

# Integrated Power Grid Expansion Planning

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In this paper, we seek to devise an integrated long term plan to increase the generation capability of power networks and gradually phase out environmentally unfriendly energy sources, under budget constraints. A mixed integer programming model is formulated with the objective of minimizing the environmental impact when expanding the grid and constraints pertaining to power flows, generator and line capacities and budget limitations. The resulting model is applied to an illustrative example showing clear benefits in clean integrated power grid expansion.

**Keywords:** Power grid expansion, mixed integer programming, environmental friendliness, generation expansion, transmission expansion.

## 1 Introduction

Long term power grid capacity expansion in an environmentally conscious way to match new load forecasts is one of the most challenging and critical problems faced by system planners in the electricity market nowadays. In this paper, we seek to devise an integrated long term plan to increase the generation capability of power networks and gradually phase out environmentally unfriendly energy sources, under budget constraints.

Today's power grids are complex systems comprising of three main components: generation, transmission, and distribution. Generation is the production of electricity (by facilities also known as power plants) from fuel sources such as coal, natural gas, solar power, and so on. Transmission is the movement of the electricity generated at high voltages over large distances to substations, whilst distribution is the process that carries electricity at lower voltages from substations to consumers. Over the years, the landscape of electricity markets around the world has shifted from a highly regulated monopoly, to a system of vertically integrated utilities, and finally to a deregulated system where the vertical integration has been unbundled to allow for expanded competitive opportunities in each of the grids' three main components.

Because of deregulation, research in optimization models for power grid capacity expansion has remained largely fragmented into generation expansion planning (GEP), transmission expansion planning (TEP), and to a lesser extent, distribution expansion planning (DEP). In GEP, the main concern is the investment in generating units to meet forecasted loads. It is usually a nonlinear discrete optimization problem which determines the size, place, technology, and the time of installing new plants. TEP has as a core concern line addition, line switching (determining which lines to use), and power dispatch. For a recent comprehensive review of the various optimization models used in GEP and TEP, see (Hemmati et al. 2013). DEP consists of determining the capacity, siting, and timing of installation of new distribution equipment.

Even in a deregulated environment, it is imperative to integrate GEP, TEP, and DEP for coordinated expansion. The choice of generating units is strongly influenced by the distance of these units from transmission substations and load centers (Sharan and Balasubramanian 2012).

This paper proposes an optimization model to integrate GEP and TEP, with environmental and budget considerations. The model is formulated as a mixed integer programming model is formulated with the objective of minimizing the environmental impact when expanding the grid and constraints pertaining to power flows, generator and line capacities and budget limitations.

## 2 Optimization Model

Consider a power grid with a set of nodes  $B$  and a set of edges  $V$ . The nodes are divided into a set of supply nodes  $S$ , a set of transmission nodes  $I$ , and a set of demand nodes  $D$ . The edges are divided into a set of existing lines  $E^L$  which are part of the original setup of the grid and a set of candidate lines  $N^L$  which can be used to make new connections. We build a multi-period model with the aim of planning four main operational decisions in each of  $T$  time periods: 1) How many of each generator type from a set  $K$  of generator types to set up at each supply node, 2) how much power to produce from each operating generator, 3) which power lines to use and to expand, and 4) how much power to send along each line. Letting  $[T]$  be the set of running indices from 1 to  $T$ , the problem formulation, together with its input data and decision variables are given as follows:

### Data

- $c_{ijt}^A$ : Cost of adding a new line on  $(i, j) \in N^L$  in time period  $t$
- $c_{ijt}^E$ : Cost of expanding the capacity of line  $(i, j) \in V$  in time period  $t$
- $c_{ikt}^G$ : Cost of adding a new generator of type  $k \in K$  at  $i \in S$  in time period  $t$
- $\gamma_k$ : Environmental cost per unit of power from a generator of type  $k \in K$
- $r_{ij}$ : Reactance of line  $(i, j) \in V$
- $M$ : A big number
- $d_{it}$ : Load at node  $i \in D$  in time period  $t$
- $n_{ik}$ : Number of existing generators of type  $k \in K$  at node  $i \in S$
- $\bar{v}_{ikt}$ : Nominal capacity of a type  $k \in K$  generator at node  $i \in S$  in time period  $t$
- $f_{ij}^A$ : Capacity of line  $(i, j) \in V$
- $f_{ij}^E$ : Capacity expansion amount for line  $(i, j) \in V$
- $\mu_t$ : Budget for time period  $t$
- $s_k$ : Space occupied by a generator of type  $k \in K$
- $b_i$ : Space available at node  $i \in S$

### Decision Variables

- $\theta_{it}$ : Angle at node  $i \in B$  in time period  $t$
- $p_{ijt}$ : Power along line  $(i, j) \in V$  in time period  $t$
- $g_{ikt}$ : Power generated by generator of type  $k \in K$  at node  $i \in S$  in time period  $t$

$y_{ijt}^A$ : Is 1 if a line is at  $(i, j) \in V$  in time period  $t$ , 0 otherwise

$y_{ijt}^E$ : Is 1 if line  $(i, j) \in V$  is expanded in time period  $t$ , 0 otherwise

$x_{ikt}^G$ : Number of generators of type  $k$  set up at node  $i \in S$  in time period  $t$ .

$$\min \sum_{i \in S} \sum_{k \in K} \sum_{t \in [1, T]} \gamma_k g_{ikt}$$

$$s.t. \quad |\theta_{it} - \theta_{jt} - r_{ij} p_{ijt}| \leq M[1 - \sum_{\tau \in [t]} y_{ij\tau}^A] \quad \forall (i, j) \in V, t \in [T] \quad (1)$$

$$\sum_{(i, j) \in \sigma_i^-} p_{ijt} - \sum_{(j, i) \in \sigma_i^+} p_{jit} = \sum_{k \in K} g_{ikt} \quad \forall i \in S, t \in [T] \quad (2)$$

$$\sum_{(j, i) \in \sigma_i^+} p_{jit} - \sum_{(i, j) \in \sigma_i^-} p_{ijt} = d_{it} \quad \forall i \in D, t \in [T] \quad (3)$$

$$\sum_{(j, i) \in \sigma_i^+} p_{jit} - \sum_{(i, j) \in \sigma_i^-} p_{ijt} = 0 \quad \forall i \in I, t \in [T] \quad (4)$$

$$g_{ikt} \leq \bar{v}_{ikt} [n_{ik} + \sum_{\tau \in [t]} x_{ik\tau}^G] \quad \forall i \in S, k \in K, t \in [T] \quad (5)$$

$$|p_{ijt}| \leq f_{ij}^A \sum_{\tau \in [1, t]} y_{ij\tau}^A + f_{ij}^E \sum_{\tau \in [1, t]} y_{ij\tau}^E \quad \forall (i, j) \in V, t \in [T] \quad (6)$$

$$\sum_{\tau \in [T]} [\sum_{(i, j) \in N^L} c_{ij\tau}^A y_{ij\tau}^A + \sum_{(i, j) \in V} c_{ij\tau}^E y_{ij\tau}^E + \sum_{i \in S} \sum_{k \in K} c_{ik\tau}^G x_{ik\tau}^G] \leq \sum_{\tau \in [t]} \mu_\tau \quad \forall t \in [T] \quad (7)$$

$$\sum_{k \in K} \sum_{t \in [T]} s_k x_{ikt}^G \leq b_i \quad \forall i \in S \quad (8)$$

$$y_{ijt}^E \leq y_{ijt}^A \quad \forall (i, j) \in V, t \in [T] \quad (9)$$

$$\sum_{t \in [T]} y_{ijt}^E \leq 1 \quad \forall (i, j) \in V \quad (10)$$

$$\sum_{t \in [T]} y_{ijt}^A \leq 1 \quad \forall (i, j) \in V \quad (11)$$

All  $y_{ijt}^A, y_{ijt}^E \in \{0, 1\}$ , all  $x_{ikt}^G \geq 0$ , integer, all  $p_{ijt} \in \circ$ , all  $g_{ikt} \geq 0$ .

The objective of the model is to minimize the total environmental cost of generated power. Constraint (1) is the DC flow constraint. Constraints (2), (3), and (4) are flow balance constraints, where  $\sigma_i^+$  is the set of lines entering node  $i \in B$  and  $\sigma_i^-$  is the set of lines leaving node  $i \in B$ . Flow balance constraints ensure that: 1) the net outflow at a supply node is the amount of power generated by operating generators at that node, 2) the net inflow at a demand point is equal to its demand, and 3) the power inflow at a transmission node equals the power outflow. Constraints (5) and (6) are generator supply capacity constraints and line capacity constraints, respectively. Constraint (7) imposes budgetary restrictions on the total investment for each time

period. Constraint (8) adds spatial restrictions on supply nodes. Constraint (9) ensures that a line can only be expanded if it has been set up. Constraints (10) and (11) make sure that a line cannot be added or expanded more than once.

### 3 Illustrative Example

We use the example in Figure 1 to show how the model works. In the example shown in Figure 1, there are 4 supply nodes, 4 transmission nodes, and 7 demand points. Initially, the grid only has a gas turbine with nominal capacity of 300 MW set up at node 1 and supplying nodes 11, 12, and 13 whose loads are 60 MW, 60 MW, and 80 MW, respectively. All power values are snapshot values.

Over the next 3 years, demand is expected to grow. In time period 1, new loads of 70 MW and 80 MW are expected at nodes 9 and 10, respectively. We expect a new load of 70 MW at node 14 in time period 2 and a new load of 60 MW at node 15 in time period 3. Suppose a central planner has budgets of \$100 million for period 1, \$150 million for period 2, and \$200 million for period 3 to invest in generation and transmission expansion to match the new loads and reduce carbon emissions.

Three types of generators are available to the planner: photovoltaic (PV) blocks, wind turbines, and gas turbines. Data on these generators are shown in Table 1. Investment cost and supply capacity data are based on the 2013 report by the U.S. Energy Information Administration (U.S. Energy Information Administration 2013). The social cost of carbon (SCC) is given a value of \$43 per tonne based on peer-reviewed estimates (Parry 2007). The carbon emission for each generator type is measured with the median global warming potential (GWP). The summary of the data for the median GWP from the Intergovernmental Panel on Climate Change is taken from (Wiki 2013). We assume that every supply node has the same limited available physical space for generator installation. A percentage of 50% of supply node space occupied by a gas generator, as shown in Table 1, means that if a gas generator is installed at a supply node, it would take up half of the available space at the node. There are 47 possible lines for the central planner to add or expand. The distances covered by each line are shown in Table 2. Lines are either high-voltage or low-voltage. Adding or expanding a high-voltage line (supply-to- transmission, transmission-to-transmission) costs \$80,000 per unit distance, whilst for a low-voltage line the cost is \$20,000 per unit distance. The capacity of a high-voltage line is 300 MW and that of a low-voltage line is 100 MW. All line reactances have a value of 1.

**Table 1: Data on generators for our illustrative example**

Generator	Investment Cost (\$ million)	% of Supply Node Space Occupied	Median GWP (gCo2eq/KWh)	Nominal Supply Capacity (MW)			
				Node 1	Node 2	Node 3	Node 4
PV	2	2.5	32	0.5	0.5	1	0.25
Wind	3	2	10	2	2	1	4
Gas	270	50	443	300	300	300	300

**Table 2: Line distances (measured in arbitrary units) for our illustrative example (Line a-b means the line that connects node a and node b)**

Line	1-5	1-6	1-7	1-8	2-5	2-6	2-7	2-8	3-5	3-6	3-7	3-8
Distance	10.5	8.0	13.0	18.5	14.0	8.5	9.5	13.0	18.0	12.0	9.0	9.5
Line	4-5	4-6	4-7	4-8	5-6	6-7	7-8	5-9	5-10	5-11	5-12	5-13
Distance	25.5	19.5	13.5	9.0	6.0	7.5	7.0	4.5	4.0	6.5	10.0	11.0
Line	5-14	5-15	6-9	6-10	6-11	6-12	6-13	6-14	6-15	7-9	7-10	7-11

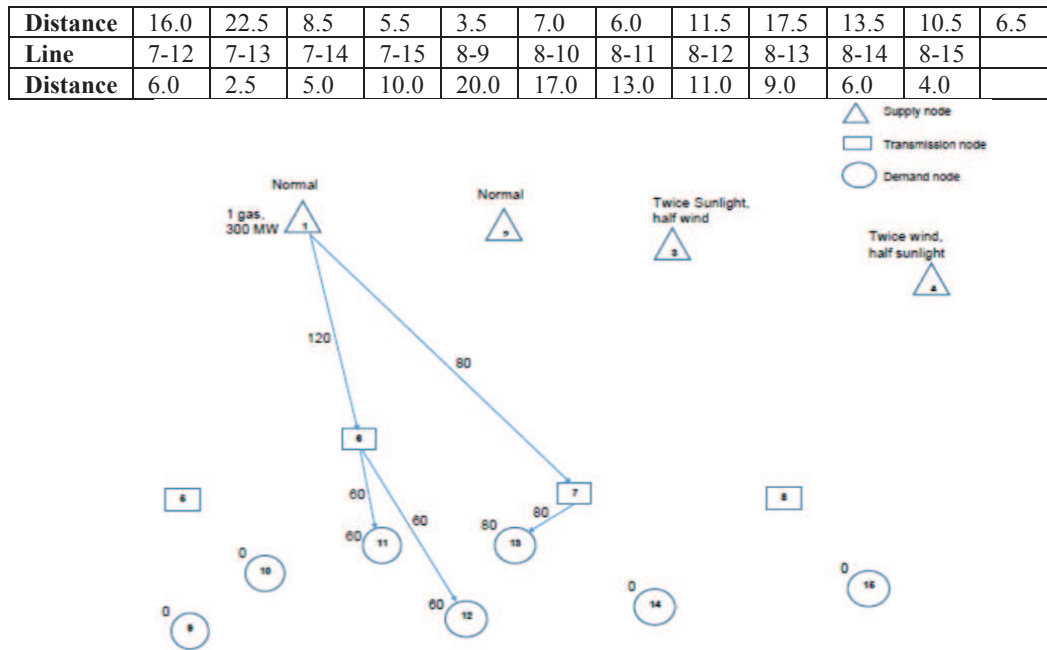


Figure 1: Initial state of power grid in our illustrative example

We code our mixed integer programming model in C++ and use Concert Technology to solve it with IBM-ILOG CPLEX 12.6 on a 2.4 GHz core i7-4500U processor with 12 GB of RAM. With the above-described data, it takes 21.0 seconds to solve the model. Table 3 shows the generation expansion plan, which consists of generator addition and power generation, and Figure 2 shows the transmission expansion plan, which consists of line setup and power dispatch.

From the expansion plans, we observe the gradual replacement of the environmentally unfriendly natural gas with cleaner sources (wind power and solar power) of energy under budget constraints. The final generation from natural gas in the third period is only 55 MW, much lower than the 200 MW generation in the initial configuration. In the transmission expansion plan in Figure 2, there are 3 numbers on every line. They denote the power flow along the line for periods 1, 2, and 3. Negative power represents power flow in a direction opposite that of the line. We observe several changes in power dispatch along lines in the transmission plan over time, together with the addition of new lines. For example, line 8-12 carries 48 MW in period 1 and 50 MW in period 2, but 10 MW in the opposite direction in period 3.

Table 3: Generation expansion plan for our illustrative example

Node	Generation Expansion Plan		
	Period 1	Period 2	Period 3
1	<ul style="list-style-type: none"> <li>• Increase gas power from 200 MW to 221 MW</li> </ul>	<ul style="list-style-type: none"> <li>• Install 32 wind turbines and generate 64 MW from wind power</li> <li>• Lower gas power to 155 MW</li> </ul>	<ul style="list-style-type: none"> <li>• Install 2 PV blocks and generate 1 MW from solar power</li> <li>• Install 15 wind turbines and generate 94 MW from wind power</li> <li>• Lower gas power to 55 MW</li> </ul>
2	No action	No action	<ul style="list-style-type: none"> <li>• Install 4 PV blocks and generate 2</li> </ul>



## 6 References

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