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DYNAMIC USER EQUILIBRIUM DEPARTURE TIME AND MODE CHOICES IN A BI-MODAL CORRIDOR WITH RISK-AVERSE TRAVELERS

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This paper examines dynamic user equilibrium in a bi-modal corridor with risk-averse travelers. Travelers make choices between a risky (stochastic) and congested highway against a reliable (deterministic) and crowded public transit line. A mean-variance approach is adopted to measure the travel cost under risk. At user equilibrium state, nobody can reduce his/her travel cost by ultimately changing his/her departure time or mode choice. We derive traffic flow distribution and congestion evolution patterns at user equilibrium state. It is found that three departure time intervals may exist in the sequence of time on the highway, i.e., intervals in which travelers always arrive early, may be early or late, or always arrive late at the destination. The middle interval becomes longer when travelers are more risk-averse. Unlike the risk-neutral case where the departure rate of highway traffic decreases with departure time, the departure rate of highway traffic may first decrease and then increase with departure time when travelers are highly risk-averse. In addition, when travelers are more risk-averse, the travel cost becomes higher and the public transit ridership also increases. Finally, we prove that perfect traffic information provision is welfare-improving for risk-averse travelers.

Keywords: Bi-modal corridor, dynamic user equilibrium, risk-aversion, information provision.

1 Introduction

As transportation systems can be affected by various internal and external incidents, such as traffic accidents and weather, travel time is mostly uncertain. Plentiful empirical studies (e.g., Jackson and Jucker 1982; Small et al. 1999; de Palma and Picard 2005; Fosgerau and Karlstrm 2010) have revealed that travelers are interested not only in saving the expected travel time but also in reducing travel time variability. Travelers are found to be risk-averse when they make decisions. If a true distribution of travel time is known, risk can be explicitly evaluated with the travelers' preference towards risk. A typical approach is the mean-risk measure (Markowitz 1952) which combines the mean travel time and the risk (measured by travel time variance). Some studies extend the classic bottleneck model (Vickrey 1969) to investigate the decisionmaking of risk-averse travelers at dynamic user equilibrium (e.g., Yao et al. 2010; Siu and Lo 2009, 2013; Liu et al. 2017). However, these authors do not consider travel mode choices.

This study extends the literature in multi-modal corridor (e.g., van der Weijde et al. 2013) by considering uncertain travel time on highway and risk-averse commuters. We examine the departure time and mode choices in a bi-modal corridor with a highway and a rail transit line running in parallel. In the proposed morning commute model, the risk-averse commuters make choices between driving on a risky (stochastic) and congested highway against riding in a reliable (deterministic) and crowded rail transit line. We adopt the mean-variance measure to formulate the travel cost under risk. Specifically, the travel time on the highway has a random component, which is assumed to follow a uniform probability distribution. We show that the congestion queuing patterns are different from those with risk-neutral travelers in literature. Last, the welfare gain from information provision is discussed. The remainder of this paper is organized as follows. Section 2 provides model setting and assumptions. Section 3 derives the equilibrium solutions and reveals some important properties of the equilibrium travel patterns. Section 4 examines the benefit from traffic information provision. Section 5 illustrates the equilibrium and value of information in numerical examples. Finally, Section 6 summarizes the findings.

2 Model Settings

We consider a bi-modal corridor, which consists of a highway h and a rail transit line m running in parallel. The one-to-one corridor connects a residential district H with a central business district W. During the morning peak-hour, a fixed number of N travelers commute from home H to workplace W with identical work start time t^*. Each traveler chooses a travel mode (driving or transit) and a departure time t from home so as to minimize his/her travel cost. The highway congestion pattern is characterized by the classic bottleneck model. The actual travel cost of a commuter who chooses to depart from home at time t is formulated as:

$$C_h(t) = \alpha_h\, q_h(t)/s_h + \beta SDE_h(t) + \gamma SDL_h(t) + P_h\,, \tag{1}$$

which includes the congestion cost, the schedule delay cost, and the free-flow travel cost P_h. $q_h(t)$ denotes the length of the queue before the highway bottleneck. When the arrival rate exceeds the capacity of the highway bottleneck s_h, a queue forms. Specifically, $q_h(t)$ can be formulated as $q_h(t) = \max\{\int_{t_{ho}}^{t} n_h(t)dt - s_h(t - t_{ho}), 0\}$, where $n_h(t)$ is the departure rate of highway traffic flow from H and t_{ho} is the earliest time with positive departure rate. And a traveler departing at time t experiences a queuing delay $q_h(t)/s_h$. $SDE(t)$ and $SDL(t)$ denote the schedule delay of being early and late (the difference between the actual arrival time and the work start time), respectively. α_h is the unit cost of travel time on the highway. β and γ are the unit penalty cost of being early and late at work. Similarly, the actual travel cost of a commuter who chooses the transit line and departs from home at time t is:

$$C_m(t) = \alpha_m\, n_m(t)/s_m + \beta SDE_m(t) + \gamma SDL_m(t) + P_m\,, \tag{2}$$

which includes the crowding cost, the schedule delay cost, and the free-flow travel cost P_m. $n_m(t)/s_m$ represents the in-vehicle passenger density, where $n_m(t)$ is the departure rate at time t and s_m denotes transit capacity. α_m is the crowding cost per unit in-vehicle passenger density in transit line.

We extend the existing studies in multi-modal corridor by considering uncertain travel time on the highway and the effects of risk-averse preferences. We assume that the travel time on the highway is random. Specifically, the travel time on highway consists of a constant free-flow travel time, a deterministic queuing delay, and a random travel delay (see, e.g., Siu and Lo

2009). Let τ_h denote the stochastic non-queuing travel time, which is the sum of the free-flow travel time and the random travel delay. Whereas, the travel time in transit τ_m, is fixed and deterministic. It is noted that the free-flow travel cost $P_h = \alpha_h \tau_h + p_h$ and $P_m = \alpha \tau_m + p_m$, where α is the unit cost of travel time in the transit, p_h and p_m are the out-of-pocket cost of using highway and transit,. Along the lines of (Yao et al. 2010; Xiao et al.2017), the following assumptions are imposed:

A1 The non-queuing travel time τ_h follows a uniform distribution within interval $[\overline{\tau}_h - \theta, \overline{\tau}_h + \theta]$.
A2 The travel cost by driving under risk is measured by mean-variance approach, i.e.,

$$C_h^R(t) = \mathbb{E}[C_h(t)] + \lambda \mathbb{V}[C_h(t)], \tag{3}$$

where $C_h(t)$ is uncertain travel cost, $\mathbb{E}[C_h(t)]$ is expected travel cost, and $\mathbb{V}[C_h(t)]$ is variance of travel cost at departure time t. λ is the risk-aversion parameter. $\lambda \mathbb{V}[C_h(t)]$ can be considered as the risk cost with the degree of risk-aversion λ. Because the travel time in mass transit is deterministic, the mean-variance cost $C_m^R(t)$ is reduced to the actual travel cost $C_m(t)$.

3 Equilibrium Solutions and Properties

3.1 *Equilibrium Solutions*

We derive the mode-specific flow distribution and congestion evolution patterns. At equilibrium state, no one can reduce his/her travel cost by unilaterally altering his/her departure time t from home and the traffic mode i. This condition implies that the mean-variance travel cost is constant over time if the departure rate is positive. That is for any $i \in \{h, m\}$, $dC_i^R(t)\big/dt = 0$ if $n_i(t) > 0$.

Due to the random travel time on the highway, we find that there may be three departure time intervals: 1) if $t_{ho} \le t < t_{h1}$, travelers always arrive early, 2) if $t_{h1} \le t \le t_{h2}$, travelers may arrive early or late depending on actual non-queuing travel time , i.e., , and 3) if $t_{h2} < t \le t_{he}$, travelers always arrive late. The equilibrium departure rates on the highway are derived as:

$$n_h(t) = \begin{cases} \dfrac{\alpha_h s_h}{\alpha_h - \beta}, & t_{ho} \le t < t_{h1}; \\ \dfrac{\alpha_h s_h}{(\alpha_h - \beta P_E + \gamma P_L)(1 + 2(\beta+\gamma)\theta\lambda P_E P_L)}, & t_{h1} \le t \le t_{h2}; \\ \dfrac{\alpha_h s_h}{\alpha_h + \gamma}, & t_{h2} < t \le t_{he}, \end{cases} \tag{4}$$

where $P_E = \left(t^* - t_{ho} - \overline{\tau}_h + \theta - \int_{t_{ho}}^{t} n_h(\omega)d\omega \big/ s_h\right)\big/2\theta$ and $P_L = \left(t_{ho} + \overline{\tau}_h + \theta + \int_{t_{ho}}^{t} n_h(\omega)d\omega \big/ s_h - t^*\right)\big/2\theta$, and t_{he} denotes the end time of congestion on the highway.

For transit, the equilibrium departure rates are derived as:

$$n_m(t) = \begin{cases} \dfrac{\beta s_m (t - t_{mo})}{\alpha_m}, & t_{mo} \le t \le t^* - \tau_m; \\ \dfrac{\gamma s_m (t_{me} - t)}{\alpha_m}, & t^* - \tau_m \le t \le t_{me}. \end{cases} \tag{5}$$

where t_{mo} and t_{me} are the earliest and latest departure times of transit commuters, respectively.

If both highway and transit are used, the total number of commuters choosing highway is:

$$N_h = \frac{\beta+\gamma}{\beta\gamma}\frac{s_h}{s_m}(X^R - (\bar{P}_h + (\alpha_h^2 + \beta\gamma)\lambda\frac{\theta^2}{3} - P_m)s_m - \alpha_m s_h), \tag{6}$$

where $X^R = \sqrt{\alpha_m^2 s_h^2 + 2\alpha_m(\bar{P}_h + (\alpha_h^2 + \beta\gamma)\lambda\frac{\theta^2}{3} - P_m)s_h s_m + 2\alpha_m\frac{\beta\gamma}{\beta+\gamma}s_m N}$ and $\bar{P}_h = \alpha\bar{\tau}_h + p_h$.

Finally, the equilibrium travel cost under risk can be expressed as:

$$C^R = P_m - \alpha_m\frac{s_h}{s_m} + \frac{X^R}{s_m}. \tag{7}$$

3.2 *Equilibrium Properties*

We present the important properties of user equilibrium with risk-averse commuters here.

Proposition 1. *The highway demand N_h monotonically decreases with risk parameter λ. The travel cost under risk C^R monotonically increases with λ. The length of interval $|t_{h2} - t_{h1}|$ increases with λ.*

Proposition 1 implies that when travelers are more risk-averse, the travel cost under risk becomes higher and the public transit ridership increases. And the middle interval in which travelers may arrive early or late at destination becomes longer.

Proposition 2. *On the highway, the departure rate decreases with departure time if $\lambda \le \frac{1}{2\theta(\alpha_h+\gamma)}$* holds*; otherwise, the departure rate first decreases and then increases with departure time.*

It indicates that the congestion queuing patterns are different from those with risk-neutral travelers in literature. Unlike the risk-neutral case where the departure rate of highway traffic flow decreases with departure time, the departure rate of highway traffic here may first decrease and then increase with departure time when the degree of risk-aversion is higher than a threshold.

4 Information Provision

With technologies, such as the global navigation satellite system, the traffic information can be effectively collected and disseminated. Therefore, commuters can obtain the information about τ_h within the current day in advance and choose their departure times and travel modes. We hereby investigate the situation in which all commuters know the actual non-queuing travel time τ_h explicitly before departure. That is the case with the full/perfect information. The expected travel cost $\mathbb{E}[C^I]$ can be expressed as:

$$\mathbb{E}[C^I] = \int_{\bar{\tau}_h-\theta}^{\bar{\tau}_h+\theta}(P_m - \alpha_m\frac{s_h}{s_m} + \frac{X}{s_m})\frac{1}{2\theta}d\tau_h = P_m - \alpha_m\frac{s_h}{s_m} + \frac{\mathbb{E}[X]}{s_m}, \tag{8}$$

where $X = \sqrt{\alpha_m^2 s_h^2 + 2\alpha_m(P_h - P_m)s_h s_m + 2\alpha_m\frac{\beta\gamma}{\beta+\gamma}s_m N}$.

The value of information provision can be measured by $C^R - \mathbb{E}[C^I]$, which is derived as:

$$C^R - \mathbb{E}[C^I] = \frac{X^R - \mathbb{E}[X]}{s_m}. \tag{9}$$

We prove that $X^R \geq \mathbb{E}[X]$ always holds, and thus it has $C^R - \mathbb{E}[C^I] > 0$. This suggests that the perfect information provision is always welfare-improving when travel time on the highway is uncertain, and travelers are risk-averse.

5 Numerical Example

This section illustrates the congestion patterns in the bi-modal corridor with risk-averse commuters. Following Small (1982), we use the parameter values: $\alpha = 6$ dollar/hour, $\alpha_h = 6.4$ dollar/hour, $\beta = 3.9$ dollar/hour and $\gamma = 15.21$ dollar/hour. We set $\alpha_m = 0.4$ dollar·m^2, t^* is 9:00 am, the total demand $N = 100000$, $s_h = 4000$ vehicles/hour, $s_m = 2000$ m^2/hour, $\overline{\tau}_h = 0.5$ hour, $\tau_m = 1$ hour, $p_h = 2$ dollar and $p_m = 1$ dollar. Finally, we set risk parameter $\theta = 0.1$ and risk-aversion parameter $\lambda = 1$.

Figure 1(a) (left) shows the departure rates on highway and transit. It shows that the departure rate on the highway first decreases and then increases with departure time from home. The departure rate in the transit line linearly increases and then linearly decreases with departure time. Figure 1(b) (right) illustrates each travel cost component. The expected travel time cost $\mathbb{E}[TTC_h(t)]$ increases at first and then decreases with departure time, which is positively associated with the expected queuing length in highway bottleneck. Conversely, the expected schedule delay cost $\mathbb{E}[SDC_h(t)]$ decreases at first and then increases. The risk cost $\lambda\mathbb{V}[C_h(t)]$ strictly increases with departure time. It is worthwhile to note that the curves of the cost components on the highway are not piecewise linear here, which are different from piecewise linear curves in the deterministic bottleneck model (Vickrey 1969).

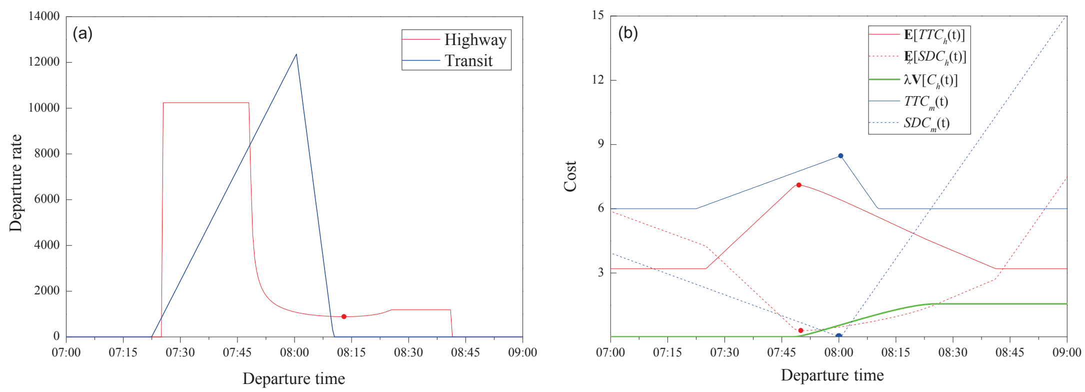

Figure 1. Equilibrium departure rates and components of travel costs.

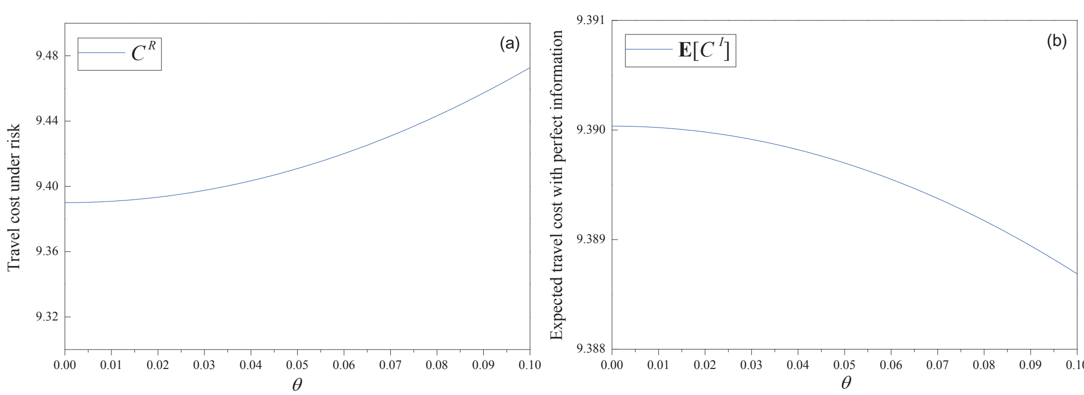

Figure 2. Value of full traffic information provision.

We next investigate the value of traffic information provision. Let the uncertainty value θ vary from 0 to 1. When $\theta = 0$, the travel cost is deterministic and $C^R = \mathbb{E}[C^I]$ holds. Figure 2(a) (left) shows that the travel cost under risk C^R increases with travel time uncertainty θ. Figure 2(b) (right) shows the expected travel cost with perfect information $\mathbb{E}[C^I]$ decreases with θ. Therefore, $C^R > \mathbb{E}[C^I]$ always holds if $\theta > 0$. It verifies our conclusion in Section 4. The perfect traffic information provision is welfare-improving if travelers are risk-averse.

6 Conclusion

This paper examines the dynamic user equilibrium in a bi-modal corridor with risk-averse travelers, in which travelers have to choose between a risky and congested highway against a reliable and crowded transit. Travelers choose departure times from home and travel modes so as to minimize their own travel cost under risk. We derive the analytical solutions of equilibrium departure rates and travel costs. Unlike the risk-neutral case where the departure rate of highway traffic decreases with departure time, the departure rate of highway traffic may first decrease and then increase with departure time when travelers are highly risk-averse. Furthermore, when travelers are more risk-averse, the travel cost under risk becomes higher and the public transit ridership also increases. We prove that perfect traffic information provision is always welfare-improving for risk-averse travelers. The congestion pattern and the value of information are illustrated in the numerical experiments.

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