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PROBABILISTIC RESPONSE ANALYSIS OF OFFSHORE STRUCTURAL RISERS WITH UNCERTAINTIES

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Probabilistic response analysis of offshore structural risers is conducted considering the uncertainties in system material properties. Both Gaussian and lognormal uncertainties in the mass density and elastic modulus are considered in this study. The random input and output are represented by using Karhunen–Loeve (KL) and Polynomial Chaos (PC) expansions, respectively. An effective model reduction technique, namely, Iterated Order Reduced (IOR) method, is employed to reduce the dimension of the stochastic system response analysis. The coefficients of PC expansions of the slave Degrees-of-Freedom (DOFs) are eliminated to improve the computational efficiency. The response statistics are obtained and compared with those from Monte Carlo Simulation (MCS). Numerical studies on an offshore riser modelled as a beam structure are conducted. Computational results demonstrate that a higher order PC expansion is required to represent the random output, and using the model reduction technique has no significant effect on the accuracy in the probabilistic response analysis but significantly reduces the computational demand.

Keywords: Uncertainty, Karhunen–Loeve Expansion, Polynomial Chaos Expansion, Stochastic Response Analysis, Model Reduction.

1 Introduction

Marine riser is a type of drilling riser, which provides a temporary extension of a subsea oil well to a surface processing facility. With the increasing demand for oil exploitation in deep water areas, the industry has to design and install very long, flexible catenary pipelines. Excess vibration of risers is considered as one of the main reasons for the degradation and failure of marine risers. Since the damage of marine riser may lead to economic losses and catastrophic environmental pollution, it is essential to monitor and maintain the operational conditions of marine risers. Many studies on vibration problems of marine riser have been conducted. A summarized review of analysis techniques for marine riser can be found in Ref. (Patel and Seyed, 1995). Most of these works are based on deterministic analysis only.

The uncertainties inevitably exist in a structural system, such as, variations of the material properties and geometric properties. These may affect the structural responses, which has not been well considered. For example, the growing marine organisms increase the mass density and have some adverse effects on offshore structures. The long-term corrosion in aggressive sea environment may significantly change the stiffness of risers. These effects should be properly considered for a better understanding of the lifetime performance of marine structures.

Stochastic dynamic response analysis gains significant attention in the last decades (Stefanou, 2009). Many methods have been proposed to represent the random field, i.e. Karhunen–Loeve (KL) expansion method, orthogonal series expansions and optimal linear estimation method. To evaluate the statistics properties of output responses, numerical methods, such as Neumann series expansion, Polynomial Chaos (PC) expansion and the first-order second-moment method, have been proposed. Stochastic Finite Element Method (Ghanem and Spanos, 2003) has been successfully applied in many engineering problems, i.e., bridge–vehicle interaction analysis (Wu and Law, 2012), structural response analysis (Anders and Hori, 1999), convergence analysis (Huang *et al.*, 2001), etc. However, the studies on marine riser is limited and worth to be explored.

This paper investigates the effect of uncertainties in system material properties on the response analysis of offshore structural risers. Both Gaussian and lognormal distributions are considered. The Gaussian random field is represented by using KL expansion, and the lognormal random field is represented by using KL and PC expansions. The random output is evaluated with PC expansion since the covariance is unknown. Stochastic response analysis is usually time consuming since the dimension of the stochastic system matrices is very large. To improve the computational efficiency, a model reduction technique, i.e. Iterated Order Reduced (IOR), is used (Xia and Lin, 2004). Numerical studies on an offshore riser are conducted. The riser is modelled with beam elements. The response statistics are obtained and compared with those from Monte Carlo Simulation (MCS).

2 Representation of stochastic processes

2.1 *KL expansion*

The theoretical background of KL expansion (Huang *et al.*, 2001) will be briefly reviewed. Let $H(\chi, \theta)$ be a second-order random process, $\bar{H}(\chi)$ is denoted as the mathematical expectation of $H(\chi, \theta)$ over all possible realizations of the process. In practice, the random field $H(\chi, \theta)$ is approximated by $\hat{H}(\chi, \theta)$ after truncating the expansion at the M-th term, i.e.,

$$H(\chi,\theta) = \hat{H}(\chi,\theta) = \bar{H}(\chi) + \sum_{n=1}^{M} \sqrt{\lambda_n}\varphi_n(\chi)\xi_n(\theta) \tag{1}$$

where λ_n and $\varphi_n(\chi)$ are the eigenvalues and eigenvectors of the covariance kernel $C_{\mathrm{cov}}(\chi_1, \chi_2)$, $\xi_n(\theta)$ is a set of uncorrelated random variables with a zero mean and an unit variance.

2.2 *PC expansion*

PC expansion is built upon the theory of the homogeneous chaos (Wiener, 1938), which is defined as elements of the space spanned by Hermite polynomials of Gaussian random variables. A general random process $\alpha(\theta)$, considered as a function of the random variable $\xi(\theta)$, can be represented in the following form

$$\alpha(\theta) = \sum_{k=0}^{\infty} \hat{a}_k \Psi_k\left(\xi(\theta)\right) \tag{2}$$

where $\hat{a}_k$ are the deterministic PC coefficients, and Ψ_k is the Hermite polynomial functional.

2.3 *Lognormal random field*

When the random field is following the lognormal marginal distribution, it can be modeled by using the exponentiation of the KL expansion as follows

$$l(\chi,\theta)=e^{\hat{H}(\chi,\theta)}=e^{\bar{H}(\chi)+\sum_{n=1}^{M}\sqrt{\lambda_n}\varphi_n(\chi)\xi_n(\theta)} \tag{3}$$

Using PC expansion, Eq. (3) can be express as

$$l(\chi)=\sum_{n=0}^{M} l_n(\chi)\Psi_n(\xi) \tag{4}$$

where M is the number of polynomial functions and depends on the number of KL expansion terms and the p order of PC expansion, the coefficients l_n can be obtained from

$$l_n(\chi)=\frac{E\left(\exp\left[\hat{H}(\chi,\theta)\right]\Psi_n\right)}{E\left(\Psi_n^2(\xi)\right)} \tag{5}$$

Since the first coefficient corresponding to $\Psi_0=1$, we have

$$l_0(\chi)=\exp\left[\bar{H}(\chi)+\frac{1}{2}\sigma_g^2(\chi)\right] \tag{6}$$

where $\sigma_g(\chi)$ is the standard deviation of $\widehat{H}(\chi,\theta)$. The other ones can be found in (Sudret and Der Kiureghian, 2000).

3 Stochastic dynamic system with uncertainties

3.1 *Equation of motion of an offshore riser under sea wave loads*

The deterministic equation of motion of an offshore riser subjected to the sea wave loads can be written as

$$\boldsymbol{m}\ddot{\boldsymbol{x}}(t)+\boldsymbol{c}\dot{\boldsymbol{x}}(t)+\boldsymbol{k}\boldsymbol{x}(t)=\boldsymbol{f}(t) \tag{7}$$

where $\boldsymbol{m}$, $\boldsymbol{c}$ and $\boldsymbol{k}$ are the mass, damping and stiffness matrices of the structure, respectively; $\boldsymbol{x}$, $\dot{\boldsymbol{x}}$ and $\ddot{\boldsymbol{x}}$ are the displacement, velocity and acceleration response vectors, respectively. $\boldsymbol{f}(t)$ is the sea wave load on the riser. In this study, only the vibration of the risers along the direction of the applied loading is considered. The transverse sea wave force per unit length of the riser can be estimated from Morison equation (Veritas, 2000).

3.2 *Representation of the system parameters*

KL expansion can be used to represent the uncertain system parameters. In this study, the uncertain mass density and elastic modulus of the system are considered as independent Gaussian random fields. Taking the mass density as an example, the mass density $\rho(\chi,\theta)$ of the structure is assumed to satisfy Gaussian distribution with the mean value $\bar{\rho}$ and a standard deviation σ_ρ. KL expansion is employed to represent the uncertain mass density as

$$\rho(\chi,\theta)=\bar{\rho}+\tilde{\rho}=\bar{\rho}+\sum_{i1=1}^{K_\rho}\xi_{i1}(\theta)\sqrt{\lambda_{i1}}\varphi_{i1}(\chi) \tag{8}$$

where $\tilde{\rho}$ denotes the random component.

The stochastic mass matrix of the system can be expressed as two parts:

$$\boldsymbol{M}(\theta)=\bar{\boldsymbol{M}}+\tilde{\boldsymbol{M}} \tag{9}$$

where

$$\bar{\boldsymbol{M}}=\sum_{i=1}^{ne}\bar{\boldsymbol{M}}_i^e=\sum_{i=1}^{ne}\left(\int \boldsymbol{N}^T(\bar{\rho}A)\boldsymbol{N}dl\right) \tag{10}$$

$$\tilde{\boldsymbol{M}} = \sum_{i=1}^{ne} \tilde{\boldsymbol{M}}_i^e = \sum_{i=1}^{ne} \left(\int \boldsymbol{N}^T \left(\sum_{i1=1}^{K_\rho} \xi_{i1}(\theta) \sqrt{\lambda_{i1}} \varphi_{i1}(\boldsymbol{\chi}) A \right) \boldsymbol{N} dl \right) \tag{11}$$

Similarly, the stiffness matrix and damping matrix can be obtained (Ni *et al.*, 2018).

3.3 *Representation of responses*

The output stochastic displacement, velocity and acceleration responses may not follow the Gaussian distributions (Xiu and Karniadakis, 2003). A random dimension, denoted as the parameter θ, is introduced in addition to the spatial–temporal dimension. The response vectors of the system can be represented as $\boldsymbol{x}(t,\theta)$, $\dot{\boldsymbol{x}}(t,\theta)$, and $\ddot{\boldsymbol{x}}(t,\theta)$. Since the covariance matrix of nodal acceleration $\ddot{\boldsymbol{x}}(t,\theta)$, velocity $\dot{\boldsymbol{x}}(t,\theta)$ and displacement $\boldsymbol{x}(t,\theta)$ are not available a priori, the output responses can be expanded by using PC expansion according to Eq. (2) with truncations (Xiu and Karniadakis, 2002). Taking the displacement for example, it can be expressed as

$$\boldsymbol{x}(t,\theta) = \sum_{j=0}^{m} \Psi_j(\theta) \boldsymbol{U}^j(t),\ \dot{\boldsymbol{x}}(t,\theta) = \sum_{j=0}^{m} \Psi_j(\theta) \dot{\boldsymbol{U}}^j(t),\ \ddot{\boldsymbol{x}}(t,\theta) = \sum_{j=0}^{m} \Psi_j(\theta) \ddot{\boldsymbol{U}}^j(t) \tag{12}$$

where $\boldsymbol{U}^j(t)$ is the vector of the coefficients in the PC expansion of $\boldsymbol{x}(t,\theta)$. Similarly, the velocity and acceleration can be expanded and $\dot{\boldsymbol{U}}^j(t)$ and $\ddot{\boldsymbol{U}}^j(t)$ are the corresponding PC coefficients, and m is the dimension of PC expansion.

Substituting Eq. (12) into Eq. (7) and taking the inner product on both sides of the equation with $\Psi_k(\theta)$, we have

$$\begin{aligned} &\sum_{j=0}^{m} \sum_{i1=0}^{K_\rho} \left\langle \xi_{i1}(\theta) \Psi_j(\theta), \Psi_k(\theta) \right\rangle \boldsymbol{M}_{i1} \ddot{\boldsymbol{U}}^j(t) + \sum_{j=0}^{m} \left\langle \Psi_j(\theta), \Psi_k(\theta) \right\rangle \boldsymbol{C} \dot{\boldsymbol{U}}^j(t) \\ &\quad + \sum_{j=0}^{m} \sum_{i2=0}^{K_E} \left\langle \xi_{i2}(\theta) \Psi_j(\theta), \Psi_k(\theta) \right\rangle \boldsymbol{K}_{i2} \boldsymbol{U}^j(t) = \left\langle \Psi_k(\theta), \boldsymbol{F}_f \right\rangle \\ &\quad (k = 0, 1, 2 \cdots m) \end{aligned} \tag{13}$$

Eq. (13) can be rewritten as

$$\boldsymbol{M}_s \ddot{\boldsymbol{U}}_s(t) + \boldsymbol{C}_s \dot{\boldsymbol{U}}_s(t) + \boldsymbol{K}_s \boldsymbol{U}_s(t) = \boldsymbol{F}_s(t) \tag{14}$$

3.4 *Model reduction*

When a large number of KL expansion terms and a high order of PC expansion are involved in stochastic dynamic analysis, the dimensions of the stochastic system as shown in Eq. (14) will increase significantly and the computational workload will be extremely intensive. Model reduction technique will be applied to reduce the size of system matrices and improve the computational efficiency. For a structure, the responses can be divided into three categories: i) very important nodes; ii) less important nodes; iii) no important nodes. The high order PC coefficients of less important nodes and all PC coefficients of no important nodes will be defined as slave DOFs, and the others are defined as master DOFs. After model reduction, only the PC coefficients of the master DOFs will remain. In this paper, IOR method (Xia and Lin, 2004) is used in this paper for multiscale uncertainties analysis due to its faster convergence ratio.

The reduced matrices, i.e., $\boldsymbol{M}_r$, $\boldsymbol{K}_r$, and $\boldsymbol{C}_r$, will be used for the stochastic dynamic response analysis. The associated reduced coefficients of PC expansions, $\boldsymbol{U}_r$, $\dot{\boldsymbol{U}}_r$ and $\ddot{\boldsymbol{U}}_r$ are obtained by using the mode superposition method. After obtaining the coefficients of PC expansions, the mean value of nodal displacements $MEAN_U$ can be evaluated as

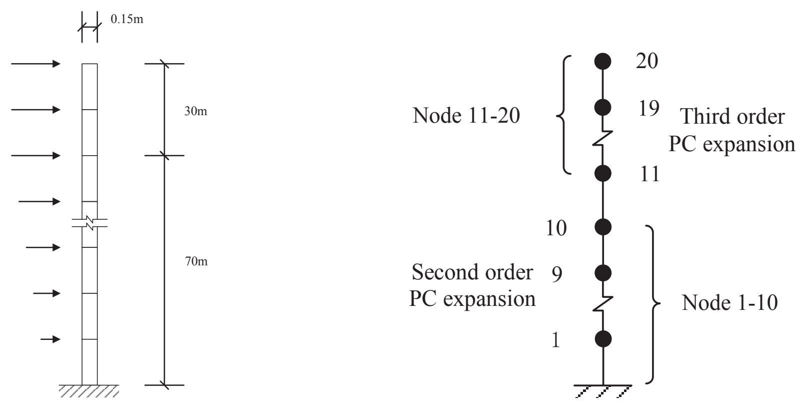
$$MEAN_{\boldsymbol{U}} = \boldsymbol{U}_r^0(t) \tag{15}$$

The variance of nodal displacements VAR_U is obtained as

$$VAR_U = \sum_{j=1}^{m} \left[\boldsymbol{U}_r^j(t) \right]^2 \Psi_j^2(\theta) \tag{16}$$

4 Numerical Studies

The numerical model of the simulated marine riser in this study is shown in Figure 1(a). It has a length of 100m and outer diameter is 0.1524m. This structure is modeled with 20 Euler beam elements and 21 nodes. Each node has 2 DOFs. The bottom of the riser is considered as a fixed support. The flexural rigidity (*EI*) and the mass density (mass per unit length) are assumed as two independent random variables with mean values of 4.0×10^{10} N·m² and 15 kg/m, respectively. In this study, 20% uncertainties in the mass density and flexural rigidity are considered. The spatial correlation is represented as an exponential covariance kernel (Ghanem and Spanos, 2003) and the correlation length is defined as 50m. The Gaussian random field is represented by using the first six terms of KL expansion, while lognormal random field is represented by using the first six terms of KL expansion and the third order PC expansion.In this example, the output responses are represented with the third order PC expansion. Nodes 1-10 are defined as the less important nodes as shown in Figure 1(b).



(a) Numerical model (b) Selection of orders in PC expansions

Figure 1. Numerical model of the marine riser

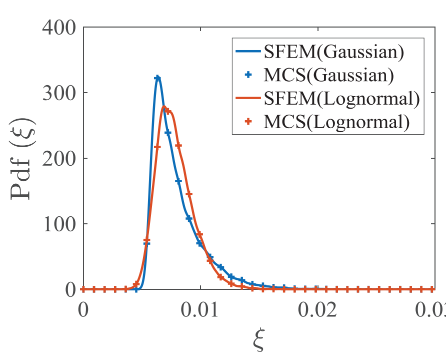


Figure 2. PDF of the horizontal response at the top of the riser

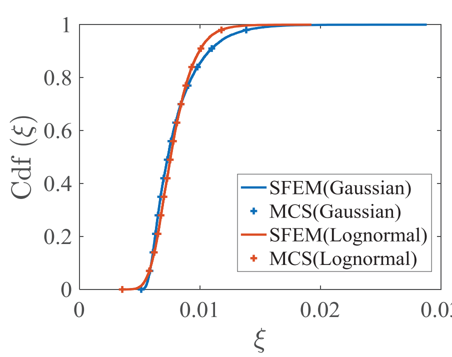


Figure 3. CDF of the horizontal response at the top of the riser

The third order PC coefficients of less important nodes and all the PC coefficients of rotational DOFs are defined as the slave DOFs in the model reduction. The horizontal displacement of the top is selected in this study. Probability Density Function (PDF) and Cumulative Density Function (CDF) of the calculated horizontal displacement response at the top of the riser are shown in Figures 2 and 3, respectively. The results considering the Gaussian and lognormal uncertainties respectively match well with the corresponding results from MCS, indicating the accuracy of the proposed approach. The computational time of using the proposed approach is about 1 hour, however it is more than 100 hours when using MCS. These results verify that the proposed stochastic response analysis approach with IOR model reduction technique is more efficient.

5 Conclusions

The effect of uncertainties in system material properties on the dynamic response statistics of a marine riser is investigated in this paper. The uncertainties are considered as Gaussian and lognormal distribution, respectively. KL expansion is used to represent the Gaussian random field, and the random output is represented by PC expansion. To improve the computational efficiency, IOR model reduction technique is used in this paper. The response statistics obtained by using the proposed approach agree well with those from MCS. It is demonstrated that using the model reduction technique has no significant effect on the accuracy of stochastic response analysis but significantly reduces the computational time.

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