting Kau Bridge (TKB), as shown in Figure 1, serves a key part of important transport network linking the Hong Kong International Airport on Lantau Island to the rest of Hong Kong. The TKB is a three-tower cable-stayed bridge with a total length of 1177 m. More specifically, the two main bridge spans have length of 448 m and 475 m respectively, while two side spans are both 127 m. The three towers are 170 m, 194 m and 158 m respectively and composed of concrete structure with steel boxes attached to the top section. Since 1999, a long-term SHM system has been devised, installed and operated by the Highways Department of the Government of Hong Kong SAR. The SHM system deployed on the TKB is to monitor the structural health and performance of the bridge under in-service conditions (Ko and Ni 2005). The SHM system is an integration of six systems, namely, sensory system, data acquisition and transmission system, data processing and control system, structural health evaluation system, structural health data management system, and inspection and maintenance system. A total of 238 sensors are permanently deployed on the TKB, consisting of 45 accelerometers (including uniaxial, biaxial, and triaxial), 7 anemometers (including ultrasonic-type and propeller-type), 2 displacement transducers, 83 temperature sensors, 88 strain gauges (including 66 linear and 22 rosette), 7 global positioning system (GPS) receivers, and a weigh-in-motion (WIM) sensing system with 6 sensors (Ni et al 2011).

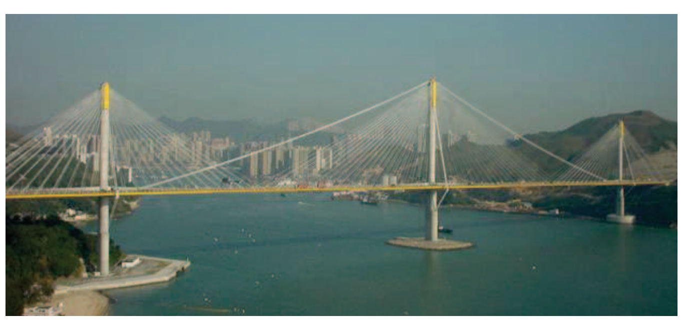


Figure 1. Ting Kau Bridge.

3.2 Characterization of Structural Strain Responses

Without loss of generality, two strain gages are chosen to illustrate the present GPM-based approach. The method discussed in this work can be readily employed to model the strains of any other measured points. The sampling rate of the strain gages is set as 25.6 Hz. For the sake of demonstration convenience, the first selected strain gage is denoted “SG1” and the second one is denoted “SG2”. One-month real-time monitoring strain data is used for demonstration. A total of 590 sets of strain data are collected, as shown in Figure 2. The whole data is randomly divided into two groups with identical number: 50% training data and 50% test data. The modeling results of GPMs with SE and MA covariance functions are given in Figure 3, from which one can see that the MA covariance function-based GPM has a better prediction performance than SE covariance function-based GPM. One can also find that the differences between the predicted and the measured strains at untried sites are greater than those at tried sites. This phenomenon can be explained by the fact that for the untried sites, we are more uncertain about the strain behavior of this large-scale, complex bridge, which results in higher prediction deviation and uncertainty (variance).

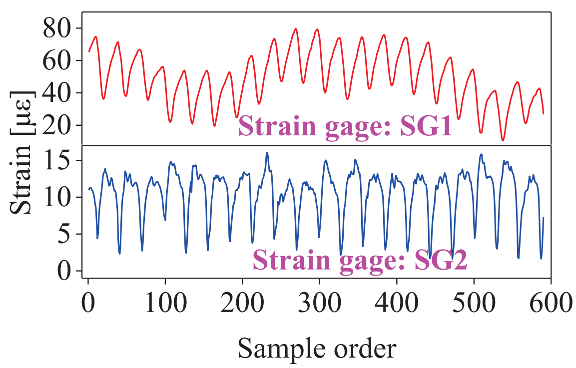
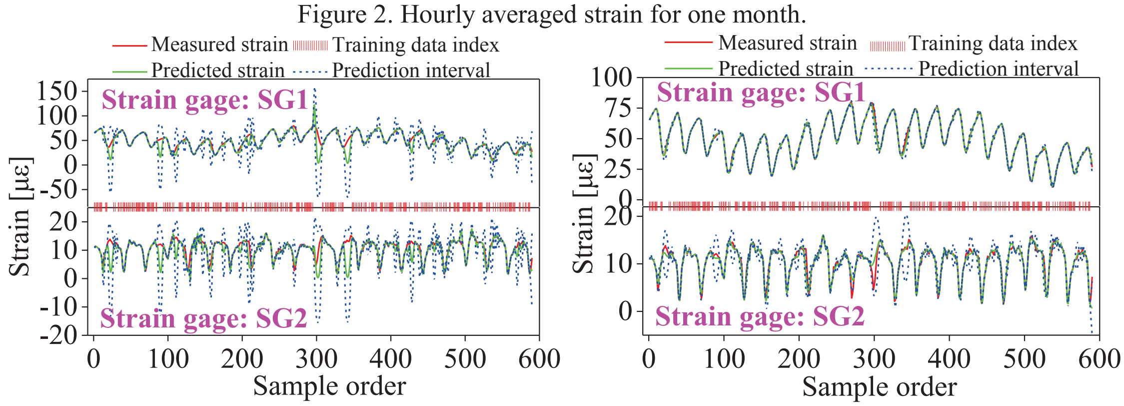


Figure 2. Hourly averaged strain for one month.



(a) SE (b) MA

Figure 3. GPM prediction with different covariance functions.

To assess the prediction capabilities of GPM quantitatively, we adopt the root mean square relative error (RMSRE) measure criteria. The results of the RMSRE metrics for the GPMs with SE and MA covariance function are demonstrated in Figure 4. Again, the quantitative metric results confirm that the GPM with MA covariance function performs better than that with the SE covariance function. This higher prediction capability owned by MA covariance function is probably attributable to its own feature that unlike SE covariance function, the MA one does not impose strong assumption of the high smoothness on the latent relationship under investigation. The SE covariance function has the appeal of the analytical capability that can be perfectly used for solving versatile engineering problems efficiently, such as uncertainty quantification (Wan et al 2014, 2017a) and sensitivity analysis (Wan et al 2015, 2017b). In addition, the assumption that the physical systems to be modeled are quite smooth usually holds. Therefore, the SE covariance function is popularly used in engineering literature. However, the assumption of the high smoothness of the latent function imposed by the SE covariance function may be too restrictive for certain practical problems. In this circumstance, the MA covariance function is a preferred choice for the construction of GPM.

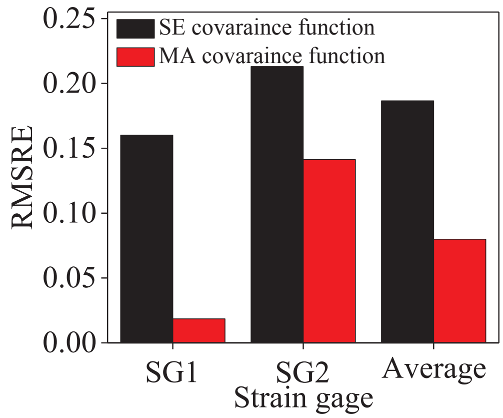


Figure 4. Prediction performance of GPMs with different covariance functions.

4 Conclusions

A GPM-based Bayesian approach is presented to characterize the strain responses of bridge. The admirable feature of GPM-based approach that the analytical calculations of predictive mean and variance are both available makes it very efficient for Bayesian predictions of the structural strain responses. As an important ingredient to specification of GPM, the covariance function significantly affects the performance of characterizing the structural strain responses. In this regarding, we discuss in detail two popularly used types of covariance functions, namely, SE and MA covariance functions. The monitored strain data recorded from TKB is used to demonstrate the present approach and to investigate the predication capability of the GPMs with SE and MA covariance functions. The results show that the GPM with MA covariance function performs better than the GPM with SE covariance function. The reason is that the development process of the structural strain is not very smooth, and compared to the SE covariance function, the MA one is more suitable for modeling the not very smooth functions. This study demonstrates the application of the GPM-based Bayesian approach to the characterization of bridge strain responses, and also offers modelers useful guidance on how to select an appropriate covariance function for their specific modeling problems.

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