

A STRAIGHTFORWARD MOMENT METHOD TO ESTIMATE THE LOAD AND RESISTANCE FACTORS

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The load and resistance factor design is widely applied in engineering, where the load and resistance factors are introduced to account for the uncertainties in practice. Accurate estimation of the load and resistance factors on the basis of specified reliability-based requirement is important for an appropriate design, which is generally conducted by the First Order Reliability Method (FORM). However, since the design point must be determined and derivative-based iterations have to be used, FORM is not practical for engineers. Furthermore, distributions of the load and resistance are required in FORM, which are usually unknown in practice. To overcome these deficiencies, moment methods are proposed, which estimate the load and resistance factors based on their moments. Although, the existing moment methods significantly reduce the iteration numbers, iterations are still necessary when higher accuracy is required. Therefore, a straightforward moment method is proposed in the present paper to estimate the load and resistance factors without iteration. Explicit and simplistic expressions of the proposed method are given. The procedure of the proposed method is summarized in a flowchart and the accuracy of the proposed method is examined by comparison study among existing methods. It is shown that the proposed method is both accurate and efficient in estimating the load and resistance factors with no iteration required and thus it is simple to perform for engineers.

Keywords: Load and resistance factor, moment method, mean resistance, iteration number.

1 Introduction

In designing a structure, the insurance of the safety is the most important task that should be accomplished. To achieve this, the Load and Resistance Factor Design (LRFD) (Ang and Tang 1984; AIJ 2002) format is proposed and has been widely applied in practical engineering, where the safety of a structure is assured by making the nominal design resistances reduced by the resistance factors no less than the nominal design loads amplified by the load factors. The load and resistance factors are introduced to account for the uncertainties inherent in the determination of the nominal strength, the load effects due to natural variation in the loads, the material properties, the accuracy of the theory, the precise of the analysis, etc. Accurate predetermination of the load and resistance factors on the basis of specified reliability-based requirement is important for an appropriate design, where probabilistic analysis is necessary.

Generally, the load and resistance factors can be obtained by using the first order reliability method (FORM) (Melchers 1999; Nowak and Collins 2000), in which the “design point” must be determined, derivative-based iterations have to be used, and the problem of multiple design points has to be dealt with (Der-Kiureghian and Dakessian 1998; Barranco-Cicilia *et al* 2009). The complexity of FORM would prevent the practicing engineers in general to perform it in engineering designs. To conduct structural design more flexible, it is necessary to propose suitable and simple methods to determine the load and resistance factors for practical engineering. AIJ (2000) recommendation has provided a simple method based on the proposal of Mori (2002), in which all the random variables are assumed to have known probability density function (PDF) and required to be transferred into lognormal random variables. However, in reality, the PDFs of some of the basic random variables are often unknown due to the lack of statistical data. ASCE (2010) standard propose simple equations to determine the load and resistance factors, where the sensitivity coefficients in the formulation uses approximate value. To obtain suitable load and resistance factors including random variables with unknown PDFs, Lu *et al* (2010) proposed a method based on the moment method, where the load and resistance factors can be obtained including random variables with unknown PDFs. This moment method expands the application of LRFD into areas where the PDFs of random variables are unknown. However, it still needs iteration when high accuracy is required.

The objective of the present paper is thus to propose a straightforward moment method to determine the load and resistance factors with no iteration required. This paper is organized as follows. Firstly, the FORM and existing moment methods to determine the load and resistance factors are reviewed. Then the proposed method is deduced with explicit formulas presented. The procedure of the proposed method is summarized in flowchart. The accuracy of the proposed method is investigated with numerical examples. Finally, the findings of the present paper are concluded.

2 Review of existing estimating methods

The LRFD format is expressed as follows

$$\phi R_n \geq \sum \gamma_i S_{ni} \quad (1)$$

where ϕ is the resistance factor; γ_i is the partial load factor to be applied to load effect S_i ; R_n is the nominal value of the resistance of a structural component; and S_{ni} is the nominal value of the load effect S_i which has the same dimension with R .

To account for the uncertainties in reality, the appropriate expressions of ϕ and γ_i are determined for specified reliability-based requirement, which are expressed as

$$P_f \leq P_{fT} \quad (2a)$$

$$\beta \leq \beta_T \quad (2b)$$

where P_{fT} and β_T are the target probability of failure and target reliability index, respectively; P_f and β are the probability of failure and reliability index corresponding to the performance function, expressed as

$$G(\mathbf{X}) = R - \sum S_i \quad (3)$$

where R and S_i are the random variables representing the uncertainty in the resistance and load effects.

2.1 FORM for LRFD

To fulfill the reliability-based requirement, i.e., Eqs. (2a) and (2b), FORM is performed for the performance function $G(\mathbf{X})$, then the LRFD format can be expressed as

$$R^* \geq \sum S_i^* \quad (4)$$

where R^* and S_i^* are the values of resistance R and load S_i at the design point of FORM, respectively. And the load and resistance factors can be obtained as (Ang and Tang 1984)

$$\phi = R^* / R_n, \quad \gamma_i = S_i^* / S_n \quad (5)$$

Since R^* and S_i^* are obtained using derivative-based iterations, explicit expressions of R^* and S_i^* are not available. Some simplifications have been proposed in order to avoid iterative computations (Ugata 2000; Mori 2002).

2.2 Existing Moment method

2.2.1 Estimation of the load and resistance factor

By using the second moment method, the reliability index can be obtained as

$$\beta = \beta_{2M} = \mu_G / \sigma_G \quad (6a)$$

$$\mu_G = \mu_R - \sum \mu_{Si}, \quad \sigma_G^2 = (\mu_R V_R)^2 + \sum \sigma_{Si}^2 \quad (6b)$$

where β_{2M} is the second moment reliability; μ_G and σ_G are the mean and standard deviation of the performance $G(\mathbf{X})$, respectively; μ_R and μ_{Si} are the mean value of the resistance R and load S_i , respectively; V_R and σ_{Si} are the coefficient of variation (COV) of R and standard deviation of S_i , respectively. To make a balance between the economical and the reliability-based requirements, the second moment index β_{2M} is set to be equal to the targeted reliability index β_T :

$$\beta_{2M} = \beta_T \quad (7)$$

Substituting Eqs. (6a) and (6b) into Eq. (7), Eq. (7) is rearranged as follows

$$\mu_R (1 - \alpha_{R_M} V_R \beta_T) = \sum \mu_{Si} (1 + \alpha_{Si_M} V_{Si} \beta_T) \quad (8)$$

where α_{R_M} and α_{Si_M} are the direction cosines (also known as separating factors) of R and S_i , respectively, and are given as

$$\alpha_{R_M} = \alpha_R V_R / \sigma_G, \quad \alpha_{Si_M} = \sigma_{Si} / \sigma_G \quad (9)$$

Comparing Eq. (8) with Eq. (1), the resistance and partial load factors can be expressed as

$$\phi = (1 - \alpha_{R_M} V_R \beta_T) \mu_R / R_n, \quad \gamma_i = (1 + \alpha_{Si_M} V_{Si} \beta_T) \mu_{Si} / S_n \quad (10)$$

If R and S_i are mutually independent normal random variables, the reliability index β can be exactly obtained by using the second moment method β_{2M} , and thus the load and resistance factors can be obtained exactly by using Eq. (10). However, in practical engineering, the distributions of R and S_i are usually non-normal and thus higher order reliability index are proposed. To take the advantage of the simple forms of Eq. (10), β_T is transformed into equivalent second-moment target reliability index β_{2T} , which is in the same space as the second moment reliability index β_{2M} , by using the higher order moment method. And then the reliability-based requirement given in Eq. (7) is transformed into

$$\beta_{2M} = \beta_{2T} \quad (11)$$

The expressions of β_{2T} obtained by using different methods are listed in Table 1, where the names of the expressions of β_{2T} are given according to the authors' name who firstly proposed

Table 1. The expressions of β_{2T} and μ_{R0} of existing moment methods

Name	Expression of β_{2T}	Expression of μ_{R0}
Zhao and Ono (2000)	$\beta_{2T} = \frac{3}{\alpha_{3G}} \left\{ 1 - \exp \left[\frac{\alpha_{3G}}{3} \left(-\beta_T - \frac{\alpha_{3G}}{6} \right) \right] \right\}$	$\sum \mu_{Si} + \sqrt{\beta_T^{3.5} \sum \sigma_{Si}^2}$
Zhao and Ono (2001)	$\beta_{2T} = \beta_T - \frac{\alpha_{3G}}{6} (\beta_T^2 - 1)$	$\sum \mu_{Si} + \sqrt{\beta_T^{3.5} \sum \sigma_{Si}^2}$
Wang (2017)	$\beta_{2T} = \frac{3.8}{\alpha_{3G}} \left[1 - \exp \left(-\frac{\alpha_{3G}}{3.8} \beta_T \right) \right]$	$\sum \mu_{Si} + \sqrt{\beta_T^5 \sum \sigma_{Si}^2}$
Zhao and Lu (2007)	$\beta_{2T} = k_2 \beta_T^3 - l_1 \beta_T^2 + k_1 \beta_T + l_1$ $l_1 = \frac{\alpha_{3G}}{6(1+6l_2)}, l_2 = \frac{1}{36} \left(\sqrt{6\alpha_{4G} - 8\alpha_{3G}^2} - 14 - 2 \right)$ $k_1 = \frac{1-3l_2}{1+l_1^2-l_2^2}, k_2 = \frac{l_2}{1+l_1^2+12l_2^2}$	$\sum \mu_{Si} + \sqrt{\beta_T^{3.3} \sum \sigma_{Si}^2}$

the expressions. In Table 1, α_{3G} and α_{4G} are the skewness and kurtosis of the performance $G(\mathbf{X})$, respectively, and expressed as

$$\alpha_{3G} = \frac{1}{\sigma_G^3} \left[\alpha_{3R} (\mu_R V_R)^3 - \sum \alpha_{3Si} \sigma_{Si}^3 \right] \quad (12a)$$

$$\alpha_{4G} = \frac{1}{\sigma_G^4} \left[\alpha_{4R} (\mu_R V_R)^4 + 6\sigma_R^2 \sum_{i=1}^n \sigma_{Si}^2 + \sum_{i=1}^n \alpha_{4Si} \sigma_{Si}^4 + 6 \sum_{i=1}^{n-1} \sum_{j>i}^n \sigma_{Si}^2 \sigma_{Sj}^2 \right] \quad (12b)$$

where α_{3R} and α_{3Si} are the skewness of the resistance R and load S_i , respectively; α_{4R} and α_{4Si} are the kurtosis of the resistance R and load S_i , respectively. With the skewness and kurtosis of the performance function $G(\mathbf{X})$ (α_{3G} and α_{4G}) given, β_{2T} can be easily determined with the aid of Table 1, and then the resistance factor ϕ and partial load factor γ_i are obtained by solving Eq. (11) and are expressed as

$$\phi = (1 - \alpha_{R_M} V_R \beta_{2T}) \mu_R / R_n \quad (13a)$$

$$\gamma_i = (1 + \alpha_{Si_M} V_{Si} \beta_{2T}) \mu_{Si} / S_n \quad (13b)$$

Eqs. (13a) and (13b) are in the same form as Eq. (10), while β_{2T} is applied in Eqs. (13a) and (13b) instead of β_T in Eq. (10). When $\alpha_{3G} = 0$ and $\alpha_{4G} = 3$, $\beta_{2T} = \beta_T$ and Eqs. (13a) and (13b) reduce to Eq. (10). Thus, Eqs. (13a) and (13b) are used to represent the expressions applied to estimate load and resistance factors by moment methods.

According to Eqs. (9)-(13) and Table 1, the values of the resistance factor ϕ and partial load factor γ_i depend on the value of μ_R , which is unknown in the design process. Therefore, Eqs. (13a) and (13b) have to be evaluated iteratively, where the initial mean resistance, μ_{R0} , is listed in Table 1. The accurate determination of the initial mean value of the resistance μ_{R0} is necessary for an efficient method. However, according to Table 1, μ_{R0} are defined by experienced formula in all the existing moment methods, which are not precise enough and hence cause the iterations. It is in this regard, a new formula is proposed in the present paper to avoid the iterations required in the existing moment methods.

3 The proposed method

3.1 Estimation of the load and resistance factor

To simplify the procedure of estimating the load and resistance factors, a suitable formula to predetermine the μ_R is proposed. Once μ_R is determined, the load and resistance factors can be directly obtained with the aid of Eq. (9), (13a) and (13b), thus the iterations required in the existing moment methods are avoided.

If the resistance R and partial loads S_i ($i=1, \dots, n$) are independent normal random variables, the exact value of μ_R can be obtained by solving Eqs. (6a), (6b) and (7), and expressed as

$$\mu_R = \frac{1 + \sqrt{1 - \omega_R \omega_S}}{\omega_R} \sum \mu_{Si} \quad (14a)$$

where ω_R and ω_S are coefficients, and given as

$$\omega_R = 1 - \beta_T^2 V_R^2, \quad \omega_S = 1 - \beta_T^2 V_S^2 \quad (14b)$$

where $V_S = \sum \sigma_{Si}^2 / \sum \mu_{Si}$ is the COV of S , when considering all the partial loads S_i ($i=1, \dots, n$) as a whole load effect S .

In general, the resistance R and partial loads S_i ($i=1, \dots, n$) are not normally distributed, and then the exact value of μ_R is obtained by solving Eqs. (6a), (6b) and (11) with the aid of Table 1 and given as

$$\mu_R = \frac{1 + \sqrt{1 - \omega'_R \omega'_S}}{\omega'_R} \sum \mu_{Si} \quad (15a)$$

where ω'_R and ω'_S are coefficients, and given as

$$\omega'_R = 1 - \beta_{2Tcheck}^2 V_R^2, \quad \omega'_S = 1 - \beta_{2Tcheck}^2 V_S^2 \quad (15b)$$

where $\beta_{2Tcheck}$ is the a checking value of β_{2T} , obtained at the checking value of μ_R with the aid of Table 1. Comparing Eqs. (15a) and (15b) with Eqs. (14a) and (14b), the expression of μ_R for non-normal random variables is in the same form as that for the normal random variables, except that $\beta_{2Tcheck}$ is used in (15b) instead of β_T in (14b). To predetermine the value of β_{2T} , a checking value of μ_R , named as μ_{Rcheck} , is given as

$$\mu_{Rcheck} = 2(V_R^2 + 1) \sum \mu_{Si} \quad (16)$$

As discussed above, the value of μ_R can be directly obtained in the proposed method by using Eqs. (15a), (15b), (16) and Table 1. Thus the calculation of load and resistance factors is straightforward and no iteration is required.

3.2. Procedure of the proposed method

The procedure of the proposed method is depicted in Fig. 1, and the procedure circled by bold dotted line is the first step of the proposed method to predetermine μ_R . As illustrated in Fig. 1, with μ_R predetermined, the iterations required in the existing moment methods are avoided in the proposed method. And since the step to predetermine μ_R includes only explicit expressions, the proposed method is easy to conduct in practical engineering.

4 Application in structural reliability analysis

4.1. Example 1

The first example considers the following performance function (ASCE 2010)

$$G(X) = R - (D + L + S) \quad (17)$$

where R , D , L and S are the resistance, dead load, live load and snow load, respectively. The distribution and the first four moments of the random variables are listed in Table 2.

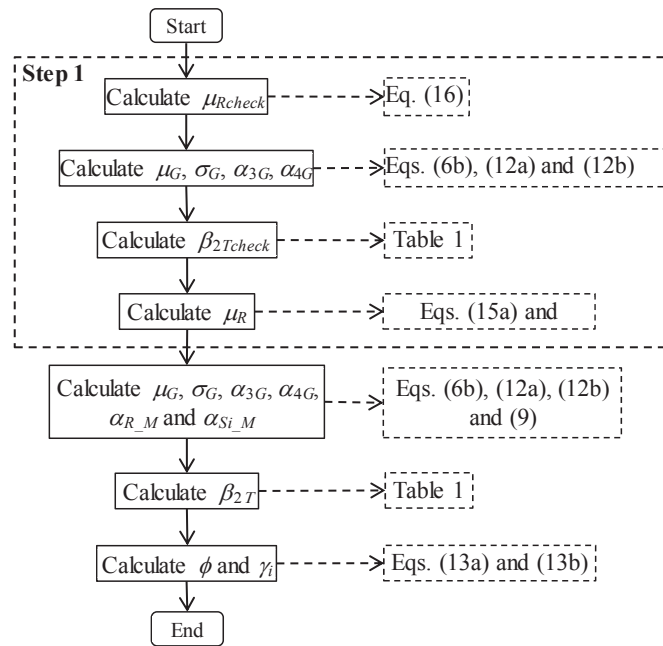


Fig. 1. Procedure of the proposed method

Table 2. Statistical information of the random variables in Eq. (17)

Variables	Distribution	μ_i/D_n	V_i	σ_i	α_{3i}	α_{4i}	μ_R/R_n or μ_{Si}/S_{ni}	S_{ni}/D_n
R	Lognormal	—	0.09	—	0.271	3.131	1.06	—
D	Normal	1	0.25	0.250	0	3	1.0	1
L	Gamma	0.175	0.59	0.103	1.180	5.089	0.35	0.5
S	Gumbel	0.6874	0.21	0.144	1.140	5.4	0.982	0.7

Note: D_n is the nominal value of D .

Table 3. Results of the proposed method by using different β_{2T} formulas

	Zhao 2000	Zhao 2001	Wang	Zhao 2007
$\beta_{2Tcheck}$	2.9202	2.9190	2.9292	2.9667
β_{2T}	2.9810	2.9810	2.9827	3.0299
μ_R	3.0665	3.0659	3.0710	3.0898
ϕ	0.8697	0.8697	0.8694	0.8657
γ_D	1.4519	1.4519	1.4518	1.4577
γ_L	0.5038	0.5038	0.5038	0.5058
γ_S	1.1967	1.1967	1.1967	1.1995

According to Fig. 1, four steps are conducted to obtain the resistance factor ϕ and partial load factor γ_i . Different formulas in Table 1 are used to calculate $\beta_{2Tcheck}$ and β_{2T} and the results are listed in Table 3.

As can be observed from Table 3, the biggest relative differences among $\beta_{2Tcheck}$ and β_{2T} obtained by different methods are relatively small (i.e., 1.6219%, between $\beta_{2Tcheck}$ obtained by Zhao 2007's formula and Zhao 2001's formula; 1.6283%, between β_{2T} obtained by Zhao 2007's formula and Zhao 2000's formula). This indicates there is no big difference in the accuracy of these expressions of $\beta_{2Tcheck}$ and β_{2T} obtained by different formulas listed in Table 1. To make the proposed method more practical, Zhao 2001's formula is applied in the following example.

4.1. Example 2

In order to examine the accuracy and efficiency of the proposed method, comparison studies among the proposed method, FORM and existing moment methods, is conducted in this section. Consider the following performance function:

$$G(X) = R - (D + L + S + W) \quad (18)$$

where R , D , L , S , W are the resistance, dead load, live load, snow load, and wind load, respectively. The distribution and the first four moments of the random variables are listed in Table 4.

Table 4. Statistical information of the random variables in Eq. (18)

Variables	Distribution	μ_i/D_n	V_i	σ_i	α_{3i}	α_{4i}	μ_R/R_n or μ_{Si}/S_{ni}
R	Lognormal	—	0.15	—	0.453	3.368	1.10
D	Normal	1	0.10	0.1	0	3	1
L	Lognormal	0.5	0.40	0.2	1.264	5.969	0.45
S	Gumbel	2.0	0.25	0.5	1.140	5.4	0.47
W	Gumbel	2.0	0.20	0.4	1.140	5.4	0.60

Since the distributions of the random variables are known, the FORM can be performed and the results of the FORM are used herein as the benchmark to evaluate the performance of different LRFD methods. By using the statistical moments of the random variables, existing moment methods and the proposed methods can be performed. The changes in the mean value of R (μ_R), the iteration numbers required, the resistance factor ϕ and partial load factor γ_i with the increase of β_T by using different methods, corresponding to a tolerable error ε of 0.03, are depicted in Figs. 2 (a-f), respectively, which reveal that:

- (i) As can be seen from Fig. 3 (a), the mean value of R (μ_R) obtained by the moment methods are slightly larger than that obtained by using FORM, this error may come from the approximation of β_{2T} in the moment methods. In practical engineering, larger μ_R can make the structure much safer, which indicates that the moment methods are much conservative compared with FORM. When β_{2T} is relative small, the results obtained by the proposed method and the existing moment methods are almost the same. When the β_{2T} becomes larger, the difference between the proposed method and the existing method becomes larger, while the μ_R obtained by the proposed method becomes closer with that obtained by the FORM, which indicates the proposed method is suitable to conduct LRFD for β_T changes from 1 to 3.
- (ii) As can be seen from Fig. 3 (b), the derivative-based iteration numbers of FORM for LRFD varied from 9 to 22 as β_T changed from 1.0 to 3.0. The iteration numbers required in the existing method reduce significantly compared with those in FORM for LRFD. However,

one may have to conduct iteration steps to satisfy the tolerable error ($\varepsilon = 0.03$), except for some particular cases. There is no need of iteration in the proposed method, and thus the proposed method is much simpler to be used in practice.

- (iii) As can be seen from Figs. 3 (c-f), the resistance factor ϕ and partial load factor γ_i ($i=1, \dots, 4$) obtained by the existing moment methods and those by using the proposed method coincide with each other, with the only exception of the resistance factor ϕ obtained by using Zhao 2007's formula. However, the difference in the resistance factor ϕ obtained by using the proposed and Zhao 2007's methods is not large. These again indicate the efficiency of the proposed method. The resistance factor ϕ and partial load factor γ_i ($i=1, \dots, 4$) obtained by the moment methods and FORM are different, this can be explained by that different combinations of load and resistances factors can result in the same design results. In design practice, if the resistance factor determined by a certain method is adopted, it is important that the corresponding load factors should be used.

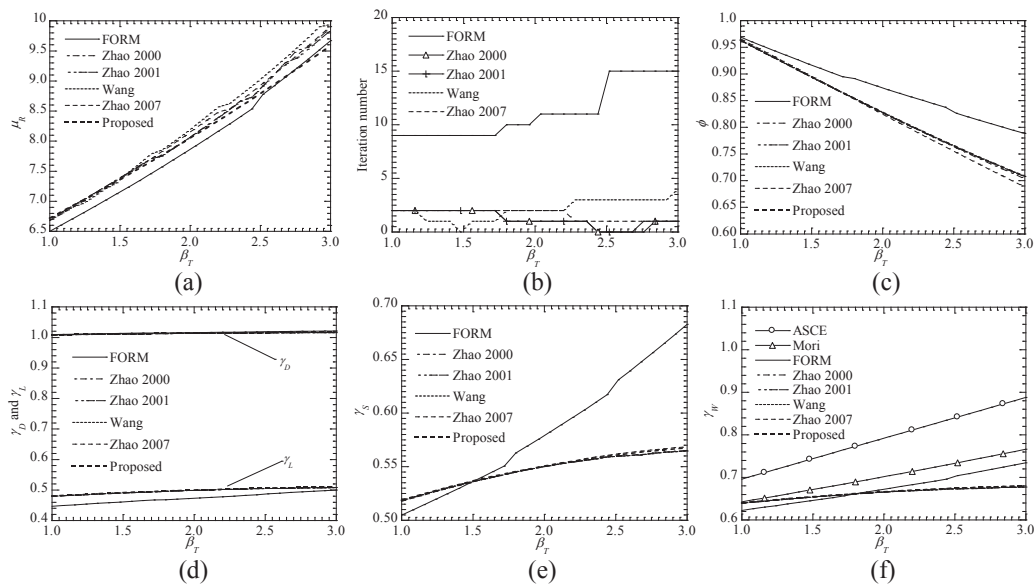


Fig. 2. Comparison of the LRFD by using different method: (a) mean value of R ; (b) iteration numbers; (c) resistance factor; (d) load factor of D and L ; (e) load factor of S ; (f) load factor of W

5 Conclusion

A straightforward moment method to estimate the load and resistance factors for reliability-based structural design without iterations is proposed. The procedure of the proposed method is summarized in flowchart. It is shown that the proposed method avoids iterations, which are needed by the existing moment methods and FORM. And thus the proposed method is much easier to apply. Comparison studies among the proposed method, the existing moment methods and the FORM are conducted and the results of the proposed method are similar with those of the FORM which indicates the accuracy of the proposed method.

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