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STOCHASTIC RESPONSE OF CONCRETE STRUCTURES WITH DEPENDENT RANDOM PARAMETERS

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In most stochastic response analysis of structures with random parameters, independence or complete correlation of these parameters is assumed. It is recognized that practical random parameters are partially correlated, which may have obvious influence on the statistical properties of stochastic responses. Recently, a random function model was proposed to characterize the dependence between two random parameters. In this model, the underlying physics is introduced to yield the shape of the function, then the observed data are taken to identify the parameters of the function. On this basis, the stochastic seismic response of a concrete frame structure with dependent random strength and random initial elasticity of modulus is studied. Meanwhile, in order to capture the whole propagation path of randomness, the probability density evolution method (PDEM) is employed in this paper. The results show that when the nonlinearity of structural behavior is strong under severe earthquakes, the discrepancy between the results considering and without considering the dependence of random parameters cannot be ignored.

Keywords: correlation, random function model, concrete frame structure, stochastic earthquake, probability density evolution method.

1 Introduction

Parameters of structural mechanical properties, e.g., the strength and initial modulus of elasticity of concrete, are usually dependent random variables in nature (Ang & Tang 2006, Li et al. 2007). However, in engineering practice they are usually regarded either as independent or as completely correlated, partly due to the lack of data and model characterizing such dependence and partly due to the unavailability of methods that can tackle parametric correlation in stochastic response analysis of structures. However, ignoring the parametric correlation may yield misleading results in the performance evaluation and reliability assessment of real-world structures. To involve such dependency among random parameters, a variety of investigations have been carried in the past decades. The approaches can be largely classified to those dealing with the joint probability density function directly and those transforming correlated random parameters to uncorrelated or independent variables. The copula function method belongs to the first class (Nelsen 2006). But the selection of the form of copula function depends sometimes strongly on empirical judgement (Li et al. 2015, Chen et al. 2018). To the second class belong the Rosenblatt transformation (Rosenblatt 1952) and polynomial chaos expansion (Ghanem & Spanos 1990, Das et al. 2009, Soize & Ghanem 2017). However, these latter methods introduced strong nonlinear transform that may make the problem to be tackled more ill-posed (Xu & Cheng 2003, Lebrun & Dutfoy 2009).

In a recent paper (Chen et al. 2018), a novel random function model was proposed to capture the correlation of two-dimensional dependent random variables. In this method, the underlying physical mechanism was advocated to determine the shape of the function and the data was adopted to identify the distribution parameters. In the present paper, this model will be employed to implement the stochastic response analysis and reliability assessment of concrete structures via the probability density evolution method (Li & Chen 2009). It is shown that the correlation among basic random variables can be extremely crucial in structural reliability assessment, in particular when the structure exhibits strong nonlinearity under strong earthquakes.

2 Random Function Model for Dependent Random Variables

2.1 *Fundamentals of random function model*

The main idea of the new method is to establish a random function model (Chen et al. 2018). For example, consider two dependent random variables X and Y. The random function model can be written as

$$Y = g_1(X) + \zeta g_2(X) \tag{1}$$

where ζ and X are assumed to be independent, and $\mathbb{E}[\zeta] = 0$, $\mathbb{E}\left[\zeta^2\right] = 1$, in which $\mathbb{E}[\cdot]$ is the expectation operator. By using the expectation operator on both sides of Eq. (1), we have

$$\mathbb{E}[Y \mid X] = g_1(X) \text{ and } \sqrt{\mathbb{E}\left[\left(Y - \mathbb{E}[Y \mid X]\right)^2 \mid X\right]} = g_2(X) \tag{2}$$

which means that $g_1(\cdot)$ is the conditional mean function while the latter $g_2(\cdot)$ is the conditional standard deviation function. Once these two functions are determined, the distribution of random variable ζ can be determined easily by the statistical inference method. Actually, let (x_i, y_i) be the *i*th sample value of random vector (X, Y), then the *i*th sample value of ζ is

$$\zeta_i = \frac{y_i - g_1(x_i)}{g_2(x_i)}. \tag{3}$$

In Chen et al. (2018), for the compressive strength and the initial modulus of elasticity of concrete, a viscoelasticity mechanism model is advocated to seek the forms of $g_1(\cdot)$ and $g_2(\cdot)$, then the data from concrete specimen tests were taken to specify the parameters and distribution of ζ.

2.2 *Procedure to establish random function model*

The major steps to establish a random function model is as follows:

(i) Figure out any underlying physical mechanism, which specifies the conditional mean function. If the underlying physical mechanism is still unclear, the conditional mean function can be determined just by the statistical regression method.
(ii) Separate the conditional mean function from the random function model and repeat Step (i).
(iii) Obtain the samples of ζ by Eq. (3) and determine its distribution by the statistical inference method.
(iv) Assemble Step (i) to (iii) above with Eq. (1).

Following the steps given above, a random function model can be eventually determined to describe the correlation between the compressive strength f_c and the initial modulus of elasticity E_c of concrete (Chen et al. 2018), i.e.

$$E_c = \frac{10}{a + b/f_c} + (\zeta\sigma + \mu)\frac{Bf_c}{(Af_c + B)^2} \cdot \frac{1}{\exp(Cf_c)} \quad (4)$$

where the parameters in Eq. (4) are listed in Table 1.

Table 1. The parameters in the random function model (Chen et al. 2018)

Parameter	a	b	A	B
Value	2.234	30.305	1.057	24.090
Parameter	C	μ	σ	ζ
Value/Distribution	0.00152	0.118	1.380	Normal

Since E_c is a random function of both f_c and ζ, clearly E_c and f_c are of partial correlation or dependent. Meanwhile, it is stressed that ζ and f_c are independent so that the original case has already been converted into the case involving independent random variables. It should be noted that in Chinese code an empirical relationship between E_c and f_c is employed (MOHURD 2010)

$$E_c = \frac{10}{2.2 + 34.7/f_c}. \quad (4)$$

Comparing Eq.(4) with Eq.(4) demonstrates that the variation between the two parameters is taken back in the random function model.

It is noted that in this random function model, the direct adoption of joint probability density function of random vector is avoided. In addition, this model is usually weakly nonlinear, therefore the well-posedness will not be deteriorated when converting one original problem with dependent random parameters to another problem with independent random parameters.

3 Applications

In this section, the above model is employed and the stochastic response of concrete structures with dependent random parameters is implemented. A 10-story plane frame structure with random compressive strength f_c and initial modulus of elasticity E_c of concrete is subjected to earthquake excitations (Wang & Li 2011). Two cases, i.e., the partially dependent case characterized by Eq. (4) and the complete dependent case characterized by Eq. (4), are studied. Three different peak ground accelerations, i.e., the seismic fortification intensity 7 (SFI-7) with PGA = 0.1g, SFI-9 with PGA = 0.4g and the extreme condition with PGA = 0.8g, are considered. The details are listed in in Table 2.

Table 2. Detailed description of all six cases

Case ID	Correlation of f_c and E_c	PGA of earthquake
1 / 2	Partial / Complete	0.1g
3 / 4	Partial / Complete	0.4g
5 / 6	Partial / Complete	0.8g

3.1 *Concrete frame structure within random parameters*

The geometry property of the concrete structure is illustrated in Figure 1. Each story of the structure shares the same mechanical parameters of both concrete and steel bars, while it is independent for arbitrary two different floors. Therefore, Eq. (4) is used for Case 1, 3 and 5 while Eq. (4) is for Case 2, 4 and 6.

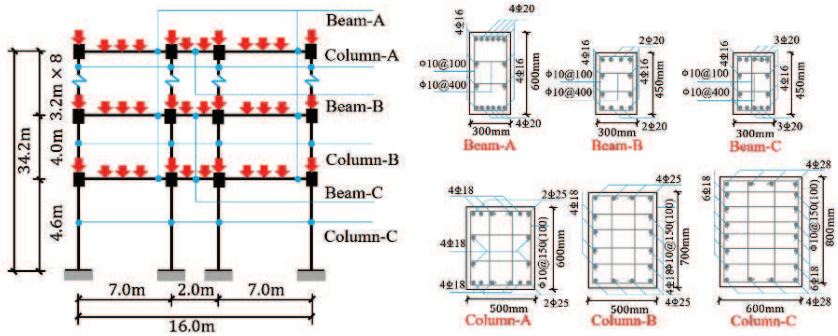

Figure 1. Geometry information of the designed concrete structure

3.2 *Stochastic model of earthquake ground motion process*

A stochastic model of earthquake ground motion process (Wang & Li 2011) is applied, which is based on seismic source-path-site physical mechanism. There are 4 basic parameters in this model, i.e., A_0, τ, ξ_g and ω_g, which are of physical meaning and of randomness as well. Values of these parameters are all available in the reference (Li & Wang 2013) with detailed descriptions.

3.3 Results and Discussions

All the six cases are analyzed via the OpenSEES software. The results are shown in Figures 2 through 6. It is clearly seen that the effect of correlation of mechanical parameters will increase as the nonlinearity of the structural behavior becomes stronger, which is embodied on the stochastic responses (Figures 2 to 4), contours of PDFs of responses (Figure 5) and the reliability (Figure 6) as well.

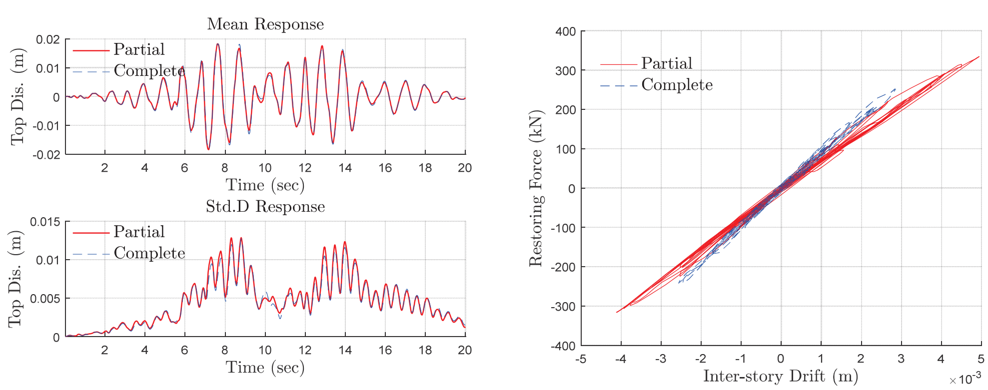

Figure 2. Top displacement (left) and typical hysteretic curve of restoring force vs. inter-story drift (right) (Case 1: solid lines; Case 2: dashed lines)

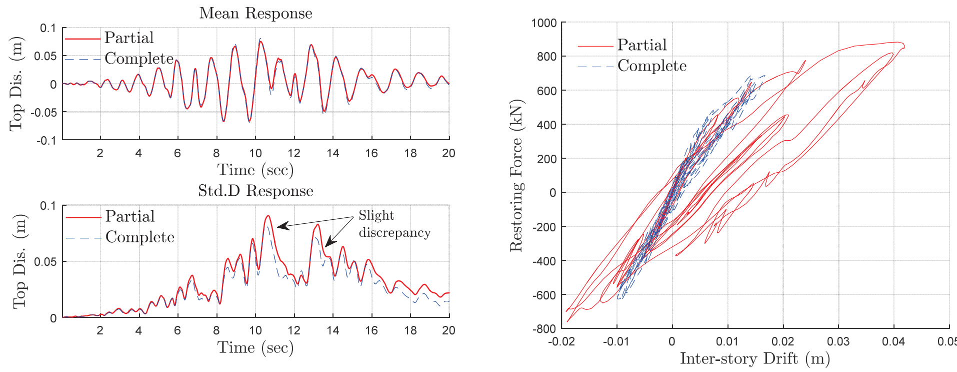

Figure 3. Top displacement (left) and typical hysteretic curve of restoring force vs. inter-story drift (right) (Case 3: solid lines; Case 4: dashed lines)

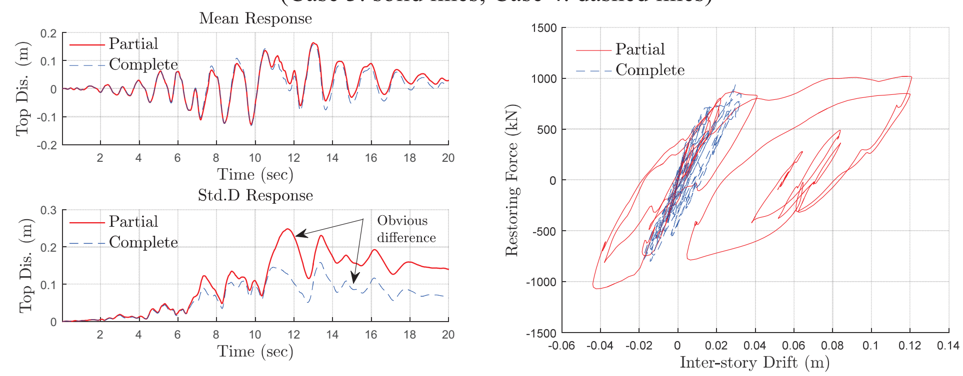

Figure 4. Top displacement (left) and typical hysteretic curve of restoring force vs. inter-story drift (right) (Case 5: solid lines; Case 6: dashed lines)

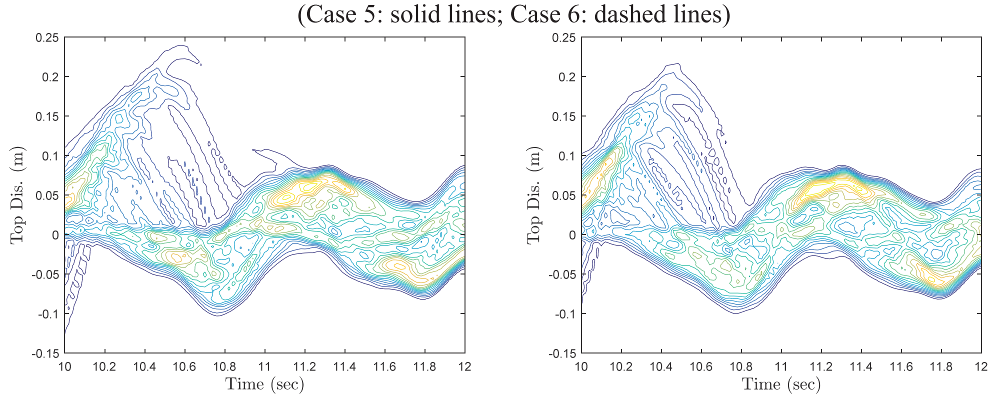

Figure 5. Case 3 (left) & Case 4 (right) – contours of PDF for the time interval [10,12] sec

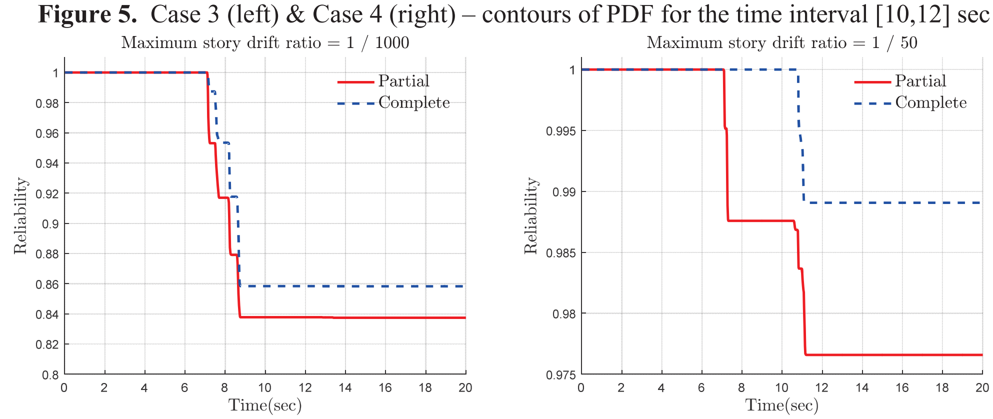

Figure 6. Time dependent reliability (Case 1 & 2: left; Case 5 & 6: right)

4 Conclusions

A new random function model characterizing the correlation among random parameters is firstly outlined. The underlying physical mechanism is advocated to capture the shape of functions, then the data are adopted to identify the parameters and distributions of the independent random variables. This method is incorporated into the probability density evolution method to implement the stochastic response analysis of concrete structures under earthquake actions.

It is demonstrated that when the earthquake action is not very strong, the effect of correlation between the compressive strength and the initial modulus of elasticity is not obvious. However, as the degree of nonlinearity increases, the effects of correlation between the basic parameters cannot be ignored. Moreover, the reliability when ignoring such effects may be non-conservative. Therefore, the correlation between the basic random variables should be characterized reasonably in the design of structures under strong earthquakes. Further investigations are urgent to capture the dependence among multiple random variables.

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