Proc. of the 6th Intl. Symposium on Reliability Engineering and Risk Management (6ISRERM)
31 May – 1 June 2018, Singapore
Editor(s) Xudong Qian, Sze Dai Pang, Ghim Ping Raymond Ong, Kok-Kwang Phoon

RELIABILITY ANALYSIS OF MOORING LINES USING SUBSET SIMULATION

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As exploration and production of hydrocarbon extend towards deep water reserves, mooring structures are increasingly exposed to harsher environments with greater uncertainties. Consequently, the interdependency among environmental loads is becoming an important consideration in design against mooring overload failures. In place of assuming design environments, methods of estimating long-term extreme responses aim to rationally account for failure probabilities across all possible sea states. However, successful industry implementation is impeded by its high computational demand. First, ultra-low industry target failure rates necessitate colossal amounts of sampling. Second, the incorporation of short-term stochastics in the long-term analysis requires integration of high dimensionality. Relative to the classic Monte Carlo approach, subset simulation offers a practical and robust alternative as an unbiased method of probabilistic evaluation. By dividing the reliability analysis into subsets of intermediate conditional probabilities, sampling requirements are greatly reduced for low probabilities whilst maintaining dimensional insensitivity as an integration technique. This paper explores its implementation on mooring reliability analysis, with its methodology illustrated on a mooring structure simulated in a site located in the Gulf of Mexico.

Keywords: Reliability, mooring, probability, environment, subset, Monte Carlo.

1 Introduction

As global energy demand increases, the demand for higher hydrocarbon extraction rates have up-surged over the past decades, accelerating the depletion of accessible land and shallow water reserves. Pushed by the imminence of diminishing supplies and pulled by an estimated abundance of uncovered resources, petroleum companies have become motivated toward explorations for future deep-water production fields. However, despite their massive potential, the industry's experience is minimal and the exploitation of deep-water reserves faces many challenges.

The structural integrity of moorings lines is an important design consideration of floating facilities. Mooring failures suffer severe consequences, such as off-station drifting, damage of risers and subsea equipment, production shut-down, and costly repairs. Unfortunately, the high occurrences of mooring failures have affected operational safety, with historical failure rates an order of magnitude greater than the industry target. Statistics suggest at least one mooring failure is to be expected for a unit deployed for a field life exceeding nine years. As the required operating envelope is being extended towards greater water depths, harsher environments, larger vessels to be moored, higher pressures and longer design lives, the number of uncertainties affecting mooring safety margins increases. Reliability-based design offers a rational account for uncertainties, presenting probabilities of structural failure for decision-making through risk matrices.

However, long lead times of sufficiently accurate dynamic analyses accumulates severe resource expenditure, rendering analyses at the low probability tail-end infeasible. The codified approach accordingly implements a semi-probabilistic alternative, where extreme value analyses are performed on design environments with metocean parameters of independently prescribed return periods as recommended by DNV (2014), otherwise known as the *design sea states* approach. Such practices not only fail to address the imperfect correlations of the environmental loads are not accounted for, but also incorrectly infers an n-year response from and n-year environmental event. Collinearity of all environmental vectors are often aligned against the lay of the mooring line, resulting in the overestimation of severity that do not account for directional differences of loads. Consequently, these shortcomings accumulate errors, with no means of assuring conservatism.

Long-term Extreme Response Analysis

The difficulty in applying probabilistic assessment methodologies on mooring structures is multi-fold. First, physical complexities arise from floating systems of numerous degrees of freedom exposed to highly dynamic loading regimes, necessitating costly structural analyses. In practice, the long-term time series of mooring responses are represented by a series of independent, stationary ergodic processes, over shorter discrete time intervals. Each stationary condition, hereby referred to as sea states, are distinctly defined by a set of metocean parameters, namely wave, wind and current vector components. In this time scale, responses are subject to short-term-stochastics due to random wave excitations despite deterministic wave spectra. Hence among independent runs of the same metocean parameters, the structure is exposed to distinct surface elevation time histories due to random phase lags and amplitudes of wave packages. Altogether, random variables of the analysis can be described by a high-dimensional vector X= [$\boldsymbol{X_l}$ $\boldsymbol{X_s}$]=[H_s T_z W Θ_w V Θ_v $\boldsymbol{A}$ $\boldsymbol{\theta}$], where $\boldsymbol{X_l}$ is the long-term vector containing the metocean parameters of significant wave height H_s, zero-upcrossing period T_z, wind speed W, wind direction Θ_w, current speed V and current direction Θ_v, and $\boldsymbol{X_s}$ embodies the short-term vector of wave package amplitudes $\boldsymbol{A}$ and phase lags $\boldsymbol{\theta}$. Evaluating a structure's extreme response requires calculating the probability of response exceedance across a given threshold, reducing to the expectation integral:

$$P(Y > y) = E\left[I_y(\boldsymbol{x})\right] = \int I_y(\boldsymbol{x}) f(\boldsymbol{x})\, d\boldsymbol{x} \tag{1}$$

Where $\boldsymbol{x}$=[x_1,...,x_n]$\in\boldsymbol{X}\subset\mathbb{R}^n$ is a vector of uncertain variables with a joint probability density function (PDF) f, I_y is an indicator function of the $\boldsymbol{X}$ domain, equating to unity where the response is greater than threshold y and null otherwise. The groundwork supporting this evaluation is the uncertainty modelling of the $\boldsymbol{X}$ domain, a distribution-fitting procedure against historical data to establish a joint probabilistic description $f(\boldsymbol{x})$ of influencing variables. Accordingly, structural response is iteratively evaluated across the variable space via the reliability algorithm for a probabilistic evaluation of the response.

2 Environmental Uncertainty Modelling

The joint environmental model in this work seeks to describe marginal distributions and partial correlations between various environmental variables, which would establish the groundwork towards assessing conservatism in traditional design methodologies. An existing joint environmental framework by Bitner-gregersen and Haver (1991) is referenced in this work. Measurement records, including significant wave height and period, wind speed and direction, and current profile, and available via the National Oceanic and Atmospheric Administration's public domain. Hereafter, the location id: 42041 would be applied as a geographical case study represented by the probabilistic environmental model.

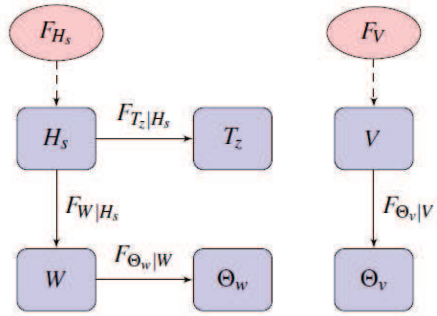

Figure 1. Conditional description of environmental variates.

The Conditional Modelling Approach (CMA) is applied to model the relationships between variable pairs (X, Y) via established idealizations of conditional distribution functions ($F_{X|Y}$). In addition to the CMA framework, wind and current directions are accounted for via established joint distributions of their vector components. By establishing the parameters of the probabilistic idealizations through data-fitting, the probabilistic descriptions of random environmental variates can be conditionally defined via the process sequence in Figure 1.

Significant Wave Height Distribution

Notorious for its severe hurricane conditions, Kwan (2005) notes that operating experience in the GoM has indicated structural overloading to be the driving causality in mooring system failures. The gulf's unique severity of storms conventional H_s distribution-fitting techniques recommended by DNV (2014) unsuitable for providing adequate representation of the data in the low probability domain. Instead, the 4-parameter distribution established by Ochi and Whalen (1980) is applied, with parameters selected via Least Squares.

Zero Upcrossing Period Distribution

The conditional density of T_z is established by DNV (2014) to observe a Log-normal distribution where the conditional mean and standard deviation of $\ln(T_z)$ observe empirical relationships against H_s. Parameters $\mu[\ln t\,|h] = a_0 + a_1 h^{a_2}$ and $\sigma[\ln t\,|h] = b_0 + b_1 \exp b_2 h$ are the mean and standard deviation of $\ln(T_z)$ at a given significant wave height, and sub-parameters a_j and b_j ($j=0,1,2$) are respectively obtained from regression power and exponential optimizations against historical data.

The direct approach of obtaining conditional samples of T_z on H_s is to gather an array of zero-upcrossing period t values in the scatter diagram corresponding to wave height h within an interval of $[h_i \pm \Delta h]$. However, the larger the width, the noisier the conditional data while selecting a narrow width would yield insufficient conditional samples. A Gaussian smoothing scheme applied by He and Low (2014) is hence adopted as rational treatment for conditional data, where a weighting function is implemented on each record of T_z to obtain conditional central moments.

Wind Speed Distribution

The marginal wind speed probability density is described by Bitner-gregersen and Haver (1991) to observe a 2-parameter Weibull distribution. Parameters $k = c_1 + c_2 h^{c_3}$ and $U_c = c_4 + c_5 h$ are distribution shape and scale parameters respectively, and sub-parameters c_j ($j=1,2,\ldots,5$) are obtained via the same weighted regression analysis applied for T_z, while simultaneously optimizing k and U_c against l^{th} ($l=1,2$) raw distribution moments:

$$m^l = U_c^l \cdot \Gamma\left(1 + \frac{l}{k}\right) = \frac{\sum w_j^l \psi_j}{\psi_j} \tag{2}$$

Where ψ_j is the weighting function. The weighted conditional moments and parameters are hence evaluated across a range of H_s before performing Least Squares optimization of the sub-parameters c_j.

Wind Direction Distribution

The anisotropic Gaussian model proposed by Weber (1991) is modified to describe the joint distribution of the Cartesian wind velocity components. Instead of aligning the component axes to the prevailing wind direction as proposed, the alignment of zero component correlation directions is applied to minimize correlation modelling errors. The probabilistic relationship between wind direction and speed is obtained from polar transforming the Cartesian joint distribution.

Current Direction Distribution

Due to significant kurtosis of current Cartesian components, the shifted generalized lognormal distribution (SGLD) proposed by Low (2013) is applied in place of the Gaussian distribution. Similar to that of the wind vector, the fitted Cartesian distribution is polar transformed to establish probabilistic relationship between direction and speed.

Current Speed Distribution

The long-term distribution of the normalized current velocity V has been established by Ashkenazy and Gildor (2011) to observe a 2-parameter Weibull distribution, with the scale and shape parameters optimized via Maximum Likelihood Estimation.

Current Profiling

The Power Law description of the two-dimensional current profile applied in Orcaflex will be fitted from temporal mean speeds across depth bins:

$$S(z) = S_b + \left(S_f - S_b\right)\left[\frac{z+d}{d}\right]^{1/Exponent} \tag{3}$$

Where $d=1316m$ is the water depth, z is the vertical distance from the still water level (negative downwards), the *Exponent* is the Power Law exponent, S_f and S_b are the current speeds at the surface and seabed respectively, optimized via Least Squares fitting. Currents amplitude and directional distributions are assumed to be governed by the flow measured at Depth Bin 2, the vertical sector with the highest current speed, with the resulting two dimensional profile modelled as $V_c(z) = V \cdot S(z)$, such that $V = V_{bin2}/E[V_{bin2}]$ is the amplitude multiplier of the current profile function, or otherwise described as the normalized speed to the temporal mean at Depth Bin 2.

3 Reliability Sampling

Monte Carlo Simulation

Monte Carlo Simulation (MCS) is a robust method of evaluating Eq. (1) by sampling the system's responses across the ***X*** domain. Naturally, simulating infrequent events require larger sample sizes, with massive required numbers of iterations on coupled time domain simulations amounting to unrealistic computational costs. Nonetheless, MCS is to provide a robust probabilistic evaluation of structural response across the 4 moorings down to moderate probability levels. A pseudo-random scatter of $N=10{,}000$ sea states, across the long-term environmental description established in the environmental model, followed by coupled time-domain dynamic evaluation of line tensile responses. The complementary cumulative distribution function of each mooring line's response is presented in Figure 2, where tensile severity is greater in Lines 2 and 3.

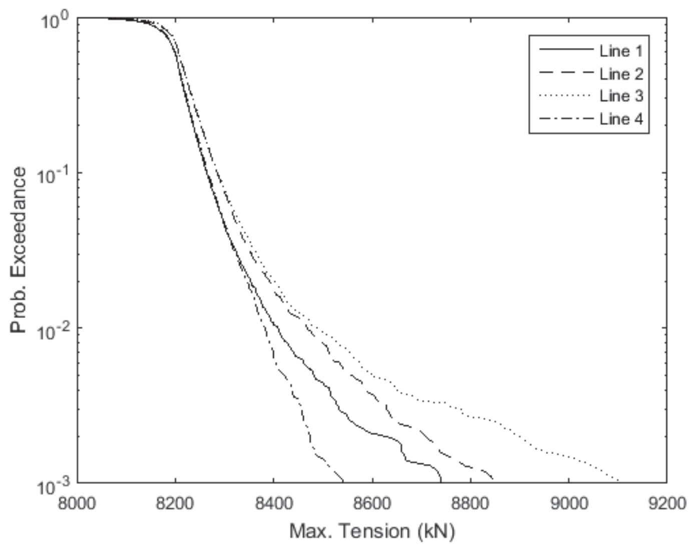

Figure 2. Conditional description of environmental variates.

Subset Simulation

Subset simulation, presented by Au and Wang (2014), is an efficient reliability method of dividing burdensome low probability problems into simpler ones of intermediate conditional probabilities. The exceedance probability P of response Y across a threshold of high severity level can be evaluated as a product of conditional probabilities of exceedance across intermediate levels:

$$P(Y > b_m) = P(Y > b_1) \cdot \prod_i^m P(Y > b_i | Y > b_{i-1}) \tag{4}$$

Where b_m is the highest threshold to be evaluated at Level m, such that $b_1<b_2<...<b_m$. All product terms are computed via a separate sets of Monte Carlo simulations, referred as a 'subsets'. The first subset is executed via the conventional MCS where samples are pseudo-randomly drawn from the established joint-distribution across the probability space. A convenient high probability $P(Y>b_1)$ of low estimation variance is selected while its corresponding response recorded as the first level threshold b_1. Subsequent subsets evaluate conditional probabilities of responses above the preceding level threshold b_{i-1}. Efficiency superiority over direct MCS becomes apparent as the evaluated probability declines exponentially while the number of evaluations increases only linearly.

In this experiment responses of return periods up to 1000 years are evaluated using 41,000 coupled time-domain simulations. The subset algorithm's advantage of evaluating low target probabilities becomes apparent in the level specifications presented in Table 1. From the ten subset levels the 10, 100 and 1000 year return period probabilities bear reasonable coefficients of variation of 11.1, 12.4 and 13.4% respectively. The response CCDF (Figure 3) is evaluated at lower probabilities, with an 'uplift' of the tail-end is observed beyond the probability of 10^{-3} owing to the unique severity of GoM hurricanes.

As a measure of maximizing sampled information, failure samples from the preceding level are included in the conditional sample set of the current subset. In this work, at every level increment the marginal computational expenditure remains a constant of 4000 samples while the evaluated cumulative probabilities decay by a factor of 0.2. Hence, the implementation of subset levels beyond the conventional Monte Carlo enables the component reliability analysis of a mooring line to access industrial target probabilities, addressing feasibility issues elaborated in Section 1.

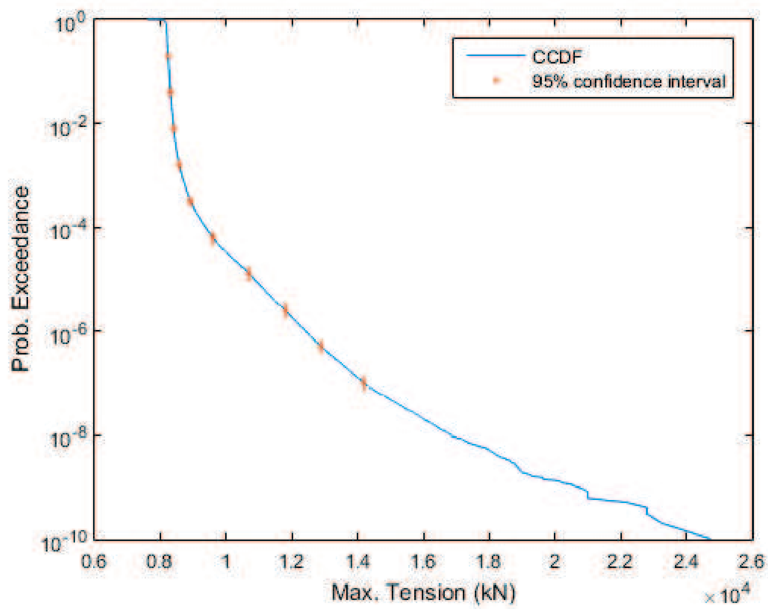

Figure 3. Subset simulation CCDF of critical Line 3.

Table 1. Subset level results of critical Line 3.

Level	Level sample size	Conditional Probability	Cummulative Probability	Cummulative COV	Return Period (years)	Tension (kN)
1	5000	0.200	$2\text{x}10^{-1}$	0.028	-	8239
2	5000	0.200	$4\text{x}10^{-2}$	0.048	-	8306
3	5000	0.200	$8\text{x}10^{-3}$	0.065	-	8403
4	5000	0.200	$1.60\text{x}10^{-3}$	0.079	-	8566
5	4995	0.200	$3.20\text{x}10^{-4}$	0.091	1	8916
6	4995	0.199	$6.35\text{x}10^{-5}$	0.100	1	9606
7	4960	0.202	$1.28\text{x}10^{-5}$	0.108	5	10695
8	5000	0.199	$2.54\text{x}10^{-6}$	0.116	23	11808
9	4965	0.201	$5.11\text{x}10^{-7}$	0.125	112	12883
10	4995	0.200	$1.02\text{x}10^{-7}$	0.132	559	14181

4 Conclusion

With a growing shift from deterministic means to probabilistic approaches in offshore structural design, advanced techniques employed in this work grant access to probabilistic evaluations on black swan events of mooring loads, otherwise regarded computationally absurd by conventional means. The environmental modelling methodology presented delivers a comprehensive long term description of the production site's characteristics, accounting for directional variability between surface and subsurface loads. By representing joint behavior between metocean variates with a series of conditional distributions, the probabilistic space can be conveniently transformed into standard normal space, allowing the approach to be an ideal uncertainty model for a wide array of reliability algorithms. One such application is subset simulation, where mooring overload reliability is assessed with low error margins, even at extremely low industrial target failure rates. With a means of unbiased probabilistic evaluation within reasonable lead times, the presented methodology is a suitable candidate for probabilistic verification against approximate reliability methods for long-term extreme response analysis, enabling disruptive strides towards offshore reliability research.

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